

LEBANESE AMERICAN UNIVERSITY  
DEPARTMENT OF COMPUTER SCIENCE AND MATHEMATICS  
**MTH 201 - CALCULUS 3**  
EXAM 1 – FALL 2011

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Duration: 75 minutes

**Name:**

**ID#:**

- **This exam consists of 9 pages and 8 problems.**
- **Answer the questions below on the space provided. You can use the back pages for scratch or for more space for your answers. Please specify.**
- **Make sure you justify all your answers.**

<u>Question Number</u>	<u>Grade</u>
<b>1. 10%</b>	
<b>2. 12%</b>	
<b>3. 20%</b>	
<b>4. 12%</b>	
<b>5. 12%</b>	
<b>6. 10%</b>	
<b>7. 12%</b>	
<b>8. 12%</b>	
<b>TOTAL</b>	

**Problem 1:** (10%) Evaluate the following limits

(a)  $\lim_{x \rightarrow \infty} \sin^{-1}\left(\frac{x^2}{1 + 2x^2}\right)$

(b)  $\lim_{x \rightarrow \infty} \frac{\tan^{-1}(e^x)}{e^{2x} + x}$

**Problem 2:** (12%)

(a) Simplify the following expression

$$\ln(\cosh 6x - \sinh 6x) + \ln(\cosh 3x + \sinh 3x)$$

(b) Evaluate

$$\int \frac{\sinh(\ln x)}{x} dx$$

Simplify your answer.

**Problem 3:** (20%) Evaluate the following integrals

(a)  $\int \tan^{-1}(4x) dx$

(b)  $\int \frac{1}{x^2+4x+5} dx$

**Problem 4:** (12%) Evaluate the following improper integrals

(a)  $\int_2^4 \frac{1}{x^2-x-2} dx$

(b)  $\int_{-\infty}^{\infty} \frac{1}{(e^x+e^{-x})} dx$

**Problem 5:** (12%) Determine the convergence or divergence of the following improper integrals. Justify your answers.

(a)  $\int_0^{\infty} \frac{1}{(e^x+1)^2} dx$

(b)  $\int_1^{\infty} \frac{\ln x}{e^x} dx$

**Problem 6:** (10%) Show that

$$\int_1^{\infty} \frac{\sin x + 2}{x^2} dx$$

converges, whereas

$$\int_1^{\infty} \frac{\sin x + 2}{x} dx$$

diverges.

**Problem 7:** (12%)

Find the values of  $p$  for which

$$\int_1^{\infty} \frac{x}{\sqrt{x^p + 1}} dx$$

converges.

Justify your answer.



**Problem 8:** (12%) Determine if the following sequences converge or diverge. Justify your answers.

(a)  $a_n = \frac{n \sin\left((2n-1)\frac{\pi}{2}\right)}{n+1}$

(b)  $a_n = \left(1 + \frac{2}{n}\right)^n \frac{1}{\sqrt[n]{n^2}}$