Sample for review....

Calculus III ......Hamdan, Fall 2011

1. Integrate the following

   (a) $\int \frac{x\,dx}{4+9x^4}$

   (b) $\int \cos^{-1} x\,dx$

2. Simplify

   (a) $\cos\left(\tan^{-1}\left(\frac{5}{\sqrt{x}}\right)\right)$.

   (b) $10\cosh(lnx)$

   Evaluate the following (improper) integrals:

   (a) $\int_0^\infty \frac{2dx}{4+x^2}$

   (b) $\int_8^\infty \frac{dx}{(x-7)(x-6)}$

3. Determine whether the following improper integrals converge or diverge:

   (a) $\int_1^\infty \frac{\sqrt{x+2}}{x^5}dx$

   (b) $\int_0^3 \frac{dx}{\sqrt{3-x}}$

   (c) $\int_0^\infty \frac{\sqrt{x}}{1+x^2}dx$

   (d) $\int_1^\infty \frac{|\ln x|}{x}dx$

   (e) $\int_1^\infty \frac{\ln x}{(x+\sqrt{x-1})}dx$

   (f) $\int_1^\infty \frac{e^x}{\sqrt{1+x^2}}dx$

   (g) $\int_0^\infty \frac{|\sin x|}{1+x^2}dx$

4. Find the value of $p > 0$ for which the integral $\int_2^\infty \frac{1}{x\,(\ln x)^p}dx$ converges.

5. Determine whether the improper integrals converge or diverge:

   (a) $\int_{-\infty}^{-5} \frac{8}{x^3}dx$

   (b) $\int_1^\infty \frac{e^x}{\sqrt{1+x^2}}dx$

6. Determine whether the following series converges or diverges. Name the test you are using:

   (a) $\int_1^\infty \frac{2x^2-3x+\ln x}{2x-1}dx$

   (b) $\int_1^\infty \frac{e^{-x}}{x^3}dx$

   (c) $\int_1^\infty \frac{\ln x}{x^{1.5}}dx$

   (d) $\int^0 \frac{e^{-x}}{[illegible]}dx$

(d) $\displaystyle\int_0^{\pi} \frac{dx}{x+\sin x}$

9. Sort from slowest to fastest the following functions: No need to justify:

$$x^e, \quad e^x, \quad (\ln x)^x, \quad \ln(x^x), \quad \ln(x^2)$$

Consider the improper integral $\displaystyle\int_1^{\infty} \frac{dx}{(x^2+1)^3}$.

(a) Determine whether it converges or diverges, **using the comparison test.**

(b) What can you say about $\displaystyle\int_0^{\infty} \frac{dx}{(x^2+1)^3}$?