## EECE 290, Problem solving

Session 7
Review quiz 1

## Summary opamps

- Analysis with ideal opamps:
- $\mathrm{v}_{\mathrm{p}}-\mathrm{V}_{\mathrm{n}}=0$,
- $i_{p}=i_{n}=0$
$V_{\mathrm{CC}}^{-} \leqslant V_{0} \leqslant V_{\mathrm{CC}}^{+}$
- Implies negative feedback.



## Basic configurations

- Non-inverting:
- Unity-gain:
- Inverting:



## Basic configurations cont'd

- Adder:

- Difference amplifier:


$$
v_{0}=?
$$


A. 12 V
B. -12 V
C. 2 V
D. -2 V
E. 6 V
F. -6 V

## Complex power

using rms values: $S=V I^{*}=P+j Q=Z|I|^{2}=(R+j X)|I|^{2}=Y^{*}|V|^{2}=(G-j B)|V|^{2}$
$|S|=|V||I|=|Z||I|^{2}=|\mathrm{Y}||\mathrm{V}|^{2}$
$L: S=j Q=\left.j X| |\right|^{2}=\left.j \omega L| |\right|^{2}=j|V|| | \mid$
$C: S=j Q=-j B|V|^{2}=-j \omega C|V|^{2}=-j|V|| | \mid$
$R: S=P=R\| \|^{2}=G|V|^{2}=|V|\| \|$

power factor: $\cos \left(\theta_{v}-\theta_{i}\right)$ lagging or leading (current with respect to voltage)


Maximum power transfer: $Z_{L}=Z_{\text {src }}^{*}$
using rms values: $S=V I^{*}=P+\mathrm{jQ}=Z|I|^{2}=(R+j \mathrm{j})|I|^{2}=Y^{*}|V|^{2}=(G-\mathrm{jB})|V|^{2}$
A current source of $0.1 \mathrm{~A}_{\text {rms }}$, a capacitance of $-j 2 \Omega$ and an inductance of $j 4 \Omega$ are connected in series, what is the value of $S$ ?
A. j 20 mVA
B. -j 20 mVA
C. -j 2 mVA
D. j 2 mVA

Step function: $u(t)= \begin{cases}0 & t<0 \\ 1 & t>0\end{cases}$
Impulse (Dirac, delta) function: $\left\{\begin{array}{l}\delta(t)=0 \\ \int_{-\infty}^{\infty} \delta(t) d t=1\end{array} \quad t \neq 0\right.$
$\mathrm{u}(\mathrm{t})=\int_{-\infty}^{t} \delta(\tau) d \tau, \delta(\mathrm{t})=\mathrm{du}(\mathrm{t}) / \mathrm{dt}$

## Properties:

If t is time in s , then $\delta(\mathrm{t})$ has dimension of $\mathrm{s}^{-1}$ !
Sifting property: $\int_{a}^{b} f(t) \delta\left(t-t_{0}\right) d t= \begin{cases}f\left(t_{0}\right) & a<t_{0}<b \\ 0 & \text { elsewhere }\end{cases}$
Even symmetry: $\bar{\delta}(-\mathrm{t})=\bar{\delta}(\mathrm{t})$
Scaling: $\delta(\mathrm{at})=\frac{1}{|a|} \delta(t)$



## Laplace Transform

- Complex domain: $s=\sigma+j \omega ; \mathrm{e}^{\text {st }}$

$$
\begin{aligned}
& \mathcal{L}\{f(t)\}=\mathrm{F}(\mathrm{~s})=\int_{0^{-}}^{\infty} f(t) \mathrm{e}^{-\mathrm{st}} d t \\
& \mathfrak{L}^{-1}\{F(s)\}=f(t)=\frac{1}{2 \pi j} \int_{\sigma-j \infty}^{\sigma+j \infty} F(s) e^{s t} d s \\
& \mathcal{L}\{\delta(\mathrm{t})\}=1 ; \\
& \mathcal{L}\{\mathrm{u}(\mathrm{t})\}=1 / \mathrm{s} ; \\
& \mathcal{L}\left\{\mathrm{e}^{-\mathrm{at}}\right\}=1 /(\mathrm{s}+\mathrm{a})
\end{aligned}
$$

