$$f(x,y) = \begin{cases} \dfrac{x^2 - y^2}{x+y} & (x,y) \neq (0,0) \\ \alpha & (x,y) = (0,0). \end{cases}$$

Find the value of $\underline{\underline{\alpha}}$ for which f is cont.
(if possible)

<u>Strategy</u>: $\lim_{(x,y)\to(0,0)} \left(\dfrac{x^2 - y^2}{x+y} \right)$ if exists,

make α equal to it !!

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x+y} = \left[\frac{0}{0}\right] = \lim \left(\frac{(x-y)\cancel{(x+y)}}{\cancel{(x+y)}}\right)$$

$$= \lim_{(x,y)\to(0,0)} (x-y) = 0$$

$$\therefore \lim_{(x,y)\to(0,0)} f(x,y) = 0$$

$d = 0$

Cond : Make $d = 0$

This will establish continuity

From Sample

$$\boxed{\int_0^1 \int_y^{\sqrt{2y-y^2}} x\,dx\,dy} = \int_0^1 \left.\frac{x^2}{2}\right|_{x=y}^{x=\sqrt{2y-y^2}} dy.$$

R

$$= \int_0^1 \left(\frac{2y-y^2}{2} - \frac{y^2}{2}\right) dy = \int y - y^2\, dy.$$

