Lebanese American University — **Calculus III**

Fall 2005

Byblos — **Final Exam**

2 Hours

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1. Evaluate the integral $I = \int \frac{1}{x(x-1)(x+2)}\,dx$ **(7 pts)**

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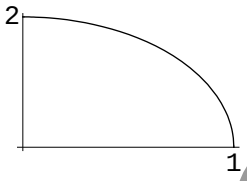
2. Let D be the region bounded by the x-axis, the y-axis and the curve $y = 2\sqrt{1-x^2}$, $0 \leq x \leq 1$.

a) Find the area of D. **(8 pts)**

b) Find the volume generated by revolving D about the:

(i) x-axis **(ii)** y-axis **(12 pts)**



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3. Determine if the series is convergent or divergent:

a) $\sum_{n=1}^{+\infty} a_n = \sum_{n=1}^{+\infty} \frac{1+\sin^2 n}{n\sqrt{n}}$

b) $\sum_{n=1}^{+\infty} a_n = \sum_{n=1}^{+\infty} \frac{(-5)^n}{3^{n+1}}$

c) $\sum_{n=1}^{+\infty} a_n = \sum_{n=1}^{+\infty} \frac{3^n}{n!}$ **(15 pts)**

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4. Using the binomial series, find the first 4 nonzero terms of the Maclaurin series of $f(x) = \frac{1}{\sqrt{1+x^2}}$ and $g(x) = \text{Ln}\left|x + \sqrt{1+x^2}\right|$ **(10 pts)**

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5. a) Find the MacLaurin series of $f(x) = \frac{\tan^{-1} x}{x}$

b) Estimate the integral $\int_0^1 \frac{\tan^{-1} x}{x}\,dx$ with an error $|R_n| \leq \frac{1}{50}$ **(10 pts)**

Turn Over

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6. Let $f(x, y) = \frac{x^2+y^2-y}{x^2+2y}$.

a) Using polar coordinates, is it possible to find the limit of f as $(x, y) \to (0, 0)$

b) Determine if this limit exists or not. **(8 pts)**

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7. Let $f(x, y) = \sin(x^2 y)$. Verify that $f_{xy} = f_{yx}$ **(7 pts)**

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8. Find the cartesian equation and sketch the ellipse

$$r = \frac{2}{\sqrt{1+3\sin^2\theta}}$$ **(8 pts)**

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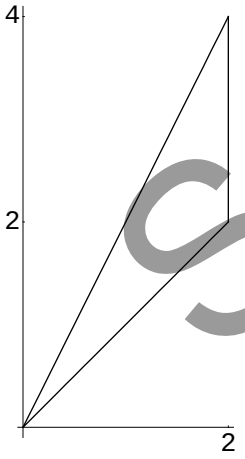
9. Evaluate the double integral $I = \iint_D xy\, dx\, dy$ where D is the domain bounded by the lines $y = x$, $y = 2x$ and $x = 2$. **(7 pts)**



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10. Evaluate the double integral $I = \int_0^1 \int_x^1 e^{y^2}\, dy\, dx$.

Hint: the function e^{y^2} can't be integrated. **(8 pts)**

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Spring 2002

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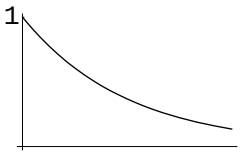
1. Let D be the region in the 1st quadrant bounded by the coordinate axes and the curve $y = e^{-x}$, $0 \leq x < \infty$.



Find the volume generated by revolving D about:

a) the x-axis.

b) the y-axis. **(15 pts)**

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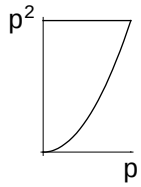
2. Let D be the region in the 1st quadrant bounded by the y-axis, the line $y = \pi^2$ and the parabola $y = x^2$, $0 \leq x \leq \pi$.



Find the mass of the plate D, if the density is:

a) $\delta = y$

b) $\delta = \sin x$ (Tabular integration) **(15 pts)**

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3. Evaluate the improper integral $I = \int_1^{+\infty} \frac{1}{x^2(x+1)}\, dx$ **(10 pts)**

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4. a) Find the sum of the geometric series $\sum_{n=1}^{+\infty} x^{n-1}$

b) Integrate both sides then deduce the sum of the series $\sum_{n=1}^{+\infty} \frac{1}{n\,2^n}$ **(10 pts)**

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5. Find the interval of convergence of the series $\sum_{n=1}^{+\infty} \frac{(2x-1)^n}{\sqrt{n}}$ **(10 pts)**

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6. Estimate the integral $I = \int_0^1 \frac{\sin x}{x}\, dx$ with an error $\left|R_n\right| \leq \frac{1}{5000}$ **(10 pts)**

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7. Find the Taylor polynomial of order 3 of $f(x) = \text{Ln}(\sec x)$ at $a = \frac{\pi}{4}$ **(6 pts)**

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8. Use series to evaluate the limit $L = \lim_{x \to 0} \frac{x - \text{Ln}(1+x)}{1 - \cos(5x)}$ **(7 pts)**

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9. Find the cartesian equation and sketch the curve $r = \frac{3}{2 - \sin\theta}$ **(7 pts)**

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10. Discuss and sketch the curve $r = 2 + 2\sin\theta$. (1 unit = 1 cm) **(10 pts)**