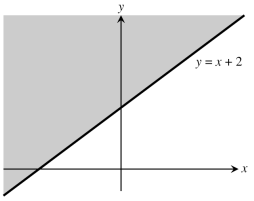
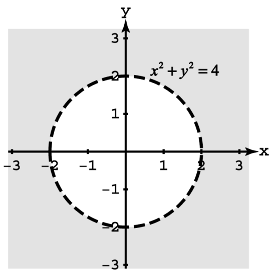
MATH 201. Homework 8 Solution.

Section 14.1:

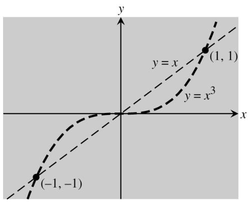
5. Domain: all points (x, y) on or above the line $y = x + 2$



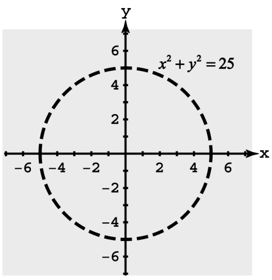
6. Domain: all points (x, y) outside the circle $x^2 + y^2 = 4$



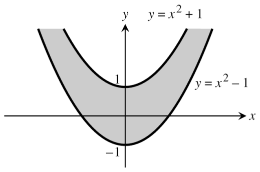
7. Domain: all points (x, y) not liying on the graph of $y = x$ or $y = x^3$



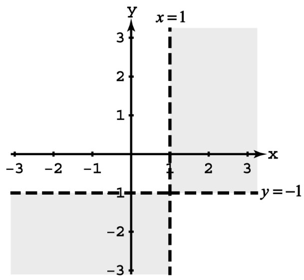
8. Domain: all points (x, y) not liying on the graph of $x^2 + y^2 = 25$



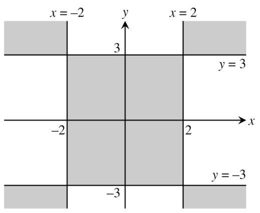
9. Domain: all points (x, y) satisfying
$x^2 - 1 \leq y \leq x^2 + 1$



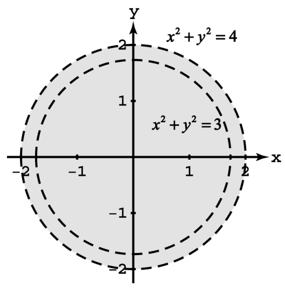
10. Domain: all points (x, y) satisfying
$(x - 1)(y + 1) > 0$



11. Domain: all points (x, y) satisfying
$(x - 2)(x + 2)(y - 3)(y + 3) \geq 0$



12. Domain: all points (x, y) inside the circle
$x^2 + y^2 = 4$ such that $x^2 + y^2 \neq 3$



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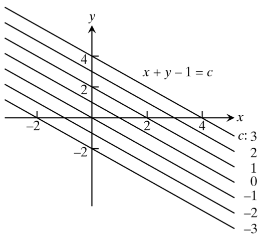
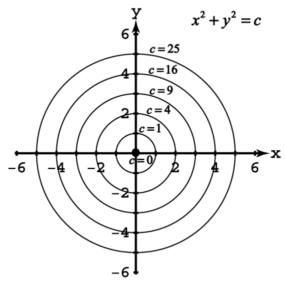


14.



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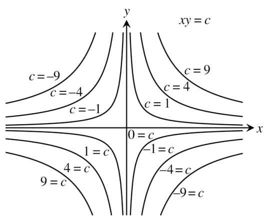

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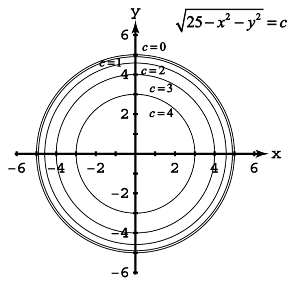


18. (a) Domain: set of all (x, y) so that $y - x \geq 0 \Rightarrow y \geq x$
    (b) Range: $z \geq 0$
    (c) level curves are straight lines of the form $y - x = c$ where $c \geq 0$
    (d) boundary is $\sqrt{y - x} = 0 \Rightarrow y = x$, a straight line
    (e) closed
    (f) unbounded

19. (a) Domain: all points in the xy-plane
    (b) Range: $z \geq 0$
    (c) level curves: for $f(x, y) = 0$, the origin; for $f(x, y) = c > 0$, ellipses with center $(0, 0)$ and major and minor axes along the x- and y-axes, respectively
    (d) no boundary points
    (e) both open and closed
    (f) unbounded

20. (a) Domain: all points in the xy-plane
    (b) Range: all real numbers
    (c) level curves: for $f(x, y) = 0$, the union of the lines $y = \pm x$; for $f(x, y) = c \neq 0$, hyperbolas centered at $(0, 0)$ with foci on the x-axis if $c > 0$ and on the y-axis if $c < 0$
    (d) no boundary points
    (e) both open and closed
    (f) unbounded

22. (a) Domain: all $(x, y) \neq (0, y)$
    (b) Range: all real numbers
    (c) level curves: for $f(x, y) = 0$, the x-axis minus the origin; for $f(x, y) = c \neq 0$, the parabolas $y = c\,x^2$ minus the origin
    (d) boundary is the line $x = 0$
    (e) open
    (f) unbounded

23. (a) Domain: all (x, y) satisfying $x^2 + y^2 < 16$
    (b) Range: $z \geq \frac{1}{4}$
    (c) level curves are circles centered at the origin with radii $r < 4$
    (d) boundary is the circle $x^2 + y^2 = 16$

(e) open
(f) bounded

24. (a) Domain: all (x, y) satisfying $x^2 + y^2 \leq 9$
(b) Range: $0 \leq z \leq 3$
(c) level curves are circles centered at the origin with radii $r \leq 3$
(d) boundary is the circle $x^2 + y^2 = 9$
(e) closed
(f) bounded

25. (a) Domain: $(x, y) \neq (0, 0)$
(b) Range: all real numbers
(c) level curves are circles with center $(0, 0)$ and radii $r > 0$
(d) boundary is the single point $(0, 0)$
(e) open
(f) unbounded

26. (a) Domain: all points in the xy-plane
(b) Range: $0 < z \leq 1$
(c) level curves are the origin itself and the circles with center $(0, 0)$ and radii $r > 0$
(d) no boundary points
(e) both open and closed
(f) unbounded

49. $f(x, y) = 16 - x^2 - y^2$ and $\left(2\sqrt{2}, \sqrt{2}\right) \Rightarrow z = 16 - \left(2\sqrt{2}\right)^2 - \left(\sqrt{2}\right)^2 = 6 \Rightarrow 6 = 16 - x^2 - y^2 \Rightarrow x^2 + y^2 = 10$

