Calc III
Sampel Final Exam
Spring 09

1. Evaluate the following limits: **(Show your work)**

   (a) $\lim_{x\to 0+}\left(1+\frac{2}{x}\right)^x$

   (b) $\lim_{n\to\infty}\sqrt[n]{n}$

   (c) $\lim_{x\to\infty}\left(1+\frac{3}{x^2}\right)^x$

2. Evaluate $\frac{dy}{dx}$ where $y=(x^3+5x^2-3x)^5(\sqrt{x}+x)^3$

3. Evaluate the following integrals:

   (a) $\int_0^a xe^{-x}dx$

   (b) $\int \frac{1}{(x+1)(x-2)}dx$

4. Determine whether the following improper integrals converge or diverge. **Justify your answer!**

   (a) $\int_0^\infty xe^{-x}dx$

   (b) $\int_1^\infty \frac{\ln x}{x^3}dx$

   (c) $\int_2^3 \frac{dx}{x(\ln x-1)}$

5. Which of the following sequences $\{a_n\}$converge or diverge. In case of convergence, find the limit:

   (a) $a_n=\frac{n^2-1}{\sqrt{9n^4-2n+\ln n-3}}$

   (b) $a_n=(-1)^n.\frac{\sin n}{n}$

6. Determine whether the following series converge or diverge. In case of an **alternating** series, determine if the series converges absolutely, conditionally or diverges:

   (a) $\sum_{n=0}^{\infty}(1/2)^n$

   (b) $\sum_{n=2}^{\infty}\frac{1}{n\ln n}$

   (c) $\sum_{n=1}^{\infty}\frac{\ln n}{n^3}$

   (d) $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n^2}\right)$

   (e) $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n^n}$

   (f) $\sum_{n=2}^{\infty}(-1)^n\frac{1}{n\ln n}$

7. For what values of x does the series $\sum_{n=1}^{\infty} 2^n(x-1)^n$ converge:

8. (a) Find the Maclaurin series for $f(x) = \frac{1}{1-x}$, where $|x| < 1$

   (b) Deduce the series representation for $\ln|1-x|$

   (c) Deduce an approximation for $\ln(3/4)$.

   (d) Deduce from (a) the derivative $f^{(12)}(0)$ of order12 of the function above at 0.

9. Determine if the following improper integrals converge or diverge: **(Explain why they are improper)**

   (a) $\int_2^{\infty} \frac{1}{x^2+1} dx$

   (b) $\int_0^{2} \frac{x}{x^2-1} dx$

   (c) $\int_1^{\infty} \frac{1}{x} e^{-\ln x} dx$

   (d) $\int_2^{\infty} \frac{\ln x}{x^{1.5}} dx$

10. Determine whether the following sequences$\{a_n\}$ converge or diverge:

   (a) $a_n = \frac{(2+n)^n}{n!}$

   (b) $a_n = \left(1 - \frac{2}{n}\right)^{2n}$

11. If the sequences $\{a_n\}$ and $\{b_n\}$ converge to A and B respectively, what can you conclude about the convergence or divergence of the following sequences? (Justify your answer if true or give a counterexample if false)

   (a) $\{a_n^2 - 3b_n\}$

   (b) $\frac{a_n}{b_n}$

12. Determine whether the following series converge or diverge:

   (a) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^{0.9} \ln n}$

   (b) $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^n$

   (c) $\sum_{n=1}^{\infty} \left(\frac{\sqrt{n^2+1}}{n^2-3n+1}\right)$

   (d) $\sum_{n=1}^{\infty} \frac{3^n}{6^{n+2}}$

13. If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B, what can you say about the convergence or divergence of the series: (Justify your answer if true or give a counterexample if false)

(a) $\sum_{n=1}^{\infty} (2a_n - 3b_n)$

(b) $\sum_{n=1}^{\infty} (a_n + 1)$

14. For what values of x does the series converge : $\sum_{n=1}^{\infty} \left(\frac{3x-5}{2}\right)^n$

converge?

15. Find the McLaurin series for $\cos(\pi x)$

16. Find a series representation for $\int e^{-x^3}\,dx$.

17. Evaluate, using power series the limit below: $\lim_{x\to 0} \frac{\sin x/x - 1}{x^2}$

18. Given the function $z = f(x,y) = \frac{x}{\sqrt{x^2+y^2}}$

(a) Find the domain and range of f.

(b) Describe the level curves for $z = 0, 1, 1/2$

(c) Determine if $\lim_{(x,y)\to(0,0)} f(x,y)$ exists. Justify your answer

19. Find $\frac{\partial w}{\partial r}$ if $w = (x+y+z)^2$, where $x = r - s$, $y = \cos(r+s)$, and $z = \sin(r+s)$.

20. Consider the surface $(S) : z = f(x,y) = \frac{1}{x^2+y^2}$.

(a) Find the rate of change of z at the point $(1,1,1/2)$,in the direction of the vector $\overrightarrow{A} = \overrightarrow{i} + \overrightarrow{j}$

(b) Write the equation of the plane tangent to (S) at $(1,1,1/2)$

(c) In which direction does z change most rapidly at $(1,1,1/2)$?

(d) Write the equation of the line (L) normal to (S) at $(1,1,1/2)$.

21. Find the volume of the solid bounded above by the surface $z = x^2 - y^3$ and below by the rectangle $2 < x < 5$, $3 < y < 7$.

22. Evaluate the integral: $\int_0^4 \int_{\sqrt{y}}^2 \frac{e^{x^2}}{\sqrt{y}}\,dx\,dy$.(Hint: reverse the order of integration.)