- MATH 201
- EXAM 2 MATERIAL

Solution to Collected Series problems from prev courses

① $\sum_{n=0}^{\infty} (a-8)^n$

Solution: This is simply a geometric series with $r=(a-8)$ and 1^{st} term $=1$ so we need $|r|<1$, meaning if $|a-8|<1$ then the series converge. So for $-1<a-8<1$ or $\boxed{\text{for } 7<a<9}$, series converge

② Determine Conv or Div :

a) $\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^n} = \sum_{n=0}^{\infty} \left(\frac{-2}{3}\right)^n$

thoughts: 1^{st} check: geometric or telescoping : geometric ✓
→ ends here :)

solution: This a geometric series wit $r=-2/3$ so $|r|<1 \Rightarrow$ it converge

b) $\sum_{n=0}^{\infty} \left(1-\frac{3}{n}\right)^{5n}$

thoughts: 1^{st} check: Not geometric Nor telescoping
2^{nd} check: n^{th} term test works here ✓

Solution: $\lim_{n\to\infty} \left(1-\frac{3}{n}\right)^{5n}$

$= \lim_{n\to\infty} \left[\left(1-\frac{3}{n}\right)^n\right]^5 = (e^{-3})^5 \neq 0$

$\Rightarrow$ Diverge by n^{th} term test.

c) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

thoughts: 1^{st} check: telescoping ✓
(write $\ln\left(\frac{n}{n+1}\right) = \ln n - \ln n+1$)

$S_n = \sum_{k=1}^{n} \ln\left(\frac{k}{k+1}\right) = \sum_{k=1}^{n} \{\ln k - \ln(k+1)\}$

$= \ln 1 - \cancel{\ln 2}$
$\cancel{\ln 2} - \cancel{\ln 3}$
$\vdots$
$\cancel{\ln(n-2)} - \cancel{\ln(n-1)}$
$\cancel{\ln(n-1)} - \ln n$

$= \ln 1 - \ln n = -\ln n$

then $\lim_{n\to\infty} S_n = \lim_{n\to\infty} -\ln n = -\infty \Rightarrow$ Series diverge

③ Find the sum of the series $\sum_{n=1}^{\infty} \frac{9}{n(n+3)}$.

<u>thoughts</u>: when the Q is find the sum, this means we can compute $S_n \Rightarrow$ either geom or telescoping.
- this is a telescoping series

<u>Solution</u>:

- We first use partial fractions to make it look like a telescoping series

$$\sum_{1}^{\infty} \frac{9}{n(n+3)} = \sum_{1}^{\infty} \frac{A}{n} + \frac{B}{n+3}$$, we find $A = 3$, $B = -3$

$$\Rightarrow \sum_{1}^{\infty} \frac{9}{n(n+3)} = \sum_{1}^{\infty} \frac{3}{n} - \frac{3}{n+3} .$$

Then we find the n^{th} partial sum S_n:

$$S_n = \sum_{k=1}^{n} \frac{3}{k} - \frac{3}{k+3} = 3 + 3/2 + 1 - \frac{3}{n+1} - \frac{3}{n} - \frac{3}{n+3}$$

$$\begin{aligned}
= &\frac{3}{1} - \cancel{\frac{3}{4}} \\
&\frac{3}{2} - \cancel{\frac{3}{5}} \\
&\frac{3}{3} - \cancel{\frac{3}{6}} \\
&\cancel{\frac{3}{4}} - \cancel{\frac{3}{7}} \\
&\cancel{\frac{3}{5}} - \cancel{\frac{3}{8}} \\
&\cancel{\frac{3}{6}} - \cancel{\frac{3}{9}} \\
&\quad \vdots \\
&\cancel{\frac{3}{n-2}} - \frac{3}{n+1} \\
&\cancel{\frac{3}{n-1}} - \frac{3}{n} \\
&\cancel{\frac{3}{n}} - \frac{3}{n+3}
\end{aligned}$$

$$\Rightarrow \lim_{n\to\infty} S_n = 4 + 3/2$$

so $\boxed{\sum_{1}^{\infty} \frac{9}{n(n+3)} = 4 + 3/2}$

④ Determine convergence or divergence.

