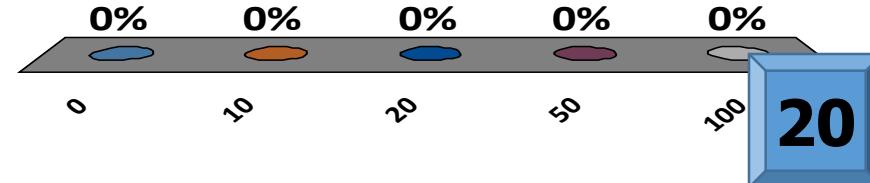


EECE 290, Problem solving

Session 11

3 filters are cascaded. At one particular frequency the gains are 46dB, -12dB and 0dB. What is the signal magnitude gain on a linear scale?

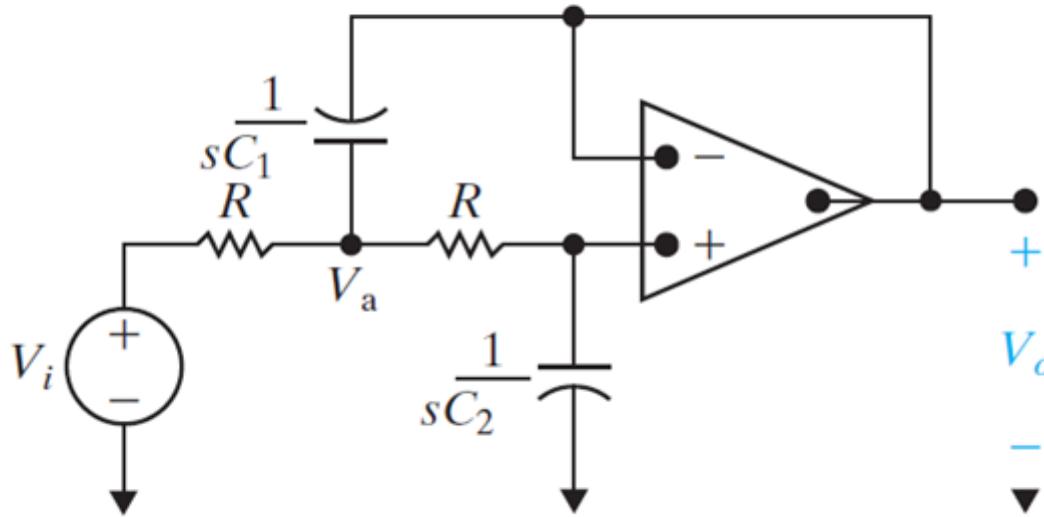
- A. 0
- B. 10
- C. 20
- D. 50
- E. 100



2nd and higher order active filters

