Problem 1: (8\%)
(a) Evaluate the following improper integral

$$
\int_{0}^{4} \frac{x}{\sqrt{16-x^{2}}} d x
$$

(b) Determine the convergence or divergence of

$$
\int_{2}^{\infty} \frac{1}{\sqrt[4]{x^{4}-x}} d x
$$

Problem 2: (12\%) Consider the following series

$$
\sum_{n=1}^{\infty} \ln \left(\frac{n}{n+1}\right)
$$

(a) Find $S_{20}$ and simplify your answer.
(b) Find the infinite sum $\sum_{n=1}^{\infty} \ln \left(\frac{n}{n+1}\right)$.

Problem 3: (20\%) Which series converge? Converge conditionally? Diverge? Justify your answer.
(a) $\sum_{n=1}^{\infty} \frac{(\sin n) \ln n}{n^{2}}$
(b) $\sum_{n=1}^{\infty}\left(1-\frac{49}{n^{2}}\right)^{n}$
(c) $\sum_{n=1}^{\infty} \frac{2^{n} n \text { ! }}{n^{n}}$
(d) $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln n)^{2}}$

Problem 4: (8\%) Find the interval of convergence of:

$$
\sum_{n=2}^{\infty} \frac{(-1)^{n}(x-2)^{n}}{4^{n} \ln n}
$$

Problem 5: (12\%)
(a) Express the indefinite integral $\int e^{-x^{3}} d x$ as an (alternating) power series.
(b) Deduce the definite integral $\int_{0}^{0.1} e^{-x^{3}} d x$ as a series.
(c) Find the appropriate value of $n$ for which $S n$ approximates the definite integral with an error of magnitude less than or equal to $10^{-11}$.

Problem 6: (8\%) Consider $f(x)=x^{2} e^{x}$.
(a) Express $f(x)$ as a power series.
(b) Deduce a value for the sum

$$
\sum_{n=1}^{\infty} \frac{n+2}{n!}
$$

(You may use term by term integration or differentiation on your result in part (a)).

Problem 7: (4\%) Find the following limit using Maclaurin series

$$
\lim _{n \rightarrow \infty}\left(n^{2} \ln \left(1+\frac{1}{n}\right)-n\right)
$$

Problem 8: $(8 \%)$ Evaluate the following or show that the limit does not exist
(a) $\lim _{(x, y) \rightarrow(0,0)} \frac{x^{3}+y^{6}}{x^{3}}$
(b) $\lim _{(x, y) \rightarrow(0,0)} \frac{x^{2} y}{x^{2}+y^{2}}$

Problem 9: (6\%) Consider the following function

$$
f(x, y)=e^{10 y} \cos (10 x)
$$

Verify whether $f(x, y)$ satisfies

$$
f_{x x}+f_{y y}=0
$$

(This is called Laplace's equation)

Problem 10: (6\%) Consider the following function

$$
f(x, y)=\sqrt{49-x^{2}-y^{2}}
$$

(a) Find the domain and range of this function.
(b) Find the equation of the level curve of $f(x, y)$ that passes through the point $(2,5)$.

Problem 11: (8\%) Evaluate the integral

$$
\int_{0}^{1} \int_{0}^{1} e^{(2 x+3 y)} d x d y
$$