· In case alternating, determine absolute convergence

(a) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n^{0.9}\ln n}$

thoughts: this is alternating,

- first test absolute convergence.

So look at $\sum_{n=2}^{\infty} \frac{1}{n^{0.9}\ln n}$

now we have many ways to do this.

1) We can use a result from class (a HWK problem) that says that $\sum \frac{(\ln n)^q}{n^p}$

→ Converges if $p>1$ - any q

→ Diverges if $p<1$ any q

→ test $p=1$. INTEGRAL TEST

2) We can use comparisons (DCT or LCT) because the series has functions with different rates of growth

3) integral ~~test~~ ~~bad~~ here is hard so NOT a good idea

Solution:

→ First test for $\sum_{n=2}^{\infty} \left|(-1)^n \frac{1}{n^{0.9}\ln n}\right|$

$= \sum_{n=2}^{\infty} \frac{1}{n^{0.9}\ln n}$

this looks like $\sum \frac{(\ln n)^q}{n^p}$

with $p = 0.9$ & $q = -1$ see

So the series does not converge absolutely.

→ Second, without absolute value this is a conditionally convergent series because $u_n = \frac{1}{n^{0.9}\ln n}$

- is >0
- $\to 0$ as $n \to \infty$
- Decreases because denominator increases.

So the Leibniz test applies, series is conditionally convergent.

Remark: for absolute convergence we can also do

1) DCT on $\sum \frac{1}{n^{0.9}\ln n}$ with $n^{0.9}\ln n < n^{0.9} n^{0.01}$

$\Rightarrow \sum \frac{1}{n^{0.9}\ln n} > \sum \frac{1}{n^{0.91}}$

or

2) LCT with $\sum \frac{1}{n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{5\sqrt{n} + 9(\ln n)^2}$

thoughts: not alternating, purely +ve.
just test regular convergence.
→ Not geom or telescoping
→ n^{th} term fails
→ think of dominant terms
use DCT or LCT
($\sqrt{n}$ dominates so suspect divergent)
→ Integral hard here

Solution:

$5\sqrt{n} + 9(\ln n)^2 \leq 9\sqrt{n} + 9\sqrt{n}$

(because $\ln n < n^{0.25} \Rightarrow (\ln n)^2 < n^{0.5}$)

$\Rightarrow \sum \frac{1}{5\sqrt{n} + 9(\ln n)^2} \geq \sum \frac{1}{18\sqrt{n}} \geq 0$

p-series $p = \frac{1}{2}$

$\Rightarrow$ By DCT, series diverge.

(c) $\sum_{n=1}^{\infty} \frac{e^{-4n^2}}{n^2}$

thoughts: +ve terms
not geom, not telescoping,
nth term fails.
→ Looks like we can compare with p-series ~> DCT or LCT

Solution: $0 \leq \sum_{n=1}^{\infty} \frac{e^{-4n^2}}{n^2} \leq \sum_{n=1}^{\infty} \frac{1}{n^2}$

Conv. p-series

$\Rightarrow$ By DCT, series converge

(d) $\sum_{n=1}^{\infty} \frac{1}{(\ln 10n)^n}$

thoughts: Same as above
So Remaining ~~tests~~ are
LCT, DCT or root ratio, integral
pick root test because
it's $(\text{something})^n$ (here comparisons fail because it's NOT a combination of different functions)

$\varphi = \lim_{n \to \infty} \sqrt[n]{\left(\frac{1}{\ln 10n}\right)^n}$

$= \lim_{n \to \infty} \frac{1}{\ln(10n)} = 0 < 1$

$\Rightarrow$ Series converge by ROOT TEST.