50. $f(x, y) = \sqrt{x^2 - 1}$ and $(1, 0) \Rightarrow z = \sqrt{1^2 - 1} = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = 1$ or $x = -1$

51. $f(x, y) = \sqrt{x + y^2 - 3}$ and $(3, -1) \Rightarrow z = \sqrt{3 + (-1)^2 - 3} = 1 \Rightarrow x + y^2 - 3 = 1 \Rightarrow x + y^2 = 4$

52. $f(x, y) = \frac{2y - x}{x + y + 1}$ and $(-1, 1) \Rightarrow z = \frac{2(1) - (-1)}{(-1) + 1 + 1} = 3 \Rightarrow 3 = \frac{2y - x}{x + y + 1} \Rightarrow y = -4x - 3$

Section 14.2:

13. $\displaystyle\lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{x^2-2xy+y^2}{x-y} = \lim_{(x,y)\to(1,1)} \frac{(x-y)^2}{x-y} = \lim_{(x,y)\to(1,1)} (x-y) = (1-1) = 0$

14. $\displaystyle\lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{x^2-y^2}{x-y} = \lim_{(x,y)\to(1,1)} \frac{(x+y)(x-y)}{x-y} = \lim_{(x,y)\to(1,1)} (x+y) = (1 + 1) = 2$

15. $\displaystyle\lim_{\substack{(x,y)\to(1,1)\\ x\neq 1}} \frac{xy-y-2x+2}{x-1} = \lim_{\substack{(x,y)\to(1,1)\\ x\neq 1}} \frac{(x-1)(y-2)}{x-1} = \lim_{(x,y)\to(1,1)} (y-2) = (1-2) = -1$

16. $\displaystyle\lim_{\substack{(x,y)\to(2,-4)\\ y\neq -4,\, x\neq x^2}} \frac{y+4}{x^2y-xy+4x^2-4x} = \lim_{\substack{(x,y)\to(2,-4)\\ y\neq -4,\, x\neq x^2}} \frac{y+4}{x(x-1)(y+4)} = \lim_{\substack{(x,y)\to(2,-4)\\ x\neq x^2}} \frac{1}{x(x-1)} = \frac{1}{2(2-1)} = \frac{1}{2}$

17. $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ x\neq y}} \frac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \lim_{\substack{(x,y)\to(0,0)\\ x\neq y}} \frac{\left(\sqrt{x}-\sqrt{y}\right)\left(\sqrt{x}+\sqrt{y}+2\right)}{\sqrt{x}-\sqrt{y}} = \lim_{(x,y)\to(0,0)} \left(\sqrt{x}+\sqrt{y}+2\right)$

$\displaystyle= \left(\sqrt{0}+\sqrt{0}+2\right) = 2$

Note: (x, y) must approach $(0, 0)$ through the first quadrant only with $x \neq y$.

18. $\displaystyle\lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2} = \lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{\left(\sqrt{x+y}+2\right)\left(\sqrt{x+y}-2\right)}{\sqrt{x+y}-2} = \lim_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \left(\sqrt{x+y}+2\right)$

$\displaystyle= \left(\sqrt{2+2}+2\right) = 2+2 = 4$

19. $\displaystyle\lim_{\substack{(x,y)\to(2,0)\\ 2x-y\neq 4}} \frac{\sqrt{2x-y}-2}{2x-y-4} = \lim_{\substack{(x,y)\to(2,0)\\ 2x-y\neq 4}} \frac{\sqrt{2x-y}-2}{\left(\sqrt{2x-y}+2\right)\left(\sqrt{2x-y}-2\right)} = \lim_{(x,y)\to(2,0)} \frac{1}{\sqrt{2x-y}+2}$

$\displaystyle= \frac{1}{\sqrt{(2)(2)-0}+2} = \frac{1}{2+2} = \frac{1}{4}$

20. $\displaystyle\lim_{\substack{(x,y)\to(4,3)\\ x-y\neq 1}} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1} = \lim_{\substack{(x,y)\to(4,3)\\ x-y\neq 1}} \frac{\sqrt{x}-\sqrt{y+1}}{\left(\sqrt{x}+\sqrt{y+1}\right)\left(\sqrt{x}-\sqrt{y+1}\right)} = \lim_{(x,y)\to(4,3)} \frac{1}{\sqrt{x}+\sqrt{y+1}}$

$\displaystyle= \frac{1}{\sqrt{4}+\sqrt{3+1}} = \frac{1}{2+2} = \frac{1}{4}$

21. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = \lim_{r\to 0} \frac{\sin(r^2)}{r^2} = \lim_{r\to 0} \frac{2r\cdot\cos(r^2)}{2r} = \lim_{r\to 0} \cos(r^2) = 1$

22. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1-\cos(xy)}{xy} = \lim_{u\to 0} \frac{1-\cos u}{u} = \lim_{u\to 0} \frac{\sin u}{1} = 0$

23. $\displaystyle\lim_{(x,y)\to(1,-1)} \frac{x^3+y^3}{x+y} = \lim_{(x,y)\to(1,-1)} \frac{(x+y)(x^2-xy+y^2)}{x+y} = \lim_{(x,y)\to(1,-1)} (x^2-xy+y^2) = \left(1^2-(1)(-1)+(-1)^2\right) = 3$

24. $\displaystyle\lim_{(x,y)\to(2,2)} \frac{x-y}{x^4-y^4} = \lim_{(x,y)\to(2,2)} \frac{x-y}{(x+y)(x-y)(x^2+y^2)} = \lim_{(x,y)\to(2,2)} \frac{1}{(x+y)(x^2+y^2)} = \frac{1}{(2+2)(2^2+2^2)} = \frac{1}{32}$

60. $|xy\,(x^2-y^2)| = |xy|\,|x^2-y^2| \le |x|\,|y|\,|x^2+y^2| = \sqrt{x^2}\,\sqrt{y^2}\,|x^2+y^2| \le \sqrt{x^2+y^2}\,\sqrt{x^2+y^2}\,|x^2+y^2|$
$= (x^2+y^2)^2 \Rightarrow \left|\frac{xy\,(x^2-y^2)}{x^2+y^2}\right| \le \frac{(x^2+y^2)^2}{x^2+y^2} = x^2+y^2 \Rightarrow -(x^2+y^2) \le \frac{xy\,(x^2-y^2)}{x^2+y^2} \le (x^2+y^2)$
$\Rightarrow \lim_{(x,y)\to(0,0)} \left(xy\,\frac{x^2-y^2}{x^2+y^2}\right) = 0$ by the Sandwich Theorem, since $\lim_{(x,y)\to(0,0)} \pm(x^2+y^2) = 0$; thus, define $f(0,0) = 0$