Band-pass $K \frac{Bs}{s^2 + Bs + \omega_0^2}$

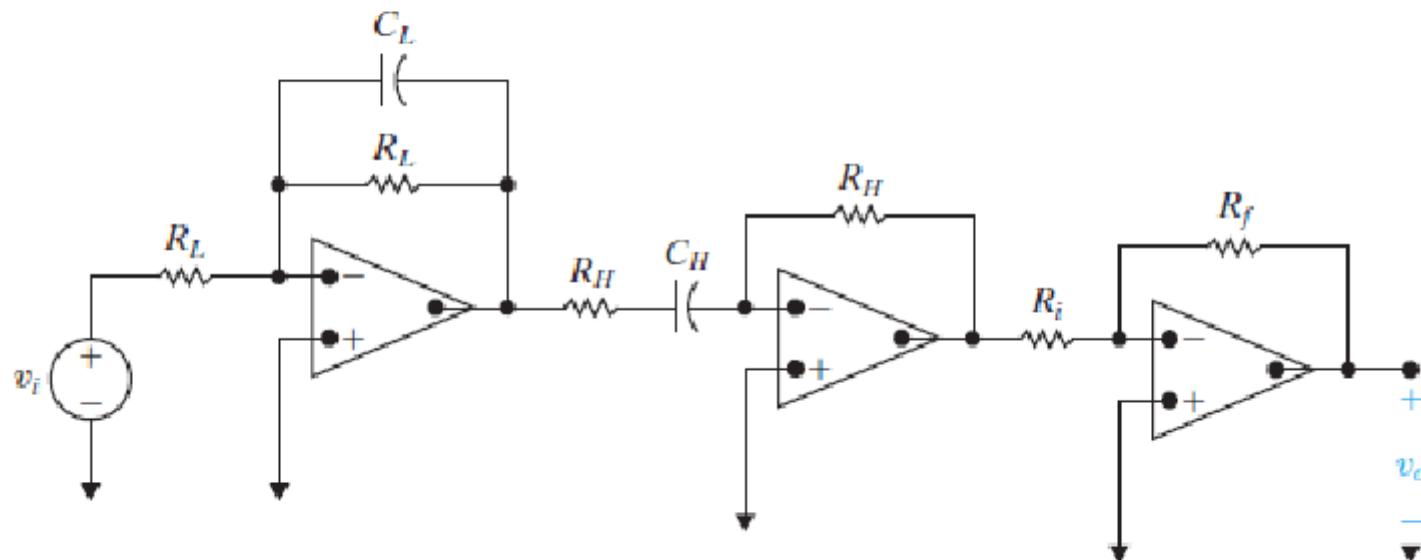
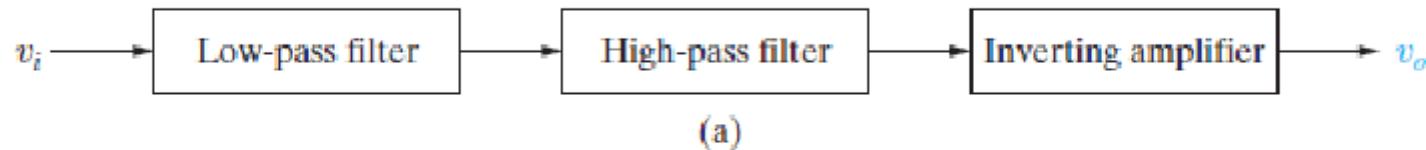
▲ LPF: $K \frac{\omega_0^2}{s^2 + Bs + \omega_0^2}$



$$B = \frac{2}{C_1}, \omega_0^2 = \frac{1}{C_1 C_2}$$

$$\text{prototype: } R=1, B = \frac{2}{C_1}, \omega_0^2 = \frac{1}{C_1 C_2} = 1$$