(e) $\sum_{n=1}^{\infty} \frac{n}{(n^{1/n}+8)^n}$

thoughts: +ve terms

Root or Ratio test, (comparisons not straight forward)

Solution:

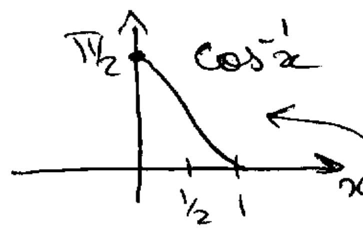
Root test:

$$\lim_{n\to 0} \sqrt[n]{\frac{n}{(n^{1/n}+8)^n}} = \lim_{n\to\infty} \frac{n^{1/n}}{n^{1/n}+8} = \lim_{n\to\infty} \frac{\cancel{n^{1/n}}}{\cancel{n^{1/n}}(1+\frac{8}{n^{1/n}})} = \frac{1}{9} < 1$$

$\Rightarrow$ Series converge

(f) $\sum_{n=1}^{\infty} 8\cos^{-1}(1/n)$

thoughts:



Solution:

$$\lim_{n\to\infty} 8\cos^{-1}(1/n) = 8 \cdot \pi/2 \neq 0$$

since $\lim_{n\to\infty} 1/n = 0$ & $\cos^{-1}$ continuous and $\cos^{-1} 0 = \pi/2$ (see graph)

$\Rightarrow$ n^{th} term test implies series diverge.

series of positive terms, $0 < 1/n \le 1$, $\cos^{-1}(1/n) \geq 0$

$\rightarrow$ not telescop nor geometric

$\rightarrow$ n^{th} term works ✓

(5) Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt[n]{n}}$

Solution: Apply ratio test to $\sum \frac{|x-2|^n}{\sqrt[n]{n}}$

$$\lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{\sqrt[n+1]{n+1}} \cdot \frac{\sqrt[n]{n}}{(x-2)^n} \right| = \lim_{n\to\infty} \underbrace{\frac{\sqrt[n]{n}}{\sqrt[n+1]{n+1}}}_{=1} \cdot |x-2| = |x-2|$$

If $|x-2| < 1 \Rightarrow$ Series is absolutely convergent $\Rightarrow$ Series converge there

so $-1 < x-2 < 1 \Rightarrow \boxed{\text{for } 1 < x < 3}$ series converge

2) Test end points: at $x=1$ & $x=3$

at $x=1$, get

$$\sum_{n=1}^{\infty} \frac{(1-2)^n}{\sqrt[n]{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[n]{n}}$$ Diverges since $\lim_{n\to\infty} \frac{(-1)^n}{\sqrt[n]{n}} = \pm 1$ DNE

by n^{th} term test:

at $x=3$, get

$$\sum_{n=1}^{\infty} \frac{(3-2)^n}{\sqrt[n]{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n}}$$ Diverges by n^{th} term test

$$\lim_{n\to\infty} \frac{1}{\sqrt[n]{n}} = 1$$

So the interval of convergence is $\boxed{(1,3)}$, i.e.

for $\boxed{1 < x < 3}$ converge

6) Which of the series converge conditionally, absolutely or diverge?

→ Here for alternating series we 1st test absolute conv if test works we stop & conclude absolute & cond. conv
If test fails, we test conditional convergence.

→ for series of +ve terms, we just test regular convergence.

(a) $\sum_{n=2}^{\infty} (-1)^{n+1} n^2 \left(\frac{3}{4}\right)^n$

Solution:

$\sum_{n=2}^{\infty} \left|(-1)^{n+1} n^2 (3/4)^n\right|$

$= \sum_{n=2}^{\infty} n^2 (3/4)^n \leftarrow u_n$

$\rho = \lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^2 \cdot (3/4)^{n+1} \cdot (3/4)^{-n}$

$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^2 \cdot (3/4)$

$= 3/4 < 1 \Rightarrow$ series is absolutely convergent ($\Rightarrow$ also convergent)

thoughts: alternating series

1) look at absolute value $\sum_{n=2}^{\infty} n^2 (3/4)^n$

→ neither geom, telesc,
→ n^{th} term fails, $(3/4)^n \to 0$ faster than n^2 grows
→ LCT, DCT or root & ratio
→ Pick root or ratio simpler...
→ get convergence ✓
→ stop here because it's abs. convergent

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

Solution:

Series is NOT abs. convergent since

$\sum_{n=2}^{\infty} \frac{1}{\ln n} \geq \sum \frac{1}{n}$ → p series $p=1$ $\quad \ln n < n$

$\Rightarrow$ By DCT, it diverges.

thoughts: Alternating

1) look at $\sum \frac{1}{\ln n}$ first ✗
DCT with $\sum \frac{1}{n} \Rightarrow$ Diverges

2) look at $\sum \frac{(-1)^n}{\ln n}$
all conditions of Leibniz series are ✓
$\Rightarrow$ Conditionally Convergent

But $\sum \frac{(-1)^n}{\ln n}$ converges conditionally

by Leibniz's test

1) $\frac{1}{\ln n} > 0$

2) $\frac{1}{\ln n} \to 0$

3) $\frac{1}{\ln n}$ decreasing as $\ln n \nearrow$.