61. $\lim_{(x,y)\to(0,0)} \frac{x^3-xy^2}{x^2+y^2} = \lim_{r\to 0} \frac{r^3\cos^3\theta - (r\cos\theta)(r^2\sin^2\theta)}{r^2\cos^2\theta + r^2\sin^2\theta} = \lim_{r\to 0} \frac{r(\cos^3\theta - \cos\theta\sin^2\theta)}{1} = 0$

62. $\lim_{(x,y)\to(0,0)} \cos\left(\frac{x^3-y^3}{x^2+y^2}\right) = \lim_{r\to 0} \cos\left(\frac{r^3\cos^3\theta - r^3\sin^3\theta}{r^2\cos^2\theta + r^2\sin^2\theta}\right) = \lim_{r\to 0} \cos\left[\frac{r(\cos^3\theta - \sin^3\theta)}{1}\right] = \cos 0 = 1$

63. $\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2} = \lim_{r\to 0} \frac{r^2\sin^2\theta}{r^2} = \lim_{r\to 0} (\sin^2\theta) = \sin^2\theta$; the limit does not exist since $\sin^2\theta$ is between 0 and 1 depending on θ

64. $\lim_{(x,y)\to(0,0)} \frac{2x}{x^2+x+y^2} = \lim_{r\to 0} \frac{2r\cos\theta}{r^2+r\cos\theta} = \lim_{r\to 0} \frac{2\cos\theta}{r+\cos\theta} = \frac{2\cos\theta}{\cos\theta}$; the limit does not exist for $\cos\theta = 0$

65. $\lim_{(x,y)\to(0,0)} \tan^{-1}\left[\frac{|x|+|y|}{x^2+y^2}\right] = \lim_{r\to 0} \tan^{-1}\left[\frac{|r\cos\theta| + |r\sin\theta|}{r^2}\right] = \lim_{r\to 0} \tan^{-1}\left[\frac{|r|\,(|\cos\theta| + |\sin\theta|)}{r^2}\right]$;
if $r \to 0^+$, then $\lim_{r\to 0^+} \tan^{-1}\left[\frac{|r|\,(|\cos\theta| + |\sin\theta|)}{r^2}\right] = \lim_{r\to 0^+} \tan^{-1}\left[\frac{|\cos\theta| + |\sin\theta|}{r}\right] = \frac{\pi}{2}$; if $r \to 0^-$, then
$\lim_{r\to 0^-} \tan^{-1}\left[\frac{|r|\,(|\cos\theta| + |\sin\theta|)}{r^2}\right] = \lim_{r\to 0^-} \tan^{-1}\left(\frac{|\cos\theta| + |\sin\theta|}{-r}\right) = \frac{\pi}{2} \Rightarrow$ the limit is $\frac{\pi}{2}$

66. $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} = \lim_{r\to 0} \frac{r^2\cos^2\theta - r^2\sin^2\theta}{r^2} = \lim_{r\to 0} (\cos^2\theta - \sin^2\theta) = \lim_{r\to 0} (\cos 2\theta)$ which ranges between -1 and 1 depending on $\theta \Rightarrow$ the limit does not exist

Section 14.3:

1. $\frac{\partial f}{\partial x} = 4x,\ \frac{\partial f}{\partial y} = -3$

2. $\frac{\partial f}{\partial x} = 2x - y,\ \frac{\partial f}{\partial y} = -x + 2y$

3. $\frac{\partial f}{\partial x} = 2x(y+2),\ \frac{\partial f}{\partial y} = x^2 - 1$

4. $\frac{\partial f}{\partial x} = 5y - 14x + 3,\ \frac{\partial f}{\partial y} = 5x - 2y - 6$

5. $\frac{\partial f}{\partial x} = 2y(xy-1),\ \frac{\partial f}{\partial y} = 2x(xy-1)$

6. $\frac{\partial f}{\partial x} = 6(2x-3y)^2,\ \frac{\partial f}{\partial y} = -9(2x-3y)^2$

7. $\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2+y^2}},\ \frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$

8. $\frac{\partial f}{\partial x} = \frac{2x^2}{\sqrt[3]{x^3+\left(\frac{y}{2}\right)}},\ \frac{\partial f}{\partial y} = \frac{1}{3\sqrt[3]{x^3+\left(\frac{y}{2}\right)}}$

9. $\frac{\partial f}{\partial x} = -\frac{1}{(x+y)^2} \cdot \frac{\partial}{\partial x}(x+y) = -\frac{1}{(x+y)^2},\ \frac{\partial f}{\partial y} = -\frac{1}{(x+y)^2} \cdot \frac{\partial}{\partial y}(x+y) = -\frac{1}{(x+y)^2}$

10. $\frac{\partial f}{\partial x} = \frac{(x^2+y^2)(1) - x(2x)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2},\ \frac{\partial f}{\partial y} = \frac{(x^2+y^2)(0) - x(2y)}{(x^2+y^2)^2} = -\frac{2xy}{(x^2+y^2)^2}$

11. $\frac{\partial f}{\partial x} = = \frac{(xy-1)(1) - (x+y)(y)}{(xy-1)^2} = \frac{-y^2-1}{(xy-1)^2},\ \frac{\partial f}{\partial y} = \frac{(xy-1)(1) - (x+y)(x)}{(xy-1)^2} = \frac{-x^2-1}{(xy-1)^2}$

12. $\frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{\partial}{\partial x}\left(\frac{y}{x}\right) = -\frac{y}{x^2\left[1+\left(\frac{y}{x}\right)^2\right]} = -\frac{y}{x^2+y^2}$, $\frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{\partial}{\partial y}\left(\frac{y}{x}\right) = \frac{1}{x\left[1+\left(\frac{y}{x}\right)^2\right]} = \frac{x}{x^2+y^2}$

13. $\frac{\partial f}{\partial x} = e^{(x+y+1)} \cdot \frac{\partial}{\partial x}(x+y+1) = e^{(x+y+1)}$, $\frac{\partial f}{\partial y} = e^{(x+y+1)} \cdot \frac{\partial}{\partial y}(x+y+1) = e^{(x+y+1)}$

14. $\frac{\partial f}{\partial x} = -e^{-x}\sin(x+y) + e^{-x}\cos(x+y)$, $\frac{\partial f}{\partial y} = e^{-x}\cos(x+y)$