$$= \frac{y^2}{2} - \frac{y^3}{3}\Big|_0^1 = \frac{1}{2} - \frac{1}{3} = 1/6$$

R

$x = y$

$x \geq 0$ $\boxed{x = \sqrt{2y - y^2}}$

$0 < y < 1$

$$x = \sqrt{2y - y^2}.$$

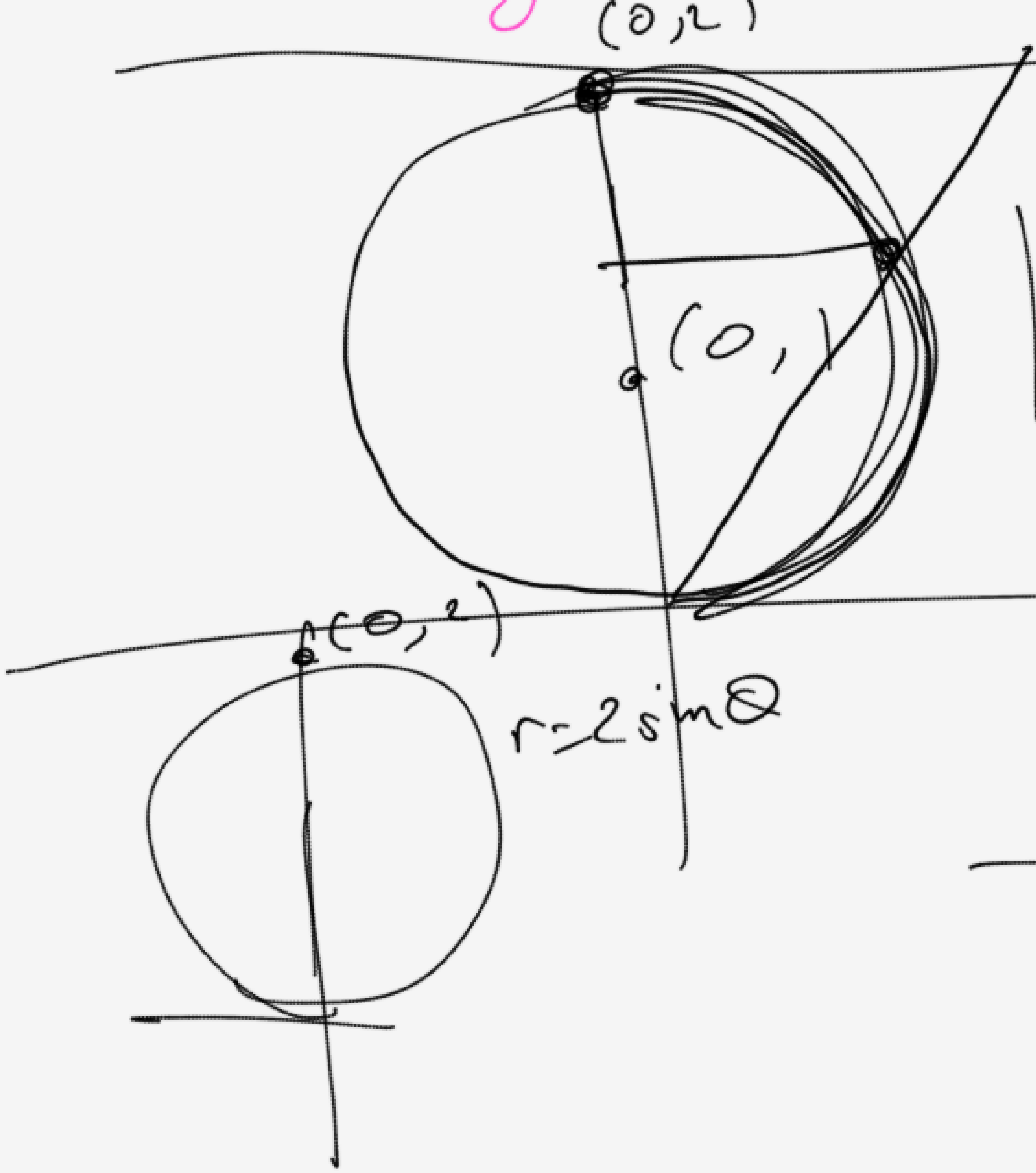
$$x^2 = 2y - y^2$$

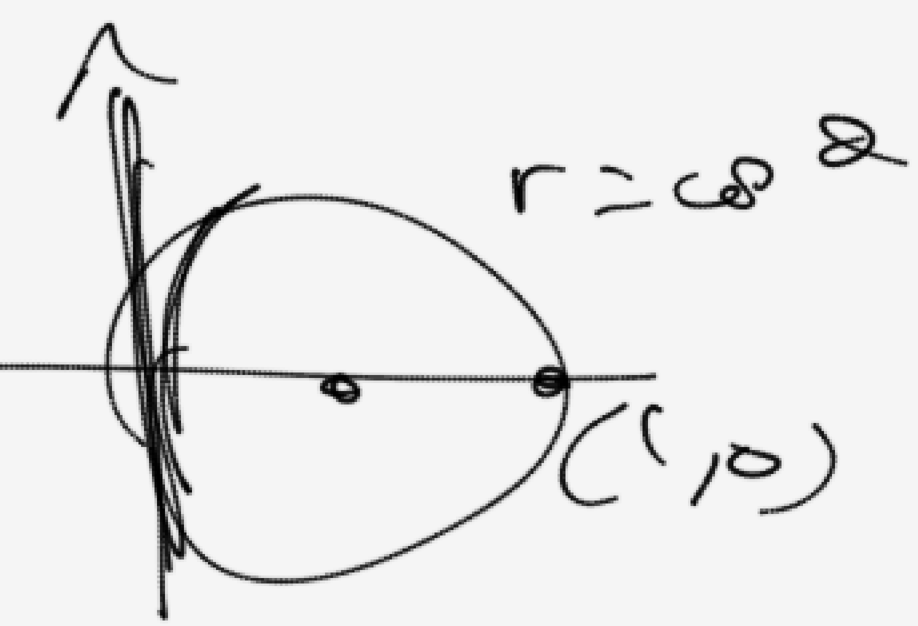
