

LEBANESE AMERICAN UNIVERSITY
Division of Computer Science and Mathematics

Calculus III

Final Exam

Spring 2009 (June 22, 2009)

Name:

Solutions

ID:

Circle the name of your instructor: Dr. Hamdan

Mr Puzantian:

<u>Question Number</u>	<u>Grade</u>
1. 8%	
2. 6%	
3. 16%	
4. 6%	
5. 7%	
6. 7%	
7. 6%	
8. 16%	
9. 6%	
10. 16%	
11. 6%	
Total	

1. (8%) Determine if the following improper integrals converge or diverge:

$$(a) \int_{15}^{\infty} \frac{1}{\sqrt{e^x - x^3}} dx \approx \int_{15}^{\infty} \frac{1}{e^{x/2}} dx < \int_{15}^{\infty} \frac{1}{x^{10}} dx$$

converges:

p-series $p > 1$

$$(b) \int_{-\infty}^{\infty} 4xe^{-x^2} dx = \int_{-\infty}^{\infty} \frac{4x}{e^{x^2}} dx \quad \text{odd fn:}$$

consider $\int_0^{\infty} \frac{4x}{e^{x^2}} dx = \int_0^{\infty} \frac{du}{e^u} \quad \text{converges.}$

\Rightarrow the whole int. converges to 0

2. (6%) Determine whether the following sequences $\{a_n\}$ converge or diverge:

$$(a) a_n = \frac{\sin n}{\ln(n+1)!} = \frac{\text{bdd}}{\infty} \rightarrow 0.$$

converges to 0.

$$(b) a_n = n \cos(1/n) = n \cos 1/n \rightarrow \infty \cdot \cos 0 = \infty \cdot 1 = \infty$$

\therefore diverges

3. (16%) Determine whether the following series converge or diverge: In case of a converging alternating series, determine the type of convergence

$$(a) \sum_{n=1}^{\infty} (-1)^n \frac{n}{7+n}$$

$$a_n = \frac{n}{7+n} \longrightarrow 1 \Rightarrow a_n \not\rightarrow 0$$

\Rightarrow Series diverges by n^{th} term test

$$(b) \sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{\sqrt{n}}$$

$$a_n = \left(1 + \frac{1}{\sqrt{n}}\right)^{\sqrt{n}} \longrightarrow e \neq 0 \Rightarrow$$

$a_n \not\rightarrow 0 \Rightarrow$ Series conv. by n^{th} term test.

$$(c) \sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{3} - \frac{6}{n}\right)^n$$

Root Test:

$$\rho = \lim (a_n)^{1/n} \longrightarrow \frac{1}{3} - \frac{6}{n} \longrightarrow \frac{1}{3} \Rightarrow \rho = \frac{1}{3}$$

$\rho < 1 \Rightarrow$ Series conv. by Root test.

$$(d) \sum_{n=1}^{\infty} \frac{n!}{6^{n+2}}$$

Ratio Test

$$\rho = \lim \frac{a_{n+1}}{a_n} = \lim \frac{(n+1)!}{6^{n+3}} \cdot \frac{6^{n+2}}{n!} \longrightarrow \frac{n}{6} \rightarrow \infty$$

$\rho = \infty > 1 \Rightarrow$ Series Conv. by Ratio test.

4. (6%) For what values of x does the series $\sum_{n=1}^{\infty} \frac{2^n(x-3)^n}{n^2}$ converge?

$$\rho = \lim \left| \frac{2 \cdot (x-3)}{n^{2/n}} \right| \rightarrow \rho = |2(x-3)|$$

$$-1 < 2(x-3) < 1 \Rightarrow \frac{5}{2} < x < \frac{7}{2}$$

Check end pt:

1) $x = 5/2$

\Rightarrow Series gives: $\sum \frac{(-1)^n}{n^2}$ conv

2) $x = 7/2$

$\Rightarrow \sum \frac{1}{n^2} \checkmark \Rightarrow$

Conv. for

$$\boxed{\frac{5}{2} \leq x \leq \frac{7}{2}}$$

5. (7%) Find the McLaurin series for $f(x) = \frac{\ln(1+x^2)}{x}$. You may use series you are already familiar with.