15. $\frac{\partial f}{\partial x} = \frac{1}{x+y} \cdot \frac{\partial}{\partial x}(x+y) = \frac{1}{x+y}$, $\frac{\partial f}{\partial y} = \frac{1}{x+y} \cdot \frac{\partial}{\partial y}(x+y) = \frac{1}{x+y}$

16. $\frac{\partial f}{\partial x} = e^{xy} \cdot \frac{\partial}{\partial x}(xy) \cdot \ln y = ye^{xy}\ln y$, $\frac{\partial f}{\partial y} = e^{xy} \cdot \frac{\partial}{\partial y}(xy) \cdot \ln y + e^{xy} \cdot \frac{1}{y} = xe^{xy}\ln y + \frac{e^{xy}}{y}$

17. $\frac{\partial f}{\partial x} = 2\sin(x-3y) \cdot \frac{\partial}{\partial x}\sin(x-3y) = 2\sin(x-3y)\cos(x-3y) \cdot \frac{\partial}{\partial x}(x-3y) = 2\sin(x-3y)\cos(x-3y)$,
$\frac{\partial f}{\partial y} = 2\sin(x-3y) \cdot \frac{\partial}{\partial y}\sin(x-3y) = 2\sin(x-3y)\cos(x-3y) \cdot \frac{\partial}{\partial y}(x-3y) = -6\sin(x-3y)\cos(x-3y)$

18. $\frac{\partial f}{\partial x} = 2\cos(3x-y^2) \cdot \frac{\partial}{\partial x}\cos(3x-y^2) = -2\cos(3x-y^2)\sin(3x-y^2) \cdot \frac{\partial}{\partial x}(3x-y^2)$
$= -6\cos(3x-y^2)\sin(3x-y^2)$,
$\frac{\partial f}{\partial y} = 2\cos(3x-y^2) \cdot \frac{\partial}{\partial y}\cos(3x-y^2) = -2\cos(3x-y^2)\sin(3x-y^2) \cdot \frac{\partial}{\partial y}(3x-y^2)$
$= 4y\cos(3x-y^2)\sin(3x-y^2)$

19. $\frac{\partial f}{\partial x} = yx^{y-1}$, $\frac{\partial f}{\partial y} = x^y \ln x$

20. $f(x,y) = \frac{\ln x}{\ln y} \Rightarrow \frac{\partial f}{\partial x} = \frac{1}{x\ln y}$ and $\frac{\partial f}{\partial y} = \frac{-\ln x}{y(\ln y)^2}$

41. $\frac{\partial f}{\partial x} = 1 + y,\ \frac{\partial f}{\partial y} = 1 + x,\ \frac{\partial^2 f}{\partial x^2} = 0,\ \frac{\partial^2 f}{\partial y^2} = 0,\ \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 1$

42. $\frac{\partial f}{\partial x} = y \cos xy,\ \frac{\partial f}{\partial y} = x \cos xy,\ \frac{\partial^2 f}{\partial x^2} = -y^2 \sin xy,\ \frac{\partial^2 f}{\partial y^2} = -x^2 \sin xy,\ \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = \cos xy - xy \sin xy$

43. $\frac{\partial g}{\partial x} = 2xy + y \cos x,\ \frac{\partial g}{\partial y} = x^2 - \sin y + \sin x,\ \frac{\partial^2 g}{\partial x^2} = 2y - y \sin x,\ \frac{\partial^2 g}{\partial y^2} = -\cos y,\ \frac{\partial^2 g}{\partial y \partial x} = \frac{\partial^2 g}{\partial x \partial y} = 2x + \cos x$

44. $\frac{\partial h}{\partial x} = e^y,\ \frac{\partial h}{\partial y} = xe^y + 1,\ \frac{\partial^2 h}{\partial x^2} = 0,\ \frac{\partial^2 h}{\partial y^2} = xe^y,\ \frac{\partial^2 h}{\partial y \partial x} = \frac{\partial^2 h}{\partial x \partial y} = e^y$

45. $\frac{\partial r}{\partial x} = \frac{1}{x+y},\ \frac{\partial r}{\partial y} = \frac{1}{x+y},\ \frac{\partial^2 r}{\partial x^2} = \frac{-1}{(x+y)^2},\ \frac{\partial^2 r}{\partial y^2} = \frac{-1}{(x+y)^2},\ \frac{\partial^2 r}{\partial y \partial x} = \frac{\partial^2 r}{\partial x \partial y} = \frac{-1}{(x+y)^2}$

46. $\frac{\partial s}{\partial x} = \left[\frac{1}{1+\left(\frac{y}{x}\right)^2}\right] \cdot \frac{\partial}{\partial x}\left(\frac{y}{x}\right) = \left(-\frac{y}{x^2}\right)\left[\frac{1}{1+\left(\frac{y}{x}\right)^2}\right] = \frac{-y}{x^2+y^2},\ \frac{\partial s}{\partial y} = \left[\frac{1}{1+\left(\frac{y}{x}\right)^2}\right] \cdot \frac{\partial}{\partial y}\left(\frac{y}{x}\right) = \left(\frac{1}{x}\right)\left[\frac{1}{1+\left(\frac{y}{x}\right)^2}\right] = \frac{x}{x^2+y^2},$

$\frac{\partial^2 s}{\partial x^2} = \frac{y(2x)}{(x^2+y^2)^2} = \frac{2xy}{(x^2+y^2)^2},\ \frac{\partial^2 s}{\partial y^2} = \frac{-x(2y)}{(x^2+y^2)^2} = -\frac{2xy}{(x^2+y^2)^2},\ \frac{\partial^2 s}{\partial y \partial x} = \frac{\partial^2 s}{\partial x \partial y} = \frac{(x^2+y^2)(-1)+y(2y)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$

47. $\frac{\partial w}{\partial x} = 2x\tan(xy) + x^2\sec^2(xy) \cdot y = 2x\tan(xy) + x^2 y \sec^2(xy),\ \frac{\partial w}{\partial y} = x^2 \sec^2(xy) \cdot x = x^3 \sec^2(xy),$

$\frac{\partial^2 w}{\partial x^2} = 2\tan(xy) + 2x\sec^2(xy) \cdot y + 2xy\sec^2(xy) + x^2 y\,(2\sec(xy)\sec(xy)\tan(xy) \cdot y)$

$= 2\tan(xy) + 4xy\sec^2(xy) + 2x^2y^2\sec^2(xy)\tan(xy),\ \frac{\partial^2 w}{\partial y^2} = x^3(2\sec(xy)\sec(xy)\tan(xy) \cdot x) = 2x^4\sec^2(xy)\tan(xy)$