$$x^2 + y^2 - 2y = 0$$

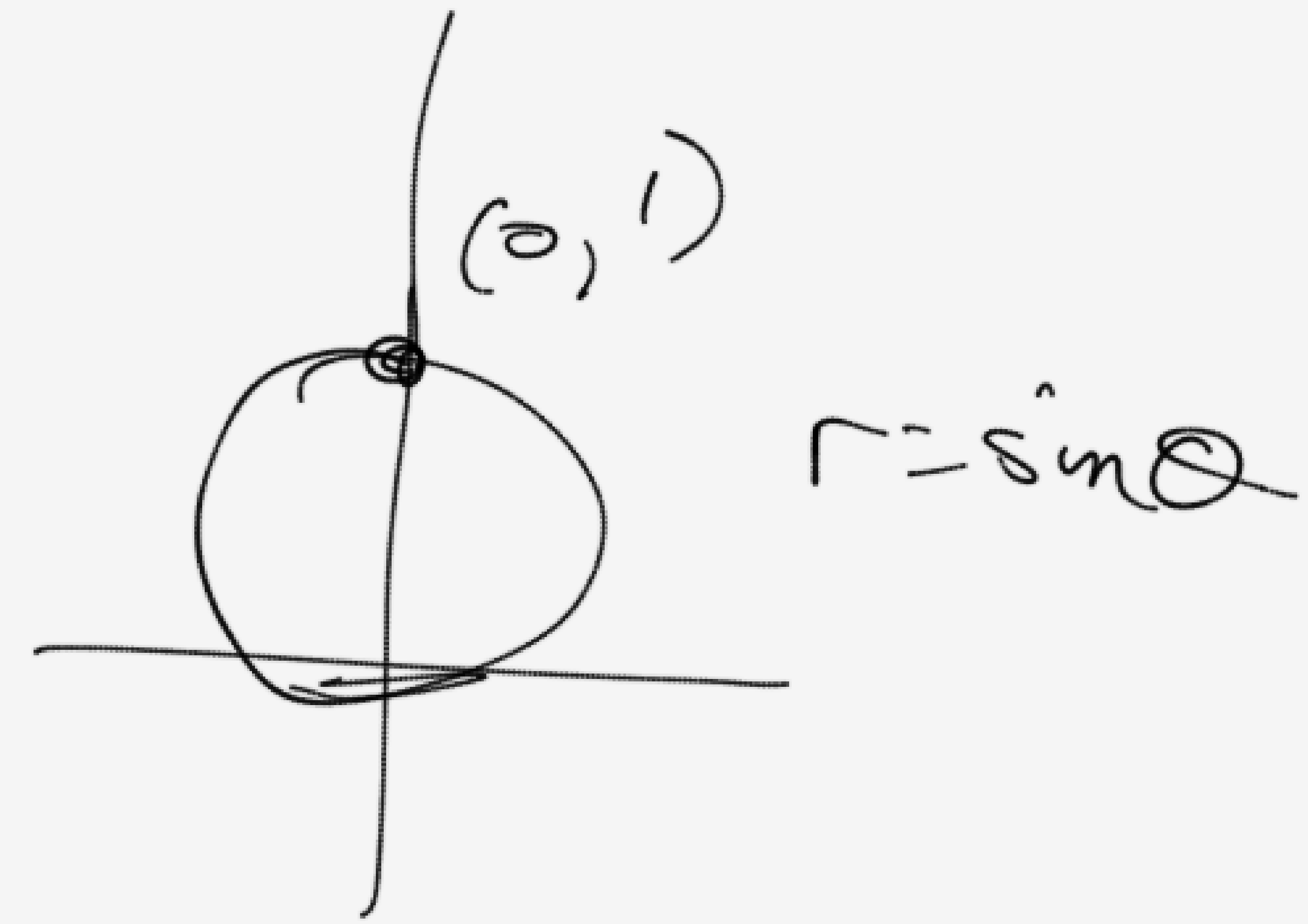
$$x^2 + (y^2 - 2y + 1 - 1) = 0$$

$$x^2 + (y - 1)^2 = 1$$

$\boxed{r = 2\sin\theta}$ F



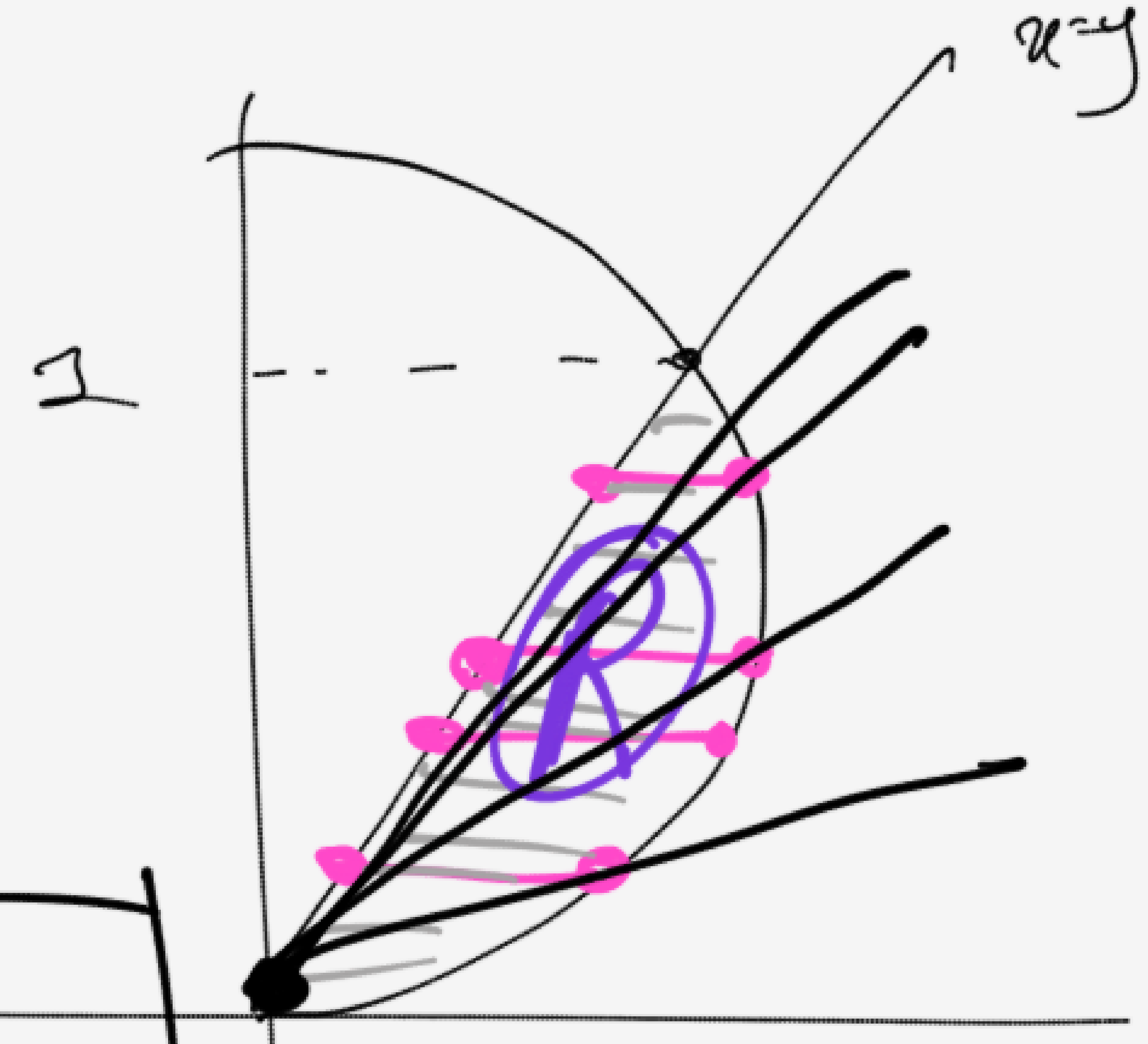




$x_{\text{entrance}} = y$

$x_{\text{exit}} = \text{circle}$