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^2}$

thoughts:
the sign changes due to $(-1)^n$ AND $\sin n$.
1) Test for abs convergence first...
• when we see $\sin n$ we think either $0 \le |\sin n| \le 1$ or $0 \le |\sin n| \le n$
here 1st option work

Solution: Series is absolutely convergent since

$$\sum \left|\frac{(-1)^n \sin n}{n^2}\right| = \sum \frac{|\sin n|}{n^2}$$

Now $0 \le \sum \frac{|\sin n|}{n^2} \le \sum \frac{1}{n^2}$ ← p series $p=2$

$\Rightarrow$ DCT implies that $\sum \frac{|\sin n|}{n^2}$ converges

$\Rightarrow$ original series $\sum \frac{(-1)^n \sin n}{n^2}$ is absolutely convergent.

(d) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{3n}\right)^n$

thoughts: series of positive terms.
n^{th} term test works

Solution: $\lim_{n\to\infty} \left(1-\frac{1}{3n}\right)^n = \lim_{n\to\infty} \left[\left(1-\frac{1}{3n}\right)^{3n}\right]^{\frac{1}{3}}$

$$= (e^{-1})^{1/3} \neq 0$$

Diverges by n^{th} term test

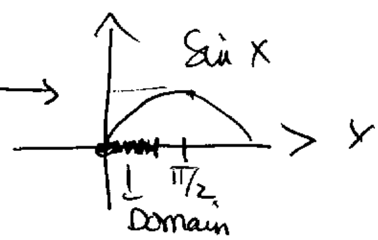
⑦ (a) $\sum \frac{(2011)^n}{n^n}$

Series of positive terms so absolute convergence is same as convergence.

<u>Root test:</u> $\rho = \lim\limits_{n\to\infty} \sqrt[n]{\frac{(2011)^n}{n^n}} = \lim\limits_{n\to\infty} \frac{2011}{n} = \text{zero} < 1$

the series converge.

(b) $\sum\limits_{n=1}^{\infty} \underbrace{\sin\left(\frac{1}{n}\right)}_{>0} \longrightarrow$



LCT with $\sum \frac{1}{n}$

let $a_n = \sin\frac{1}{n}$ $\quad b_n = \frac{1}{n}$

$\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \lim\limits_{n\to\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}} = 1 \qquad \left(\lim\limits_{x\to 0} \frac{\sin x}{x} \geq 1\right)$

$\Rightarrow$ Series behave like $\sum \frac{1}{n} \Rightarrow$ Divergent

(c) $\sum \frac{(-1)^n}{\ln n}$ conditionally convergent

1) $u_n = \frac{1}{\ln n} > 0$, 2) $u_n \to 0$

3) and u_n decreases as $\ln n \nearrow$ increases.

$\left[\sum \left|\frac{(-1)^n}{\ln n}\right| = \sum \frac{1}{\ln n} > \sum \frac{1}{n} \text{ Divergent by DCT}\right]$

(d) $\sum (-1)^n \frac{\sin n}{n^2}$

Test absolute value $\sum \frac{|\sin n|}{n^2} < \sum \frac{1}{n^2}$ ($p=2$) converges by DCT

$\Rightarrow$ Series is absolutely convergent.

(7) (e) $\sum_{n=1}^{\infty} \left(1-\frac{1}{3n}\right)^n$

$$\lim_{n\to\infty}\left(1-\frac{1}{3n}\right)^n = \lim_{n\to\infty}\left(\left(1-\frac{1}{3n}\right)^{3n}\right)^{1/3}$$

$$(e^{-1})^{1/3} \neq 0$$

Series diverge by n^{th} term test.