|  | TYPE | $f(t)\left(t>0^{-}\right)$ | F(s) |
| :---: | :---: | :---: | :---: |
|  | Impulse | $\delta(t)$ | 1 |
|  | Step | $u(t)$ | $\frac{1}{s}$ |
|  | Ramp | $t u(t)$ | $\frac{1}{s^{2}}$ |
|  | Exponential | $e^{-a t} u(t)$ | $\frac{1}{s+a}$ |
|  | Sine | $\sin \omega t u(t)$ | $\frac{\omega}{s^{2}+\omega^{2}}$ |
| $1$ | Cosine | $\cos \omega t u(t)$ | $\frac{s}{s^{2}+\omega^{2}}$ |
|  | Damped Ramp | $t e^{-a t} u(t)$ | $\frac{1}{(s+a)^{2}}$ |
|  | Damped Sine | $(\sin \omega t) \mathrm{e}^{-a t} u(t)$ | $\frac{\omega}{(s+a)^{2}+\omega^{2}}$ |
|  | Damped Cosine | $(\cos \omega t) e^{-a t} u(t)$ | $\frac{s+a}{(s+a)^{2}+\omega^{2}}$ |

## OPERATIONAL TRANSFORM

| Operation | $f(t)$ | $F(s)$ |
| :--- | :--- | :--- |
| Multiplication by a constant | $K f(t)$ | $K F(s)$ |
| Addition/subtraction | $f_{1}(t)+f_{2}(t)-f_{3}(t)+\cdots$ | $F_{1}(s)+F_{2}(s)-F_{3}(s)+\cdots$ |
| First derivative (time) | $\frac{d f(t)}{d t}$ | $s F(s)-f\left(0^{-}\right)$ |
| Second derivative (time) | $\frac{d^{2} f(t)}{d t^{2}}$ | $s^{2} F(s)-s f\left(0^{-}\right)-\frac{d f\left(0^{-}\right)}{d t}$ |
| $n$th derivative (time) | $\frac{d^{n} f(t)}{d t^{n}}$ | $s^{n} F(s)-s^{n-1} f\left(0^{-}\right)-s^{n-2} \frac{d f\left(0^{-}\right)}{d t}$ |
|  |  |  |
| Time integral | $\int_{0}^{t} f(x) d x$ | $-s^{n-3} \frac{d f^{2}\left(0^{-}\right)}{d t^{2}}-\cdots-\frac{d^{n-1} f\left(0^{-}\right)}{d t^{n-1}}$ |
| Translation in time | $f(t-a) u(t-a), a>0$ | $e^{-a s} F(s)$ |
| Translation in frequency | $e^{-a t f(t)}$ | $F(s+a)$ |
| Scale changing | $f(a t), a>0$ | $\frac{1}{a} F\left(\frac{s}{a}\right)$ |
| First derivative $(s)$ | $t f(t)$ | $-\frac{d F(s)}{d s}$ |
| $n$th derivative $(s)$ | $t^{n} f(t)$ | $(-1)^{n} \frac{d^{n} F(s)}{d s^{n}}$ |
| $s$ integral | $f(t)$ | $\int_{s}^{\infty} F(u) d u$ |

$$
\begin{aligned}
& \mathcal{L}\{\delta(t)\}=1 \\
& \mathcal{L}\{u(t)\}=\frac{1}{s} \\
& \mathcal{L}\left\{e^{-s t} u(t)\right\}=\frac{1}{s+a} \\
& \mathcal{L}\{f(t-a)\}=e^{-s s} F\{s\}
\end{aligned}
$$

$$
\mathscr{L}\{(t-1) u(t-1)\}=?
$$

A. $e^{-s} / s^{2}$
B. $e^{-s} / s$
C. $e^{-s} /(s+1)$
D. $1 /(\mathrm{s}-2)$
E. $1 / \mathrm{s}^{2}$
F. $\mathrm{e}^{-\mathrm{s}}$

## LT for circuits


$s L\}_{-}^{a}$
(a) Figure 22.1.4

(b)

(C)

- Convert circuit with initial conditions to s-domain.
- Apply circuit analysis.
- Apply ILT to result
(in s domain) to get expression in time domain.