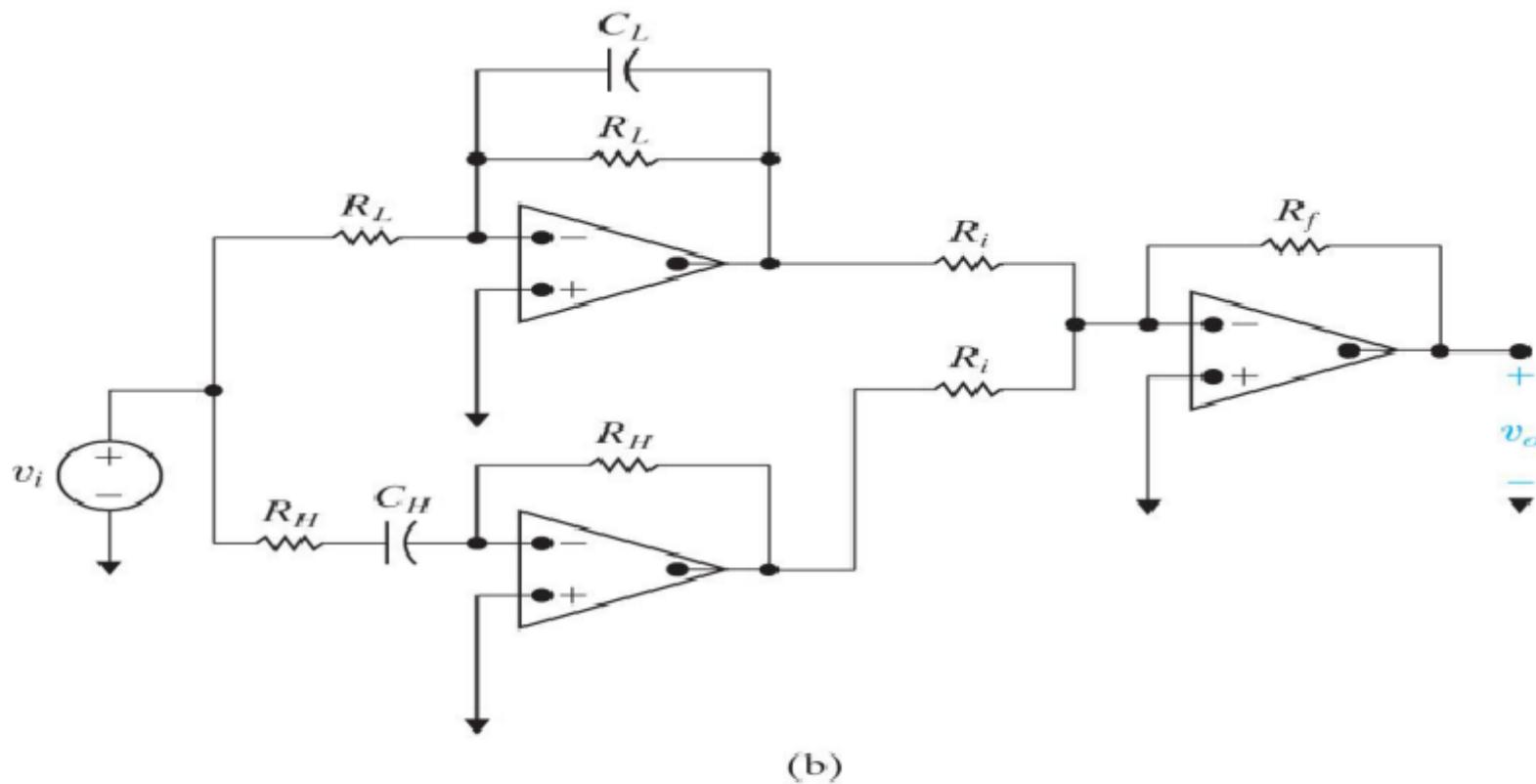
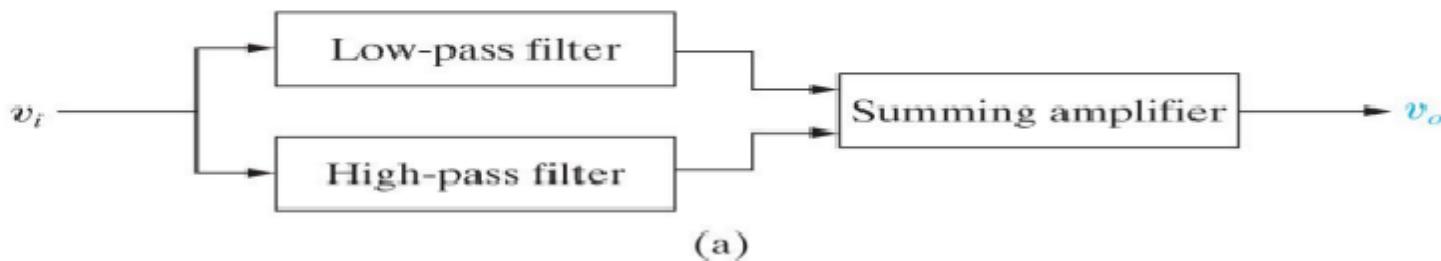
^ Broadband band-pass with cascading LPF and HPF



$$\omega_{c2} \gg \omega_{c1}$$

$$H(s) = K \frac{Bs}{s^2 + Bs + \omega_c^2}, \quad \omega_{c1} = \frac{1}{R_H C_H}, \quad \omega_{c2} = \frac{1}{R_L C_L}, \quad B = \omega_{c1} + \omega_{c2} \approx \omega_{c2}. \quad K = -\frac{R_f}{R_i}$$

^ Broadband band-reject with cascading LPF and HPF



$$\omega_{c2} \gg \omega_{c1}, \omega_{c1} = \frac{1}{R_L C_L}, \omega_{c2} = \frac{1}{R_H C_H}$$