$$\begin{aligned} \ln(1+t) &= \int \frac{dt}{1+t} = \int (1 - t + t^2 - t^3 + t^4 - \dots) \\ &= t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \frac{t^5}{5} - \dots \end{aligned}$$

$$\therefore \ln(1+x^2) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \dots$$

$$\frac{\ln(1+x^2)}{x} = x - \frac{x^3}{2} + \frac{x^5}{3} - \frac{x^7}{4} + \dots$$

6. (7%) Evaluate, using power series the limit below: $\lim_{x \rightarrow 0} \frac{\sin(x^3) - x^3}{x^9}$

$$\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \dots$$

$$\sin(x^3) = x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + \dots$$

$$\sin(x^3) - x^3 = -\frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + \dots$$

$$\frac{\sin x^3 - x^3}{x^9} = -\frac{1}{3!} + \frac{x^6}{5!} - \dots$$

$$\rightarrow \boxed{-1/6}$$

$\rightarrow 0$

9. (6%) Find the following limit, if it exists:

$$(a) \lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y}{2x^6 + y^2} = \text{Try } y = Kx^3$$

$$\lim \frac{kx^6}{2x^6 + k^2x^6} \rightarrow \frac{k}{2+k^2} \text{ depends}$$

on $k \rightarrow$ limit does not exist.

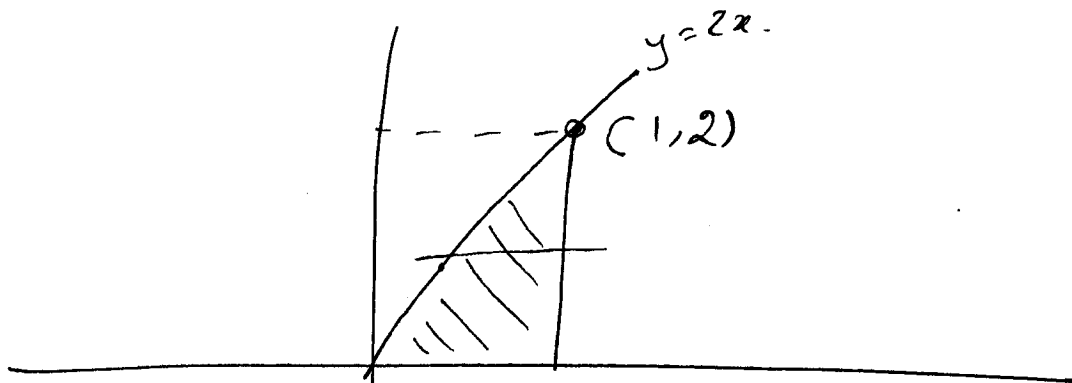
10. (16%) Evaluate the integrals:

$$(a) \int_0^2 \int_{y/2}^1 ye^{x^3} dx dy. (\text{Hint: reverse the order of integration.})$$

$$x = y/2$$

$$x = 1$$

$$0 < y < 2$$



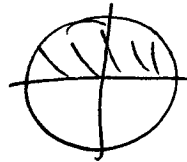
$$\therefore \int_0^1 \int_0^{2x} ye^{x^3} dy dx = \int_0^1 \frac{y^2}{2} e^{x^3} \Big|_0^{2x} dx$$

$$= \int_0^1 2x^2 e^{x^3} dx = \frac{2}{3} e^{x^3} \Big|_0^1$$

$$= \frac{2}{3} (e - 1)$$

(b) $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \frac{x^2 y}{(x^2 + y^2)^2} dy dx$. (Hint: use polar coordinates)

$$0 < y < \sqrt{1-x^2} \quad -1 < x < 1$$



$$\int_0^{\pi} \int_0^1 \frac{r^3 \cos^2 \theta \sin \theta}{r^4} r dr d\theta$$

$$\int_0^{\pi} \int_0^1 \cos^2 \theta \sin \theta dr d\theta \quad \checkmark \text{ can be done}$$

11. (6%) Discuss in a short paragraph, using your own words, the relation between the following: $f_x(1, 2)$, $f_y(1, 2)$, the surface $(S) : z = f(x, y)$ and the planes $y = 2$, and $x = 1$.

Let S be the surface $z = f(x, y)$.

If we cut (S) by the plane $y = 2$,

then $f_x(1, 2) =$ slope of the line

tangent to the curve $(S) \cap y = 2$ @ $(2, 1, f(2, 1))$

Same for cutting by the plane $x = 1 \dots$

$f_y(1, 2) =$ slope of \dots