$\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y} = 3x^2\sec^2(xy) + x^3(2\sec(xy)\sec(xy)\tan(xy) \cdot y) = 3x^2\sec^2(xy) + x^3 y\sec^2(xy)\tan(xy)$

48. $\frac{\partial w}{\partial x} = ye^{x^2-y} \cdot 2x = 2xy\,e^{x^2-y},\ \frac{\partial w}{\partial y} = (1)e^{x^2-y} + ye^{x^2-y} \cdot (-1) = e^{x^2-y}(1-y),$

$\frac{\partial^2 w}{\partial x^2} = 2y\,e^{x^2-y} + 2xy\left(e^{x^2-y} \cdot 2x\right) = 2ye^{x^2-y}(1+2x^2),\ \frac{\partial^2 w}{\partial y^2} = \left(e^{x^2-y} \cdot (-1)\right)(1-y) + e^{x^2-y}(-1)$

$= e^{x^2-y}(y-2),\ \frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y} = \left(e^{x^2-y} \cdot 2x\right)(1-y) = 2x\,e^{x^2-y}(1-y)$

49. $\frac{\partial w}{\partial x} = \sin(x^2y) + x\cos(x^2y) \cdot 2xy = \sin(x^2y) + 2x^2y\cos(x^2y),\ \frac{\partial w}{\partial y} = x\cos(x^2y) \cdot x^2 = x^3\cos(x^2y),$

$\frac{\partial^2 w}{\partial x^2} = \cos(x^2y) \cdot 2xy + 4xy\cos(x^2y) - 2x^2y\sin(x^2y) \cdot 2xy = 6xy\cos(x^2y) - 4x^3y^2\sin(x^2y),$

$\frac{\partial^2 w}{\partial y^2} = -x^3\sin(x^2y) \cdot x^2 = -x^5\sin(x^2y),\ \frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y} = 3x^2\cos(x^2y) - x^3\sin(x^2y) \cdot 2xy = 3x^2\cos(x^2y) - 2x^4y\sin(x^2y)$

50. $\frac{\partial w}{\partial x} = \frac{(x^2+y) - (x-y)(2x)}{(x^2+y)^2} = \frac{-x^2+2xy+y}{(x^2+y)^2},\ \frac{\partial w}{\partial y} = \frac{(x^2+y)(-1) - (x-y)}{(x^2+y)^2} = \frac{-x^2-x}{(x^2+y)^2},$

$\frac{\partial^2 w}{\partial x^2} = \frac{(x^2+y)^2(-2x+2y) - (-x^2+2xy+y)2(x^2+y)(2x)}{\left[(x^2+y)^2\right]^2} = \frac{2(x^3-3x^2y-3xy+y^2)}{(x^2+y)^3},$

$\frac{\partial^2 w}{\partial y^2} = \frac{(x^2+y)^2 \cdot 0 - (-x^2-x)2(x^2+y) \cdot 1}{\left[(x^2+y)^2\right]^2} = \frac{2x^2+2x}{(x^2+y)^3},\ \frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y} = \frac{(x^2+y)^2(2x+1) - (-x^2+2xy+y)2(x^2+y) \cdot 1}{\left[(x^2+y)^2\right]^2}$

$= \frac{2x^3+3x^2-2xy-y}{(x^2+y)^3}$

51. $\frac{\partial w}{\partial x} = \frac{2}{2x+3y},\ \frac{\partial w}{\partial y} = \frac{3}{2x+3y},\ \frac{\partial^2 w}{\partial y \partial x} = \frac{-6}{(2x+3y)^2}$, and $\frac{\partial^2 w}{\partial x \partial y} = \frac{-6}{(2x+3y)^2}$

52. $\frac{\partial w}{\partial x} = e^x + \ln y + \frac{y}{x},\ \frac{\partial w}{\partial y} = \frac{x}{y} + \ln x,\ \frac{\partial^2 w}{\partial y \partial x} = \ = \frac{1}{y} + \frac{1}{x}$, and $\frac{\partial^2 w}{\partial x \partial y} = \frac{1}{y} + \frac{1}{x}$

53. $\frac{\partial w}{\partial x} = y^2 + 2xy^3 + 3x^2y^4,\ \frac{\partial w}{\partial y} = 2xy + 3x^2y^2 + 4x^3y^3,\ \frac{\partial^2 w}{\partial y \partial x} = 2y + 6xy^2 + 12x^2y^3$, and $\frac{\partial^2 w}{\partial x \partial y} = 2y + 6xy^2 + 12x^2y^3$

54. $\frac{\partial w}{\partial x} = \sin y + y\cos x + y,\ \frac{\partial w}{\partial y} = x\cos y + \sin x + x,\ \frac{\partial^2 w}{\partial y \partial x} = \cos y + \cos x + 1$, and $\frac{\partial^2 w}{\partial x \partial y} = \cos y + \cos x + 1$

Section 15.1:

7. $\int_0^1\int_0^1 \frac{y}{1+xy}\,dx\,dy = \int_0^1 [\ln|1+xy|]_0^1\,dy = \int_0^1 \ln|1+y|dy = [y\ln|1+y| - y + \ln|1+y|]_0^1 = 2\ln 2 - 1$

8. $\int_1^4\int_0^4 \left(\frac{x}{2}+\sqrt{y}\right)dx\,dy = \int_1^4 \left[\frac{1}{4}x^2 + x\sqrt{y}\right]_0^4 dy = \int_1^4 (4+4y^{1/2})dy = \left[4y + \frac{8}{3}y^{3/2}\right]_1^4 = \frac{92}{3}$

9. $\int_0^{\ln 2}\int_1^{\ln 5} e^{2x+y}\,dy\,dx = \int_0^{\ln 2} [e^{2x+y}]_1^{\ln 5}\,dx = \int_0^{\ln 2} (5e^{2x} - e^{2x+1})\,dx = \left[\frac{5}{2}e^{2x} - \frac{1}{2}e^{2x+1}\right]_0^{\ln 2} = \frac{3}{2}(5-e)$

10. $\int_0^1\int_1^2 x\,y\,e^x\,dy\,dx = \int_0^1 \left[\frac{1}{2}x\,y^2e^x\right]_1^2 dx = \int_0^1 \frac{3}{2}x\,e^x\,dx = \left[\frac{3}{2}x\,e^x - \frac{3}{2}e^x\right]_0^1 = \frac{3}{2}$

11. $\int_{-1}^2\int_0^{\pi/2} y\sin x\,dx\,dy = \int_{-1}^2 [-y\cos x]_0^{\pi/2}\,dy = \int_{-1}^2 y\,dy = \left[\frac{1}{2}y^2\right]_{-1}^2 = \frac{3}{2}$