$0 < y < 1$



$r_{\text{ent.}} = 0$

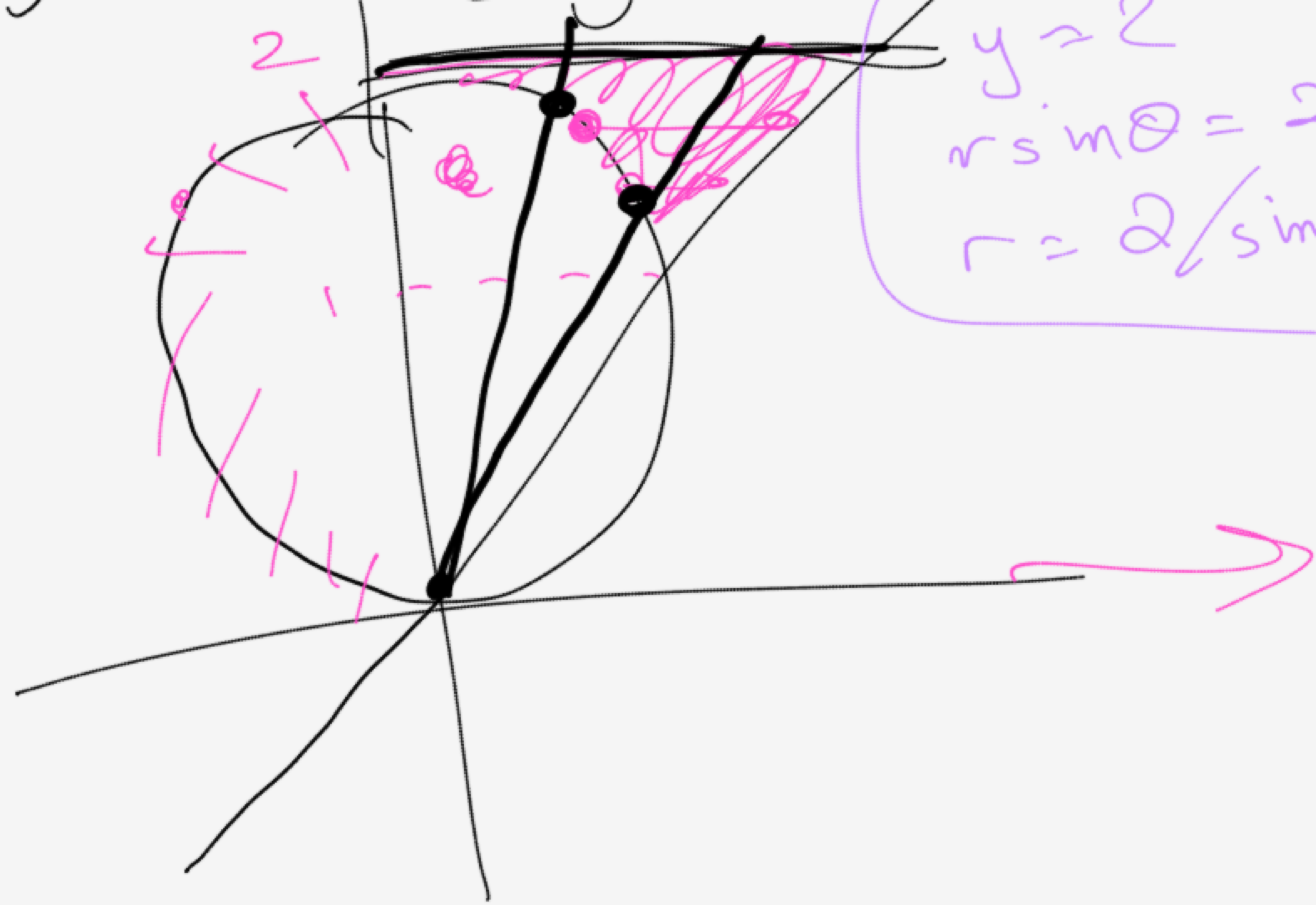
$r_{\text{exit}} = 2\sin\theta$

$$\boxed{\int_{\theta=0}^{\theta=\pi/4} \int_{r=0}^{r=2\sin\theta} (r\cos\theta) \cdot r\,dr\,d\theta}$$

No need to integrate.

Note: Redo as $dy\,dx$

New $\displaystyle\int_{y=1}^{y=2}\int_{x=\sqrt{2y-y^2}}^{x=y} dx\,dy$: Shade the region whose area is this.



Polar.

$y=2$
$r\sin\theta=2$
$r=2/\sin\theta$

$r_{\text{entrance}} = 2\sin\theta$

$r_{\text{exit}} = 2/\sin\theta$

$\pi/4 \leq \theta < \pi/2$.

