

## MATH 201. More problems on Taylor series and applications.

- a- Find the MacLaurin series for the function  $f(x) = xe^x$ .  
Does this series converge to  $f(x)$ ? If so, over which interval?  
b- Conclude a value for the sum  $\sum_{n=0}^{\infty} \frac{n+1}{n!}$ .
- Let  $f(x) = \frac{x}{1+2x}$ 
  - a) Find its Maclaurin series.
  - b) Conclude a value for  $f^{(7)}(0)$
- - a- Let  $f(x) = 2x^3 + x^2 - x + 3$ . Find its Taylor polynomial of order 3 around  $a = 1$ .
  - b- Expand the polynomial obtained. What do you observe? Why?
- Use the known geometric series  $\frac{1}{1-r} = 1 + r + r^2 + r^3 + r^4 + \dots$  for  $-1 < r < 1$  to express  $f(x) = \frac{1}{7-2x}$  as a Taylor series around  $a=3$ .
- - a- Use a known MacLaurin series to evaluate  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{1 + x - e^x}$ .
  - b- Write the indefinite integral  $\int \sin(x^2) dx$  as an infinite series.
- - a- Write the Maclaurin series for  $f(x) = \frac{1}{1-x}$
  - b- Conclude the Maclaurin series for  $\tan^{-1}(x^2)$
  - c- Conclude the value of the following sum  $\sum_{n=0}^{\infty} (-1)^n \frac{1}{4n+2}$
- - a. Write the Maclaurin series for  $e^{-x^2}$ .
  - b- Determine where the series  $\sum_{n=1}^{\infty} \frac{(-1)^n 2n}{n!}$  converges to.