12. $\int_\pi^{2\pi}\int_0^\pi (\sin x + \cos y)\,dx\,dy = \int_\pi^{2\pi} [-\cos x + x\cos y]_0^\pi\,dy = \int_\pi^{2\pi} (2+\pi\cos y)\,dy = [2y + \pi\sin y]_\pi^{2\pi} = 2\pi$

14. $\iint_R \frac{\sqrt{x}}{y^2}dA = \int_0^4\int_1^2 \frac{\sqrt{x}}{y^2}\,dy\,dx = \int_0^4 \left[-\frac{\sqrt{x}}{y}\right]_1^2 dx = \int_0^4 \frac{1}{2}x^{1/2}\,dx = \left[\frac{1}{3}x^{3/2}\right]_0^4 = \frac{8}{3}$

15. $\iint_R x\,y\cos y\,dA = \int_{-1}^1\int_0^\pi x\,y\cos y\,dy\,dx = \int_{-1}^1 [x\,y\sin y + x\cos y]_0^\pi\,dx = \int_{-1}^1 (-2x)\,dx = [-x^2]_{-1}^1 = 0$

16. $\iint_R y\sin(x+y)\,dA = \int_{-\pi}^0\int_0^\pi y\sin(x+y)\,dy\,dx = \int_{-\pi}^0 [-y\cos(x+y) + \sin(x+y)]_0^\pi\,dx$
$= \int_{-\pi}^0 (\sin(x+\pi) - \pi\cos(x+\pi) - \sin x)dx = [-\cos(x+\pi) - \pi\sin(x+\pi) + \cos x]_{-\pi}^0 = 4$

17. $\iint_R e^{x-y}dA = \int_0^{\ln 2}\int_0^{\ln 2} e^{x-y}\,dy\,dx = \int_0^{\ln 2} [-e^{x-y}]_0^{\ln 2}\,dx = \int_0^{\ln 2} (-e^{x-\ln 2} + e^x)\,dx = [-e^{x-\ln 2} + e^x]_0^{\ln 2} = \frac{1}{2}$

18. $\iint_R x\,y\,e^{xy^2}dA = \int_0^2\int_0^1 x\,y\,e^{xy^2}\,dy\,dx = \int_0^2 \left[\frac{1}{2}e^{xy^2}\right]_0^1 dx = \int_0^2 \left(\frac{1}{2}e^x - \frac{1}{2}\right)dx = \left[\frac{1}{2}e^x - \frac{1}{2}x\right]_0^2 = \frac{1}{2}(e^2-3)$

19. $\iint_R \frac{xy^3}{x^2+1}dA = \int_0^1\int_0^2 \frac{xy^3}{x^2+1}\,dy\,dx = \int_0^1 \left[\frac{xy^4}{4(x^2+1)}\right]_0^2 dx = \int_0^1 \frac{4x}{x^2+1}\,dx = [2\ln|x^2+1|]_0^1 = 2\ln 2$

20. $\iint_R \frac{y}{x^2y^2+1}dA = \int_0^1\int_0^1 \frac{y}{(xy)^2+1}\,dx\,dy = \int_0^1 [\tan^{-1}(xy)]_0^1\,dy = \int_0^1 \tan^{-1}y\,dy = \left[y\tan^{-1}y - \frac{1}{2}\ln|1+y^2|\right]_0^1 = \frac{\pi}{4} - \frac{1}{2}\ln 2$

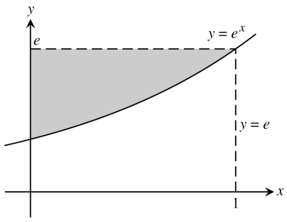
25. $V = \iint_R f(x,y)\,dA = \int_0^1\int_0^1 (2-x-y)\,dy\,dx = \int_0^1 \left[2y - xy - \frac{1}{2}y^2\right]_0^1 dx = \int_0^1 \left(\frac{3}{2} - x\right)dx = \left[\frac{3}{2}x - \frac{1}{2}x^2\right]_0^1 = 1$

26. $V = \iint_R f(x,y)\,dA = \int_0^4\int_0^2 \frac{y}{2}\,dy\,dx = \int_0^4 \left[\frac{y^2}{4}\right]_0^2 dx = \int_0^4 1\,dx = [x]_0^4 = 4$

27. $V = \iint_R f(x,y)\,dA = \int_0^{\pi/2}\int_0^{\pi/4} 2\sin x\cos y\,dy\,dx = \int_0^{\pi/2} [2\sin x\sin y]_0^{\pi/4}dx = \int_0^{\pi/2} \left(\sqrt{2}\sin x\right)dx = \left[-\sqrt{2}\cos x\right]_0^{\pi/2}$
$= \sqrt{2}$

Section 15.2:

5.



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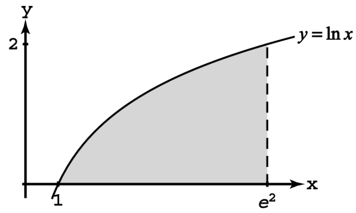

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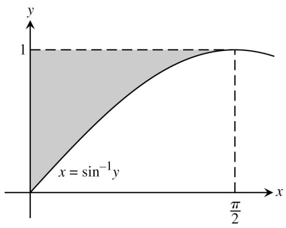

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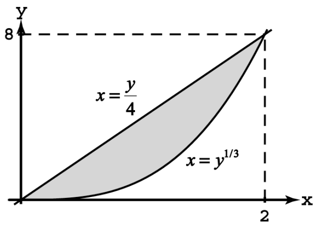
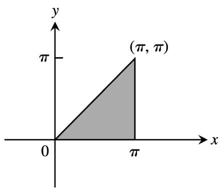


19. $\int_0^{\pi}\int_0^{x}(x \sin y)\, dy\, dx = \int_0^{\pi}[-x \cos y]_0^x\, dx$

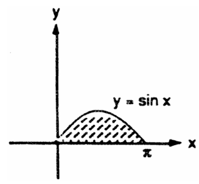
$= \int_0^{\pi}(x - x\cos x)\, dx = \left[\frac{x^2}{2} - (\cos x + x \sin x)\right]_0^{\pi}$

$= \frac{\pi^2}{2} + 2$



20. $\int_0^{\pi}\int_0^{\sin x} y\,dy\,dx = \int_0^{\pi}\left[\frac{y^2}{2}\right]_0^{\sin x} dx = \int_0^{\pi}\frac{1}{2}\sin^2 x\,dx$

