June 23, 2014 | **MATHEMATICS 218** | **NAME. --------------------**
Time : 65 minutes | **Midterm Exam** | **ID# -------------------**
Summer 2013-14

PROBLEM GRADE

PART I

1 ------- /14

2 ------- /16

3 ------- / 20

4. ------- / 10

PART II

5	**6**	**7**	**8**	**9**
a	a	a	a	a
b	b	b	b	b
c	c	c	c	c
d	d	d	d	d
e	e	e	e	e

5-9 ------- / 20

PART III Answer **True** or **False** **only** in the table below**:**

A	B	C	D	E	F	G	H	I	J

10 ------- / 20

TOTAL ------- / 100

PART I. Answer each of the following problems in the space provided for each problem (Problem 1 to Problem 4).

1. Find the values of a and b for which the system

$$x - 2y - z = 5$$
$$3x - 7y - az = 0$$
$$-2x + 3y + z = b$$

has

a. no solution
b. a unique solution
c. infinitely many solutions.

[14 points]

2. Let $A=\begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & 2 & 1 & 4 \end{pmatrix}$

(a) Find a basis of the null space N(A).

(b) Find a basis of the column space Col(A).

[16 points]

3. a. Let V be the subset of R^3 defined by:

$$V=\left\{\begin{pmatrix} x \\ y \\ 2x-y \end{pmatrix} \in R^3 \middle| x, y \in R\right\}$$

Show that V is a subspace of R^3. Find a basis for V.

[10 points]

3b. Let W be the subspace of R^3 given by

$$W=\left\{\begin{pmatrix} a \\ -2a \\ b \end{pmatrix} \in R^3 \;\middle|\; a,b \in R\right\}$$ (Do not show W is a subspace)

Find a basis for $V \cap W$.

[10 points]

4. Let V be a vector space with basis { $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}$ } . Show that every vector $v \in V$ can be written **uniquely** as a linear combination

$$\mathbf{v} = c_1\mathbf{v_1} + c_2\mathbf{v_2} + \ldots + c_n\mathbf{v_n}$$

[10 points]

PART II. Circle the correct answer for each of the following problems (Problem 5 to Problem 9) IN THE TABLE OF THE FRONT PAGE . [4 points for each correct answer].

5. The value of the number k for which the matrix

$$A=\begin{pmatrix} 1 & 2 & 3 \\ 1 & k & 1 \\ 0 & k & 2 \end{pmatrix},$$

is not invertible is:

(a) k= 2
(b) k= 0
(c) k= -1/4
(d) k= 4
(e) none of the above

[4 points]

6. Let V= $\{p(x)\in P_3(R) \mid p''(0)=0\}$
Then dim V=

(a) 1
(b) 2
(c) 3
(d) 4
(e) none of the above.

[4 points]

7. Which one of the following statements is FALSE:

(a) Any set of 6 polynomials in P_4 is linearly dependent.
(b) Any linearly independent set of 5 polynomials in P_4 is a basis of P_4.
(c) Any set of 5 polynomials in P_4 that spans P_4 is a basis of P_4.
(d) Any set of 6 polynomials in P_4 spans P_4.
(e) none of the above.

[4 points]

8. The subspace of R^3 spanned by $S=\left\{\begin{pmatrix}1\\-2\\0\end{pmatrix},\begin{pmatrix}1\\1\\3\end{pmatrix}\begin{pmatrix}2\\-1\\3\end{pmatrix},\right\}$

has dimension:

(a) 1
(b) 2
(c) 3
(d) 4
(e) none of the above.

[4 points]

9. Let {**u, v**, **w**} be a basis for a vector space V. Let U=Span {**u**}, W=Span {**v,w**}, which one of the following statements is **FALSE**

(a) {2**u, v**, **3w**} is a basis for V.
(b) V=U+W
(c) {**u+w, u+v+w, v**}is basis of V
(d) $U\cap W=\{0\}$
(e) None of the above

[4 points]

PART III. Answer **TRUE** or **FALSE** only (3 points for each correct answer, 0 for no answer, and -1 point for each wrong answer):

[20 points]

A. The set of all 2×2 invertible matrices is a subspace of $M_{2\times2}$.

B. The set of polynomials $\{2+x, 3+4x^3, x+x^3, 1+x+x^2, x^2\}$ is linearly independent in P_3.

C. Let $W=\left\{\begin{pmatrix}a\\b\\1\end{pmatrix}\in R^3 \middle| \ a,b\in R\right\}$. Then W is a subspace of R^3.

D. Let $W=\left\{\begin{pmatrix}a & 0\\b & 3b\end{pmatrix}\in M_{2\times2} \middle| \ a,b\in R\right\}$. Then W is a subspace of $M_{2\times2}$ of dimension 2.

E. The space of all 3×3 upper triangular matrices has dimension 6.

F. Let U be the subspace of P_3 given by

$$U=\{p(x)=ax^3+bx^2+cx \ +d \ \in P_3(R) \mid p(0)=0 \ \ and \ \ p''(x)=0\}.$$

Then dim $U=2$.

G. If A, B, and C are n×n invertible matrices over R such that AB=AC, then B=C.

H. The set of all solutions of a nonhomogeneous system of m linear equations in n unknowns is a subspace of R^n.

I. If A and B are n×n matrices such that ABA^T is invertible, then A must be invertible.

J. If B is a 3×3 matrix such that $B^T = -B$, then B is not invertible

[20 points]