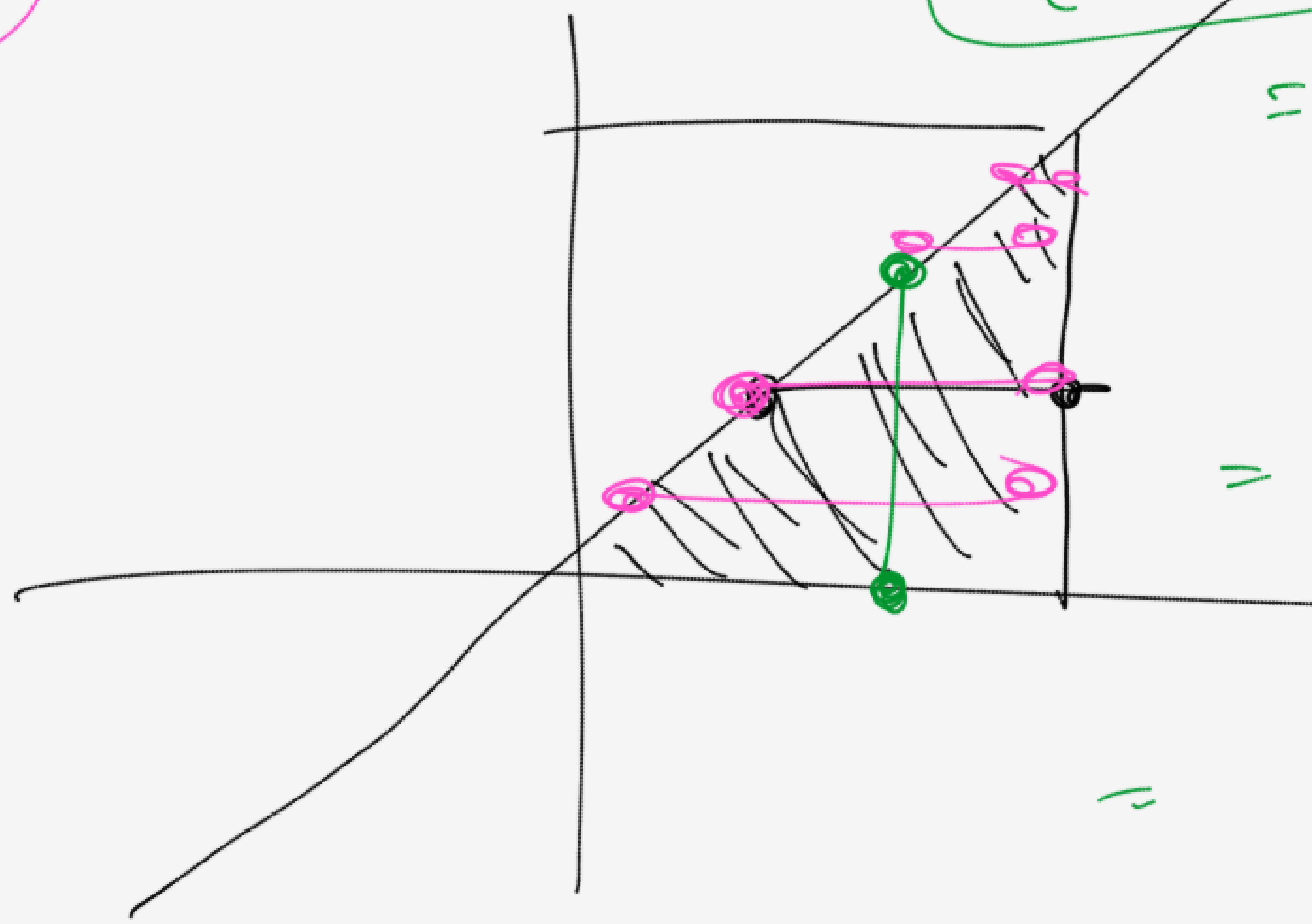
$$\int_{\pi/4}^{\pi/2}\int_{2\sin\theta}^{2/\sin\theta} r\,dr\,d\theta$$

$$\int_{y=0}^{y=1}\int_{y}^{1} 3x e^{x^3}\,\underbrace{dx\,dy} = \int_{x=0}^{x=1}\int_{y=0}^{y=x} 3x e^{x^3}\,\underline{\underline{dy\,dx}}$$