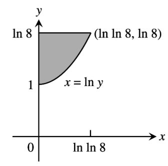
$= \frac{1}{4}\int_0^{\pi}(1-\cos 2x)\,dx = \frac{1}{4}\left[x-\frac{1}{2}\sin 2x\right]_0^{\pi} = \frac{\pi}{4}$



21. $\int_1^{\ln 8}\int_0^{\ln y} e^{x+y}\,dx\,dy = \int_1^{\ln 8}\left[e^{x+y}\right]_0^{\ln y} dy = \int_1^{\ln 8}(ye^y - e^y)\,dy$

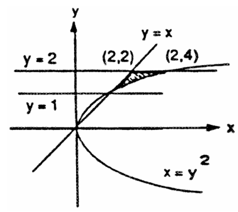
$= \left[(y-1)e^y - e^y\right]_1^{\ln 8} = 8(\ln 8 - 1) - 8 + e$

$= 8\ln 8 - 16 + e$



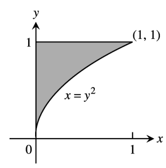
22. $\int_1^2\int_y^{y^2} dx\,dy = \int_1^2 (y^2 - y)\,dy = \left[\frac{y^3}{3}-\frac{y^2}{2}\right]_1^2$

$= \left(\frac{8}{3}-2\right) - \left(\frac{1}{3}-\frac{1}{2}\right) = \frac{7}{3}-\frac{3}{2} = \frac{5}{6}$



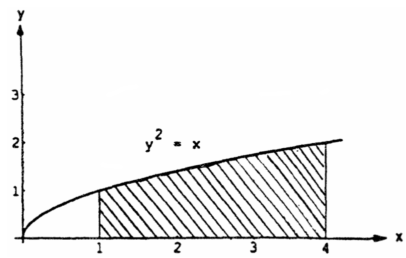
23. $\int_0^1\int_0^{y^2} 3y^3 e^{xy}\,dx\,dy = \int_0^1\left[3y^2e^{xy}\right]_0^{y^2} dy$

$= \int_0^1\left(3y^2e^{y^3} - 3y^2\right)dy = \left[e^{y^3} - y^3\right]_0^1 = e - 2$



24. $\int_1^4\int_0^{\sqrt{x}} \frac{3}{2}e^{y/\sqrt{x}}\,dy\,dx = \int_1^4\left[\frac{3}{2}\sqrt{x}\,e^{y/\sqrt{x}}\right]_0^{\sqrt{x}} dx$

$= \frac{3}{2}(e-1)\int_1^4\sqrt{x}\,dx = \left[\frac{3}{2}(e-1)\left(\frac{2}{3}\right)x^{3/2}\right]_1^4 = 7(e-1)$



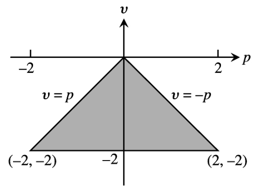
26. $\int_0^1\int_0^{1-x}(x^2+y^2)\,dy\,dx = \int_0^1\left[x^2y+\frac{y^3}{3}\right]_0^{1-x} dx = \int_0^1\left[x^2(1-x)+\frac{(1-x)^3}{3}\right]dx = \int_0^1\left[x^2 - x^3 + \frac{(1-x)^3}{3}\right]dx$

$= \left[\frac{x^3}{3}-\frac{x^4}{4}-\frac{(1-x)^4}{12}\right]_0^1 = \left(\frac{1}{3}-\frac{1}{4}-0\right)-\left(0-0-\frac{1}{12}\right) = \frac{1}{6}$

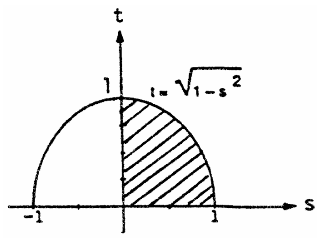
27. $\int_0^1\int_0^{1-u}\left(v-\sqrt{u}\right)dv\,du = \int_0^1\left[\frac{v^2}{2}-v\sqrt{u}\right]_0^{1-u} du = \int_0^1\left[\frac{1-2u+u^2}{2}-\sqrt{u}(1-u)\right]du$

$= \int_0^1\left(\frac{1}{2}-u+\frac{u^2}{2}-u^{1/2}+u^{3/2}\right)du = \left[\frac{u}{2}-\frac{u^2}{2}+\frac{u^3}{6}-\frac{2}{3}u^{3/2}+\frac{2}{5}u^{5/2}\right]_0^1 = \frac{1}{2}-\frac{1}{2}+\frac{1}{6}-\frac{2}{3}+\frac{2}{5} = -\frac{1}{2}+\frac{2}{5} = -\frac{1}{10}$

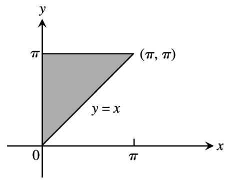
29. $\int_{-2}^{0}\int_{v}^{-v} 2 \, dp \, dv = 2\int_{-2}^{0} [p]_{v}^{-v} \, dv = 2\int_{-2}^{0} -2v \, dv$
$= -2 \left[v^2\right]_{-2}^{0} = 8$



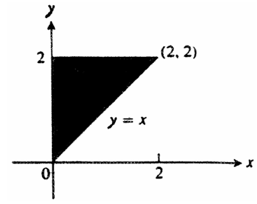
30. $\int_{0}^{1}\int_{0}^{\sqrt{1-s^2}} 8t \, dt \, ds = \int_{0}^{1} [4t^2]_{0}^{\sqrt{1-s^2}} \, ds$
$= \int_{0}^{1} 4\left(1 - s^2\right) ds = 4\left[s - \frac{s^3}{3}\right]_{0}^{1} = \frac{8}{3}$



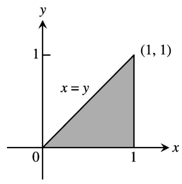
47. $\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\,dy\,dx = \int_0^{\pi}\int_0^{y} \frac{\sin y}{y}\,dx\,dy = \int_0^{\pi} \sin y\,dy = 2$



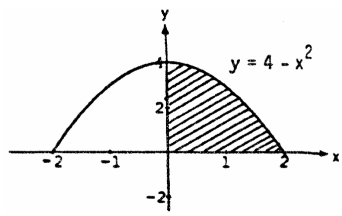
48. $\int_0^{2}\int_x^{2} 2y^2 \sin xy\,dy\,dx = \int_0^{2}\int_0^{y} 2y^2 \sin xy\,dx\,dy$
$= \int_0^{2} \left[-2y \cos xy\right]_0^y\,dy = \int_0^{2} \left(-2y \cos y^2 + 2y\right) dy$
$= \left[-\sin y^2 + y^2\right]_0^2 = 4 - \sin 4$