passband gain: $\frac{R_f}{R_i}$

~ Cascading identical filters

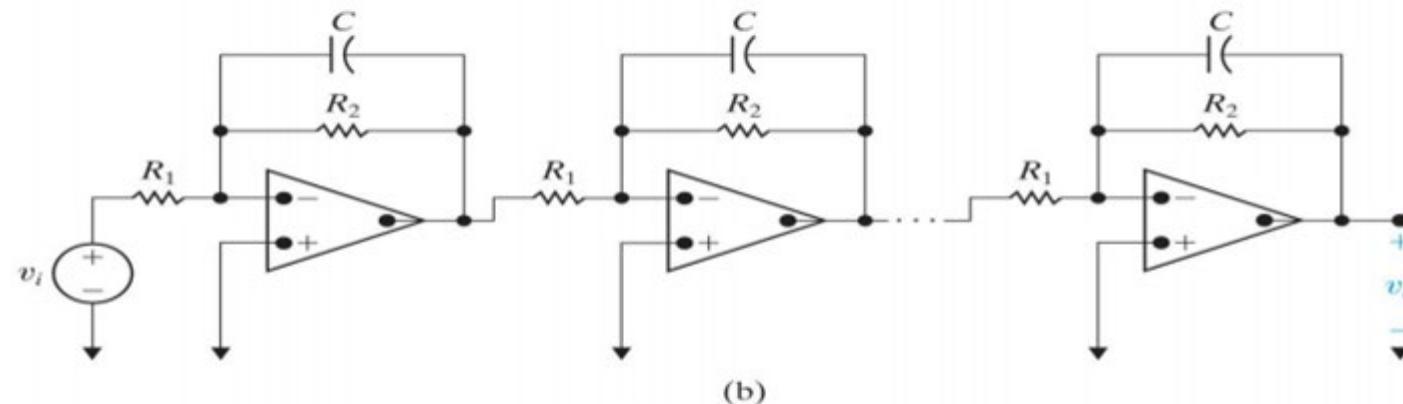
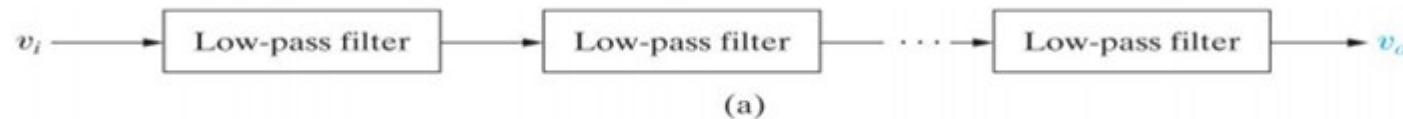


Figure: 15-17a,b

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$$\text{LPF: } H(s) = \frac{(-1)^n}{(s+1)^n}, \omega_{cn} = \sqrt{2^{1/n} - 1},$$

$$\text{HPF: } H(s) = \frac{(-1)^n s^n}{(s+1)^n}, \omega_{cn} = 1 / \sqrt{2^{1/n} - 1}$$

Butterworth filters

$$\text{LPF: } H(s) = \frac{1}{B_n(s)}, \quad |H(s)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}} = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^{2n}}}$$

$$\text{HPF: } H(s) = \frac{s^n}{B_n(s)}, \quad |H(s)| = \frac{\left(\frac{\omega}{\omega_c}\right)^n}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}} = \frac{\left(\frac{f}{f_c}\right)^n}{\sqrt{1 + \left(\frac{f}{f_c}\right)^{2n}}}$$