$x_{ent} = y$

$x_{exit} = 1$

$0 < y < 1$



$$= \int 3x e^{x^3} y \Big|_0^x dx$$

$$= \int_0^1 3x^2 e^{x^3} dx$$

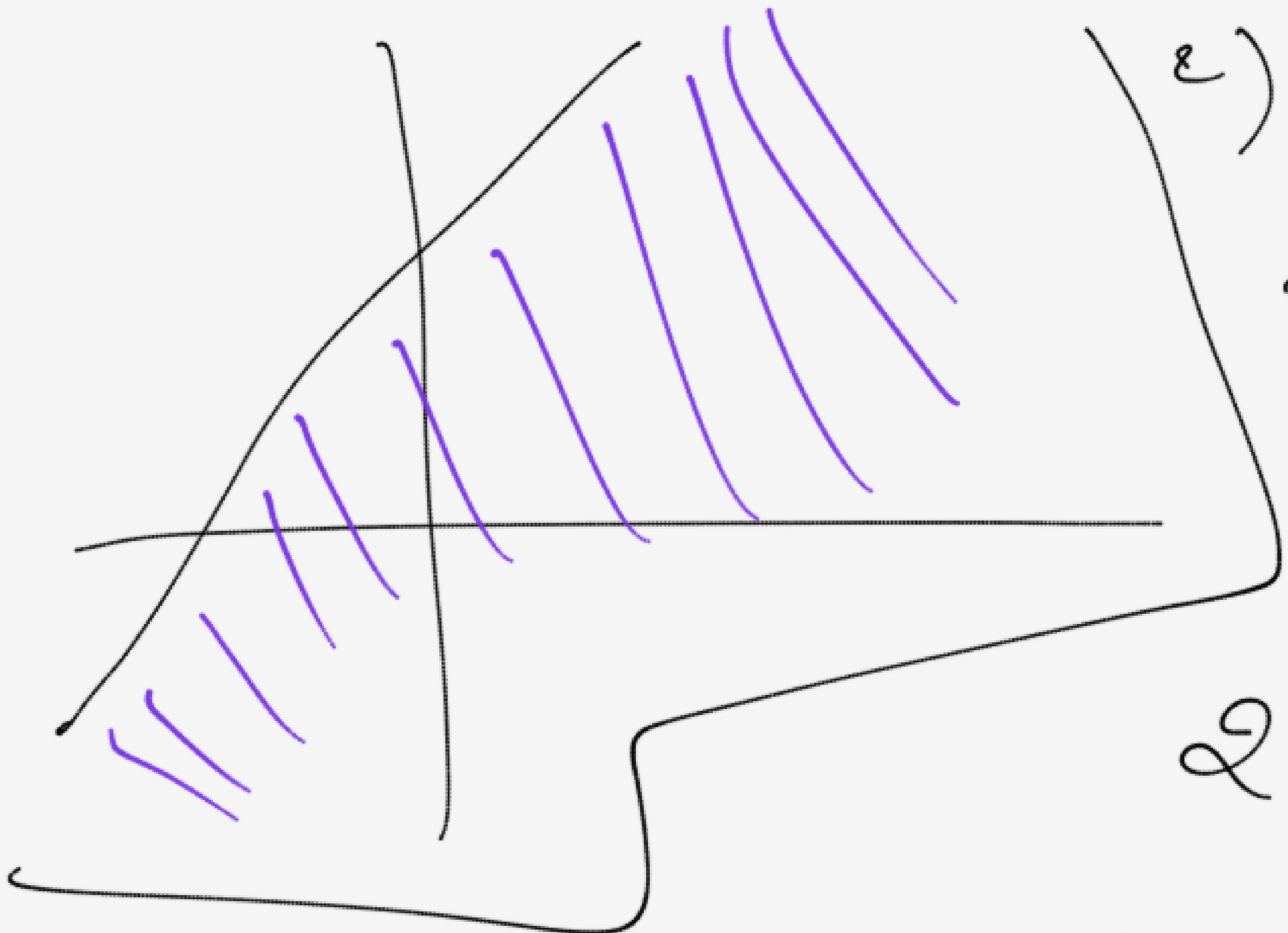
$$= e^{x^3}\Big|_{x=0}^{x=1} = \boxed{e-1}$$

$f(x,y) = \sqrt{2 + x - y}$

1) Domain

$2 + x - y \geq 0$

$y \leq x + 2$.



2) Range

$\boxed{z \geq 0}$

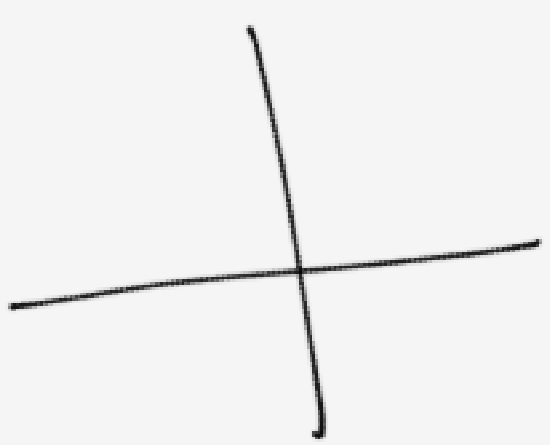
3) Level Curves:

$(S) \cap (z = c)$

$\sqrt{2 + x - y} = c \geq 0$

$2 + x - y = c^2$

$y = x + \underbrace{(2 - c^2)}$: lines // 1° bis.



$\sqrt{2 + x - y}$ $x + y$

~~slope~~ of line ~~tg to~~ curve (C) ~~slope~~ obtained by intersecting the surface (S) : $z = f(x,y)$ with the plane $x = 5$

@ $(5, 6, f(5,6))$

Answer: $f_y(5,6)$