(8) Find radius of convergence

$$\sum_{n=1}^{\infty} \frac{n^n x^n}{n!}$$

$$\rho = \lim_{n\to\infty} \left| \frac{(n+1)^{n+1} x^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n \cdot x^n} \right|$$

$$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n \cdot \frac{\cancel{(n+1)}}{\cancel{(n+1)}} \cdot |x|$$

$$\left(1+\frac{1}{n}\right)^n$$

$$= e^{-1}|x|$$

$$\rho<1 \Rightarrow e^{-1}|x|<1$$

$$\Rightarrow |x|<e$$

$\Rightarrow$ for $\boxed{-e<x<e}$ → series is absolute convergent

No matter what the end points are, $\boxed{R=e}$

⇝ for curiousity at $x=-e$, $\sum_{n=1}^{\infty} \frac{n^n(-e)^n}{n!}$

$\lim_{n\to\infty} (-1)^n \cdot e^n \cdot \frac{n^n}{n!} = \pm\infty$ DNE by n^{th} term diverges

at $x=e$ $\sum \frac{n^n e^n}{n!}$ diverges by n^{th} term test

(9) Find the sum

1) $\sum_{n=1}^{\infty} \frac{2^n}{3^{n-1}} = \sum_{n=1}^{\infty} 3\left(\frac{2}{3}\right)^n$

thoughts:
Geometric, just put it in ~~so~~ right form to find exact sum

We know $\sum_{\boxed{n=0}}^{\infty} 3 \cdot (2/3)^n = \frac{3}{1-2/3}$

So $3 + \sum_{n=1}^{\infty} 3(2/3)^n = \frac{3}{1-2/3}$

$\Rightarrow \sum_{n=1}^{\infty} 3 \cdot (2/3)^n = \boxed{\frac{3}{1-2/3} - 3}$

2) Either there's a ~~typo~~ mistake in this question and it should be $\sum \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$ or it's a trick question!

I) If there's a typo: $S_n = \sum_{k=1}^{n} \frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}$

$= \frac{1}{1} - \cancel{\frac{1}{\sqrt{2}}} + \cancel{\frac{1}{\sqrt{2}}} - \cancel{\frac{1}{\sqrt{3}}} + \cancel{\frac{1}{\sqrt{3}}} \cdots + \cancel{\frac{1}{\sqrt{n-2}}} - \cancel{\frac{1}{\sqrt{n-1}}} + \cancel{\frac{1}{\sqrt{n-1}}} - \frac{1}{\sqrt{n}}$

$= 1 - \frac{1}{\sqrt{n}} \longrightarrow 1$

OR

II trick question because it's NOT telescoping

$\sum \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n}+1}$

If you write $S_n = \frac{1}{1} - \frac{1}{\sqrt{1}+1} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}+1} \cdots \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n}+1}$

can't cancel!

But $\sum \frac{\sqrt{n}+1-\sqrt{n}}{\sqrt{n}(\sqrt{n}+1)} = \sum \frac{1}{\sqrt{n}(\sqrt{n}+1)} = \sum \frac{1}{n+\sqrt{n}} \geq \sum \frac{1}{2n}$

diverges By DCT

(10)

(a) $\sum_{n=0}^{\infty} \frac{n^n}{n!}$

~~Ratio test~~ $\cancel{\rho = \lim_{n\to\infty} \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n} = \lim \frac{1}{n+1} = 0 < 1}$

~~⇒ converges by Ratio test~~

→ $\lim \frac{n^n}{n!} = +\infty$

diverges by n^{th} term test

~~thoughts: the terms n^{th} term fails~~ ~~When we see factorials & powers ⇒ Think ratio or Root~~

(b) $\sum \left(\frac{n}{n+1}\right)^n$

$$\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n$$

$$\lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right)^n = \lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right)^{n+1} \left(1 - \frac{1}{n+1}\right)^{-1}$$

$$= e^{-1} \neq 0$$

Diverges by n^{th} term test

(c) $\sum_{n=1}^{\infty} \frac{n \sin(1/n)}{n^2+1}$

thoughts: n^{th} term test fails
comparison, integral or Root / ratio
→ See $\boxed{\sin(1/n)}$ ← think DCT ~~with 1/n~~
Since $\sin 1/n \leq 1/n$

$0 \leq \sin 1/n \leq 1/n$

$$\Rightarrow 0 \leq \sum \frac{n \sin(1/n)}{n^2+1} \leq \sum \frac{n \cdot 1/n}{n^2+1} = \sum \frac{1}{n^2+1}$$ ← converges

Either integral test or another DCT with $\sum \frac{1}{n^2}$

⇒ Series converge by DCT.