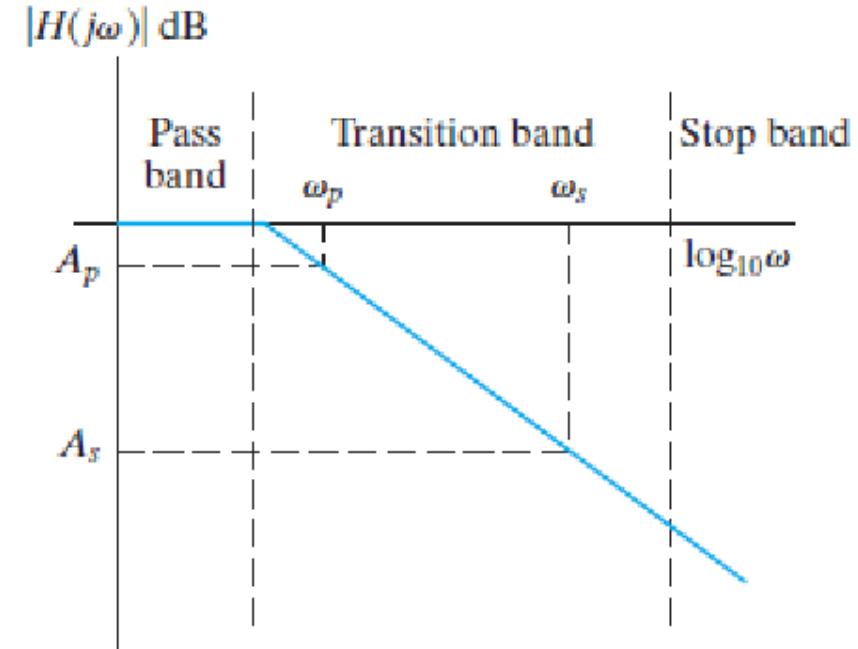


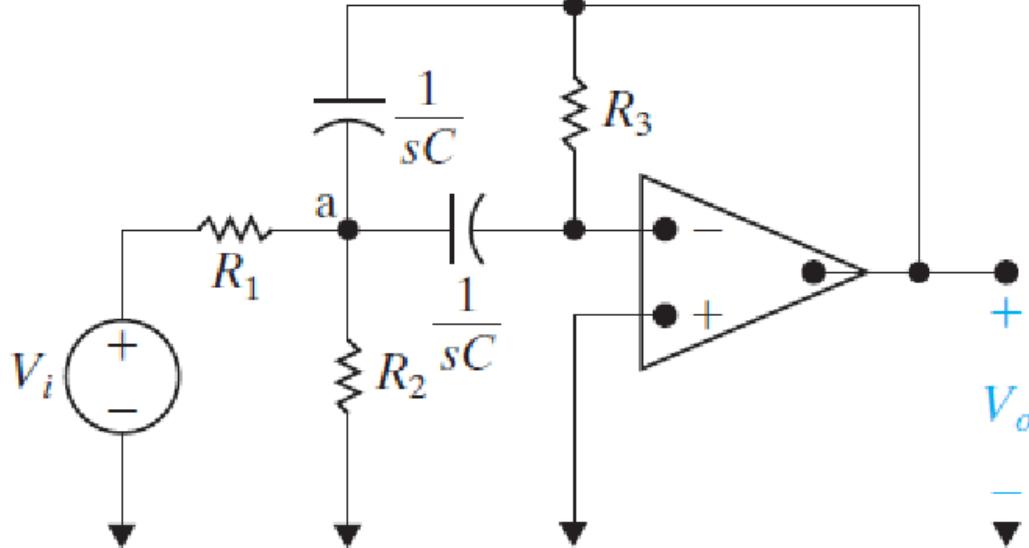
Table 15.2.2 Normalized Butterworth Polynomials of Order n

n	Factors of Polynomial $B_n(s)$
1	$(s + 1)$
2	$(s^2 + 1.414s + 1)$, where $\sqrt{2} = 1.414$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
5	$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$
6	$(s^2 + 0.518s + 1)(s^2 + 1.414s + 1)(s^2 + 1.932s + 1)$
7	$(s + 1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$
8	$(s^2 + 0.390s + 1)(s^2 + 1.111s + 1)(s^2 + 1.663s + 1)(s^2 + 1.962s + 1)$

$$n = \frac{\log_{10}\left(\frac{\sigma_s}{\sigma_p}\right)}{\log_{10}\left(\frac{\omega_s}{\omega_p}\right)}$$