An inductance of 2 H has an initial current of 4 A . In the sdomain circuit this is modeled by a voltage source

(a) Figure 22.1.4 (b)

(c)
A. of $4 / \mathrm{s}$ in parallel
B. of $4 / \mathrm{s}$ in series
C. of $2 / \mathrm{s}$ in parallel
D. of $2 / \mathrm{s}$ in series
E. of 8 in parallel
F. of 8 in series
$\mathrm{C}=0.5 \mathrm{~F}$. Right before switching $\mathrm{V}_{\mathrm{o}}=8 \mathrm{~V}$. The value of the voltage source should be
A. $8 / \mathrm{s}$
B. 8
C. 4
D. $4 / \mathrm{s}$
E. -8
F. -4
$\mathrm{C}=0.5 \mathrm{~F}$. Right before switching $\mathrm{V}_{\mathrm{o}}=8 \mathrm{~V}$. The value of the current source should be
A. $8 / \mathrm{s}$
B. 8
C. 4
D. $4 / \mathrm{s}$
E. -8
F. -4
$\mathrm{C}=0.5 \mathrm{~F}, \mathrm{R}=4 \Omega$. Right before switching $\mathrm{V}_{\mathrm{o}}=8 \mathrm{~V}$. $\mathrm{I}(\mathrm{s})=$
A. 2
B. $2 / \mathrm{s}$
C. $2 /(s+2)$
D. $2 /(\mathrm{s}+1 / 2)$
E. $4 /(s+1 / 2)$
F. $8 /(\mathrm{s}+2)$
$\mathrm{C}=0.5 \mathrm{~F}, \mathrm{R}=4 \Omega$. Right before switching
$V_{o}=8 \mathrm{~V}$. $i(t)=$

A. $4 \mathrm{e}^{\mathrm{t} / 2}$
B. $2 \mathrm{e}^{-\mathrm{t} / 2}$
C. $2 e^{-2 t}$
D. $4 e^{-2 t}$

P13.2.20 Determine $V_{o}$ in
Figure
P13.2.20.


P17.1.27 Determine the complex power delivered by $\mathbf{V}_{\text {SRC } 1}$ and $\mathbf{V}_{\text {SRC2 }}$ in Figure P17.1.27 given that $L_{1}$ absorbs 4 kW at a power factor of 0.6 lagging, $L_{2}$ absorbs 3 kW at a power factor of 0.6 leading, and the complex power absorbed by $L_{3}$ is $12+j 5 \mathrm{kVA}$. Assume that $\mathbf{V}_{\mathrm{SRC} 1}=400 \angle 0^{\circ} \mathrm{V} \mathrm{rms}$ and $V_{\text {SRC } 2}=400 \angle 90^{\circ} \mathrm{V}$ rms.


Figure P17.1.27

## $\mathbf{P}$ 12.2.9 Determine the LT of the derivative of $f(t)$ in Figure

 P21.2.9.

Determine the ILT of $F(s)=\frac{8 s^{2}+4 s+6}{(2 s+3)(s+4)}$


Figure P22.2.5

## P22.2.5 Both

switches in
Figure
P22.2.5 are opened at $t$
$=0$ after
being
closed for a long time. Determine $i_{o}(t)$.

Ans. $\quad 2\left(1-\frac{1}{s}\right)+\frac{2 e^{-2 s}}{s}\left(2-e^{-s}\right)$.
Ans. $\quad 4 \delta(t)+\left(3.6 e^{-1.5 t}-23.6 e^{-4 t}\right) \mu(t)$.

Ans. $\quad 10 e^{-5 t} \mathrm{~mA}, t$ is in ms .