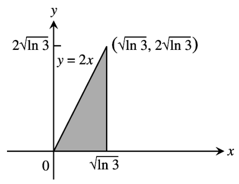
49. $\int_0^{1}\int_y^{1} x^2 e^{xy}\,dx\,dy = \int_0^{1}\int_0^{x} x^2 e^{xy}\,dy\,dx = \int_0^{1} \left[x e^{xy}\right]_0^x\,dx$
$= \int_0^{1} \left(x e^{x^2} - x\right) dx = \left[\frac{1}{2} e^{x^2} - \frac{x^2}{2}\right]_0^1 = \frac{e-2}{2}$



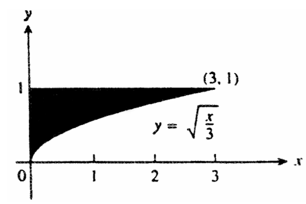
50. $\int_0^{2}\int_0^{4-x^2} \frac{x e^{2y}}{4-y}\,dy\,dx = \int_0^{4}\int_0^{\sqrt{4-y}} \frac{x e^{2y}}{4-y}\,dx\,dy$
$= \int_0^{4} \left[\frac{x^2 e^{2y}}{2(4-y)}\right]_0^{\sqrt{4-y}} dy = \int_0^{4} \frac{e^{2y}}{2}\,dy = \left[\frac{e^{2y}}{4}\right]_0^4 = \frac{e^8 - 1}{4}$



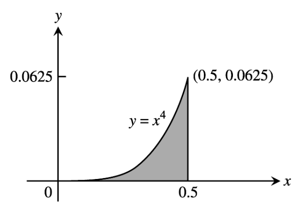
51. $\int_0^{2\sqrt{\ln 3}}\int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\,dx\,dy = \int_0^{\sqrt{\ln 3}}\int_0^{2x} e^{x^2}\,dy\,dx$
$= \int_0^{\sqrt{\ln 3}} 2x e^{x^2}\,dx = \left[e^{x^2}\right]_0^{\sqrt{\ln 3}} = e^{\ln 3} - 1 = 2$



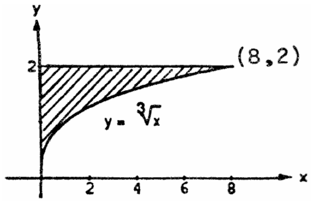
52. $\int_0^{3}\int_{\sqrt{x/3}}^{1} e^{y^3}\,dy\,dx = \int_0^{1}\int_0^{3y^2} e^{y^3}\,dx\,dy$
$= \int_0^{1} 3y^2 e^{y^3}\,dy = \left[e^{y^3}\right]_0^1 = e - 1$



53. $\int_0^{1/16}\int_{y^{1/4}}^{1/2} \cos\left(16\pi x^5\right) dx\,dy = \int_0^{1/2}\int_0^{x^4} \cos\left(16\pi x^5\right) dy\,dx$
$= \int_0^{1/2} x^4 \cos\left(16\pi x^5\right) dx = \left[\frac{\sin\left(16\pi x^5\right)}{80\pi}\right]_0^{1/2} = \frac{1}{80\pi}$



54. $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{y^4+1}\, dy\, dx = \int_0^2 \int_0^{y^3} \frac{1}{y^4+1}\, dx\, dy$
$= \int_0^2 \frac{y^3}{y^4+1}\, dy = \frac{1}{4}\left[\ln\left(y^4+1\right)\right]_0^2 = \frac{\ln 17}{4}$



57. $V = \int_0^1 \int_x^{2-x} \left(x^2+y^2\right) dy\, dx = \int_0^1 \left[x^2 y + \frac{y^3}{3}\right]_x^{2-x} dx = \int_0^1 \left[2x^2 - \frac{7x^3}{3} + \frac{(2-x)^3}{3}\right] dx = \left[\frac{2x^3}{3} - \frac{7x^4}{12} - \frac{(2-x)^4}{12}\right]_0^1$
$= \left(\frac{2}{3} - \frac{7}{12} - \frac{1}{12}\right) - \left(0 - 0 - \frac{16}{12}\right) = \frac{4}{3}$

58. $V = \int_{-2}^1 \int_x^{2-x^2} x^2\, dy\, dx = \int_{-2}^1 \left[x^2 y\right]_x^{2-x^2} dx = \int_{-2}^1 \left(2x^2 - x^4 - x^3\right) dx = \left[\frac{2}{3}x^3 - \frac{1}{5}x^5 - \frac{1}{4}x^4\right]_{-2}^1$
$= \left(\frac{2}{3} - \frac{1}{5} - \frac{1}{4}\right) - \left(-\frac{16}{3} + \frac{32}{5} - \frac{16}{4}\right) = \left(\frac{40}{60} - \frac{12}{60} - \frac{15}{60}\right) - \left(-\frac{320}{60} + \frac{384}{60} - \frac{240}{60}\right) = \frac{189}{60} = \frac{63}{20}$

59. $V = \int_{-4}^1 \int_{3x}^{4-x^2} (x+4)\, dy\, dx = \int_{-4}^1 \left[xy + 4y\right]_{3x}^{4-x^2} dx = \int_{-4}^1 \left[x\left(4-x^2\right) + 4\left(4-x^2\right) - 3x^2 - 12x\right] dx$
$= \int_{-4}^1 \left(-x^3 - 7x^2 - 8x + 16\right) dx = \left[-\frac{1}{4}x^4 - \frac{7}{3}x^3 - 4x^2 + 16x\right]_{-4}^1 = \left(-\frac{1}{4} - \frac{7}{3} + 12\right) - \left(\frac{64}{3} - 64\right) = \frac{157}{3} - \frac{1}{4} = \frac{625}{12}$

60. $V = \int_0^2 \int_0^{\sqrt{4-x^2}} (3-y)\, dy\, dx = \int_0^2 \left[3y - \frac{y^2}{2}\right]_0^{\sqrt{4-x^2}} dx = \int_0^2 \left[3\sqrt{4-x^2} - \left(\frac{4-x^2}{2}\right)\right] dx$
$= \left[\frac{3}{2}x\sqrt{4-x^2} + 6\sin^{-1}\left(\frac{x}{2}\right) - 2x + \frac{x^3}{6}\right]_0^2 = 6\left(\frac{\pi}{2}\right) - 4 + \frac{8}{6} = 3\pi - \frac{16}{6} = \frac{9\pi-8}{3}$