Narrow-band filters

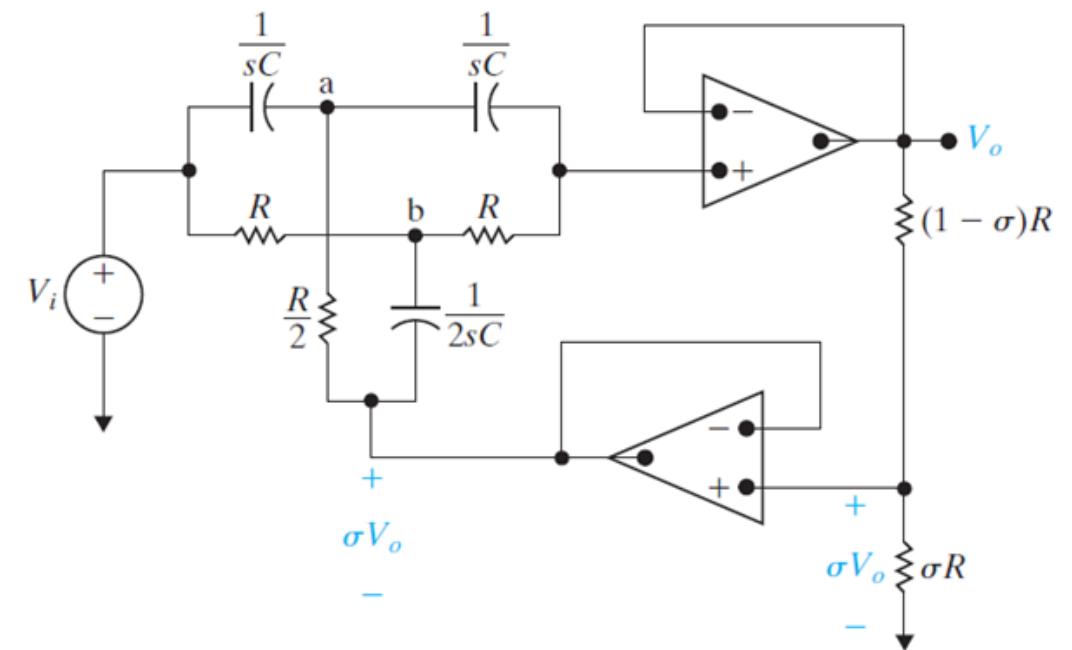
band-pass



$$K \frac{Bs}{s^2 + Bs + \omega_0^2}, B = \frac{2}{R_3 C}, KB = \frac{1}{R_1 C}, \omega_0^2 = \frac{1}{(R_1//R_2) R_3 C^2}$$

$$\text{prototype: } \omega_0 = 1 \text{ rad/s}, C = 1 \text{ F}, R_1 = \frac{Q}{K}, R_2 = \frac{Q}{(2Q^2 - K)}, R_3 = 2 \text{ Q}$$

band-reject:

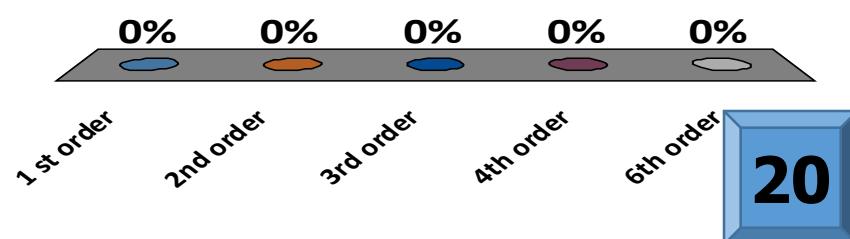


$$\omega_0^2 = \frac{1}{R^2 C^2}, B = \frac{4(1 - \sigma)}{RC}, \sigma = 1 - \frac{1}{4Q}$$

$$\frac{1}{\sqrt{1 + \left(\frac{\omega}{100}\right)^6}}$$

This is the Butterworth magnitude response of

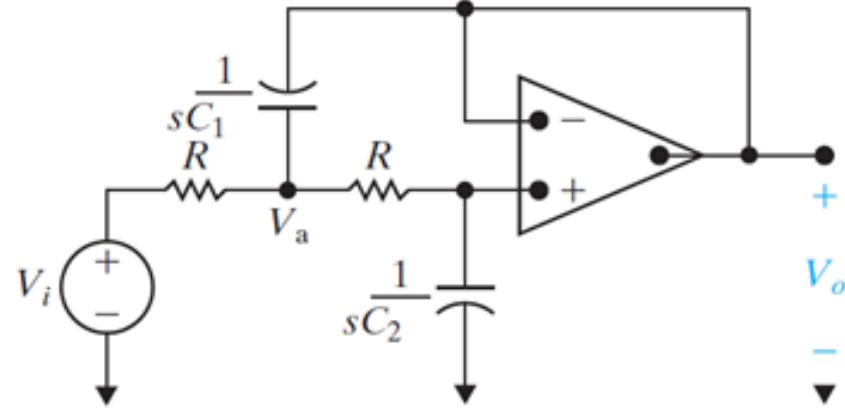
- A. 1st order
- B. 2nd order
- C. 3rd order
- D. 4th order
- E. 6th order



Design 2nd order LP Butterworth filter

Table 15.2.2 Normalized Butterworth Polynomials of Order n

n	Factors of Polynomial $B_n(s)$
1	$(s + 1)$
2	$(s^2 + 1.414s + 1)$, where $\sqrt{2} = 1.414$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
5	$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$
6	$(s^2 + 0.518s + 1)(s^2 + 1.414s + 1)(s^2 + 1.932s + 1)$
7	$(s + 1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$
8	$(s^2 + 0.390s + 1)(s^2 + 1.111s + 1)(s^2 + 1.663s + 1)(s^2 + 1.962s + 1)$