(d) Did in class, see notes

10

e) $\sum_{n=1}^{\infty} \frac{1}{n^2-\sqrt{n}}$

LCT with $\sum \frac{1}{n^2}$

$$\lim_{n\to\infty} \frac{1}{n^2-\sqrt{n}} \cdot n^2 = \lim_{n\to\infty} \frac{1}{1-\frac{\sqrt{n}}{n^2}} = 1$$

$\Rightarrow$ Original Series converge

f) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{2n^2}$

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n^2} = \lim_{n\to\infty}\left[\left(1-\frac{1}{n}\right)^n\right]^{2n} = \left(e^{-1}\right)^{2n} \to 0$$

n^{th} term test fails

But can apply root test:

$$\lim_{n\to\infty} \sqrt[n]{\left(1-\frac{1}{n}\right)^{2n^2}} = \lim_{n\to\infty}\left[\left[\left(1-\frac{1}{n}\right)^n\right]^{2n}\right]^{1/n}$$

$$= \lim_{n\to\infty}\left(\left(1-\frac{1}{n}\right)^n\right)^2 = e^{-2} < 1$$

$\Rightarrow$ Series converge.

(11) $\sum_{n=1} n(1+n^2)^p$. for what p does series converge? Verify with integral test.

(1) case $p \geq 0$, Series diverge by n^{th} term test, because $\lim_{n\to\infty} n(1+n^2)^p = \infty$ for $p \geq 0$

(2) case $p<0$: it's easier to analyze this with LCT first, then do the integral test to verify. We think of the dominant terms:

$$\frac{n}{(n^2+1)^{|p|}} \sim \frac{n}{(n^2)^{|p|}} = \frac{1}{n^{2|p|-1}}$$ so we do LCT with $\sum \frac{1}{n^{2|p|-1}}$

let $a_n = \frac{n}{(n^2+1)^{|p|}}$ Let $b_n = \frac{1}{n^{2|p|-1}}$

$\lim_{n\to\infty} a_n/b_n = \lim_{n\to\infty} \frac{n}{(1+n^2)^{|p|}} \cdot n^{2|p|-1} = 1$ so series behave alike.

Now $\sum \frac{1}{n^{2|p|-1}}$ is a p-series so we can directly say

$\Rightarrow$ it converges if $2|p|-1>1 \Rightarrow |p|>1$ but remember we assumed $p<0 \Rightarrow \boxed{\text{for } p<-1}$, the series converge.

We can now verify with the integral test

Let $p<-1$, let $f(x) = \frac{x}{(1+x^2)^{|p|}}$, $|p|>1$, $x \geq 1$

then $f'(x) = (1+x^2)^{-|p|}\left\{\frac{1+x^2-2|p|x^2}{1+x^2}\right\}$ has the same sign as $1+x^2-2|p|x^2$. Well for $|p|>1$ and $x\geq 1$, $1+x^2-2|p|x^2 < 1-x^2 \leq 0$.

then $1+x^2-2|p|x^2 \leq 0$ because

so f is decreasing.

Now $\int \frac{x}{(1+x^2)^{|p|}}dx = \int \frac{du}{u^{|p|}} = \frac{1}{1-|p|} \cdot u^{-|p|+1}$

$= \frac{1}{1-|p|}(1+x^2)^{-|p|+1}$

and this converges if $\lim_{b\to\infty} \int_1^b \frac{x}{(1+x^2)^{|p|}}dx = \lim_{b\to\infty} \frac{1}{1-|p|}(1+b^2)^{1-|p|}$ is finite. So we need $1-|p|<0 \Rightarrow 1<|p|$

agrees with LCT result!