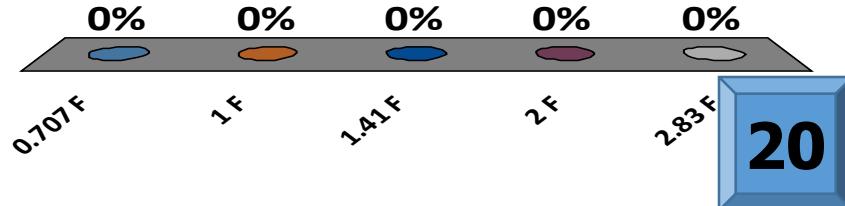
$$K \frac{\omega_c^2}{s^2 + Bs + \omega_c^2}$$

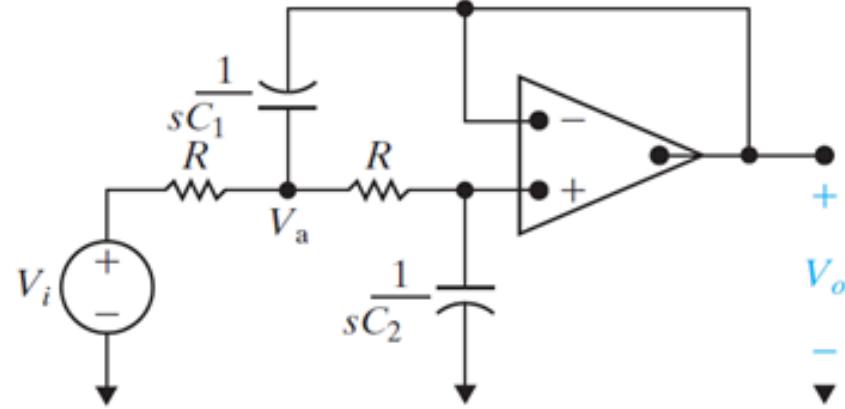
prototype: $R=1$, $B = \frac{2}{C_1}$, $\omega_0^2 = \frac{1}{C_1 C_2} = 1$

2nd order Butterworth: $B = \sqrt{2}$

Find C_1 .

- A. 0.707 F
- B. 1 F
- C. 1.41 F
- D. 2 F
- E. 2.83 F





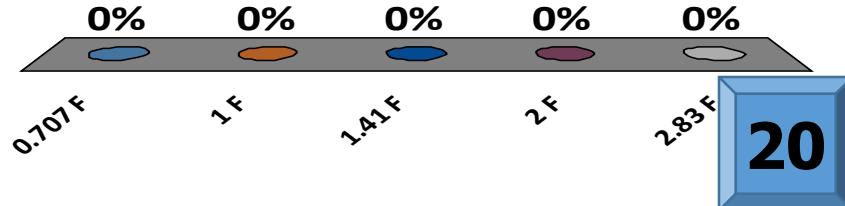
$$K \frac{\omega_c^2}{s^2 + Bs + \omega_c^2}$$

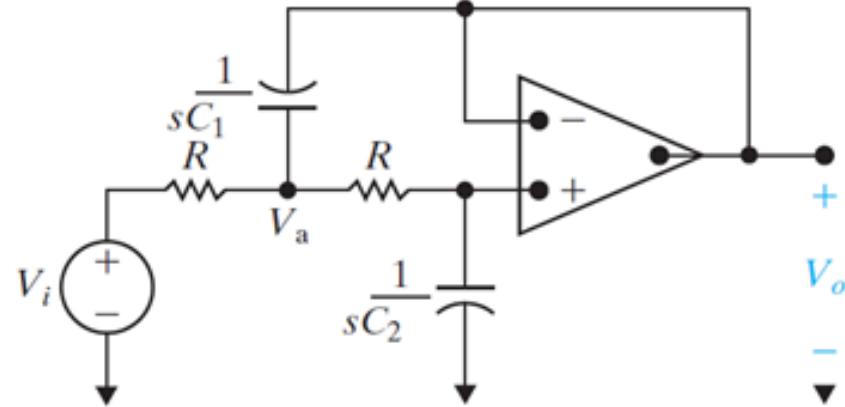
prototype: $R=1$, $B = \frac{2}{C_1}$, $\omega_0^2 = \frac{1}{C_1 C_2} = 1$

2nd order Butterworth: $B = \sqrt{2}$

Find C_2 .

- A. 0.707 F
- B. 1 F
- C. 1.41 F
- D. 2 F
- E. 2.83 F





$$K \frac{\omega_c^2}{s^2 + Bs + \omega_c^2}$$

prototype: $R=1$, $B = \frac{2}{C_1}$, $\omega_0^2 = \frac{1}{C_1 C_2} = 1$

2nd order Butterworth: $B = \sqrt{2}$

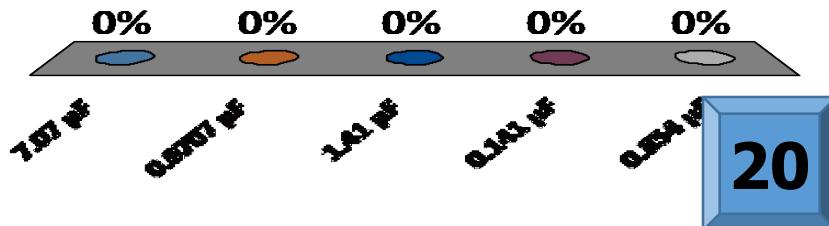
$$R' = k_m R$$

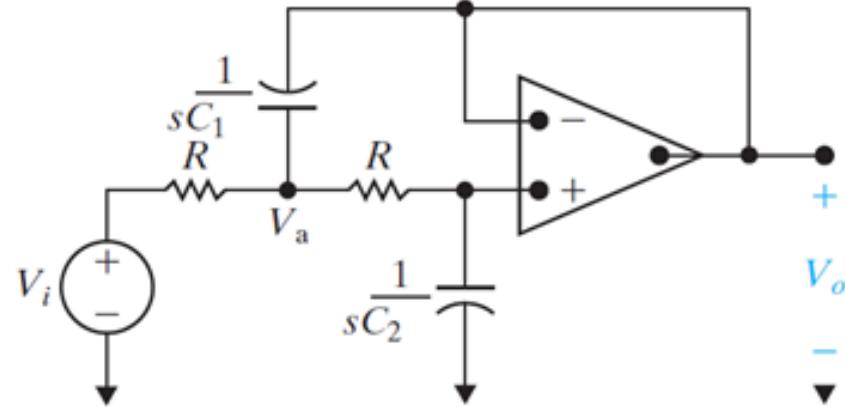
$$L' = \frac{k_m}{k_f} L$$

$$C' = \frac{C}{k_m k_f}$$

Scaling $\omega_0 \rightarrow 20$ krad/s,
 $R \rightarrow 1\text{k}\Omega$. Find C'_1 .

- A. 7.07 μF
- B. 0.0707 μF
- C. 1.41 μF
- D. 0.141 μF
- E. 0.354 μF





$$K \frac{\omega_c^2}{s^2 + Bs + \omega_c^2}$$

prototype: $R=1$, $B = \frac{2}{C_1}$, $\omega_0^2 = \frac{1}{C_1 C_2} = 1$

2nd order Butterworth: $B = \sqrt{2}$

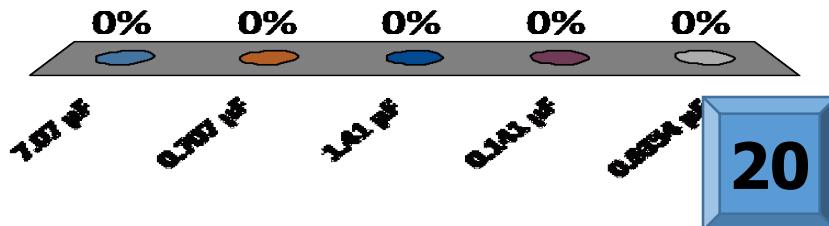
$$R' = k_m R$$

$$L' = \frac{k_m}{k_f} L$$

$$C' = \frac{C}{k_m k_f}$$

Scaling $\omega_0 \rightarrow 20$ krad/s,
 $R \rightarrow 1\text{k}\Omega$. Find C' .

- A. 7.07 μF
- B. 0.707 μF
- C. 1.41 μF
- D. 0.141 μF
- E. 0.0354 μF



Now: short quiz 2B