STAT 230

Lecture 4
Conditional Probability Examples

Naima Hammoud

June 12, 2019

Monty Hall Solution

Monty Hall Solution

$C1$ = car behind door 1
$C2$ = car behind door 2
$C3$ = car behind door 3

$D1$ = host opens door 1
$D2$ = host opens door 2
$D3$ = host opens door 3

Monty Hall Solution

C1 = car behind door 1

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$P(C1) = P(C2) = P(C3) = 1/3$

Monty Hall Solution

C1 = car behind door 1

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D1 = host opens door 1

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$P(C1) = P(C2) = P(C3) = 1/3$

I choose door 1, so now I know the host has to open either door 2 or door 3 with equal probability, but the host knows where the car is, so:

Monty Hall Solution

$C1$ = car behind door 1

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$P(D2|C1) = 0.5, \quad P(D2|C2) = 0, \quad P(D2|C3) = 1$

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$P(D2|C1) = 0.5$, $P(D2|C2) = 0$, $P(D2|C3) = 1$

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- Host opens D3 and it has a goat

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- Compare $P(C1|D3)$ and $P(C2|D3)$

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1/3 **2/3**

Monty Hall Solution

SWITCH TO DOOR 2

$$P(C1|D3) = \frac{P(D3|C1)P(C1)}{P(D3)}$$

1/3

$$P(C2|D3) = \frac{P(D3|C2)P(C2)}{P(D3)}$$

2/3

Plane Detectors

Radar Example

Suppose that the probability you see a plane when you look up to the sky is 5%, and let's say that we have a radar which registers a plane with 99% accuracy. There is also a 10% chance for the radar to sound a false alarm.

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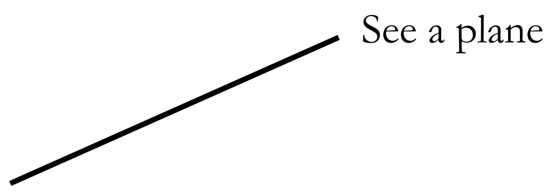

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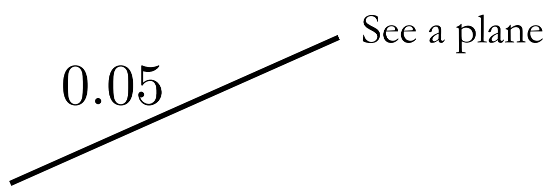

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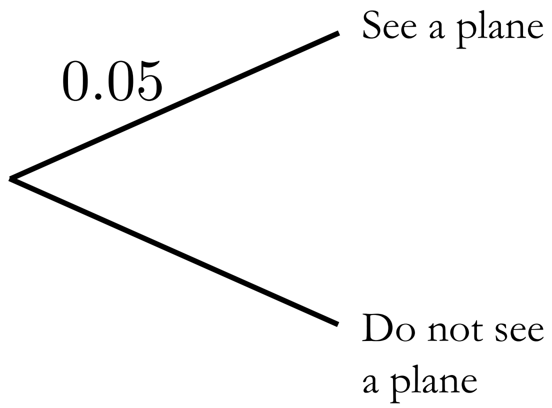

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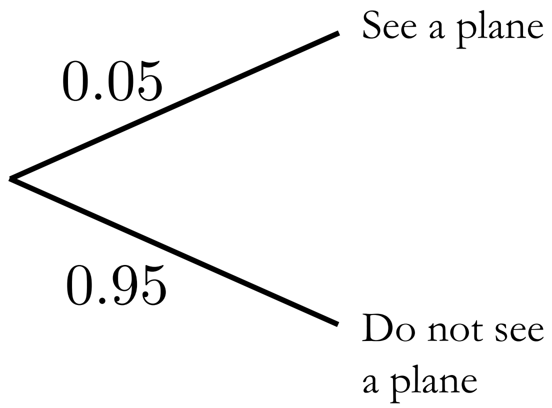

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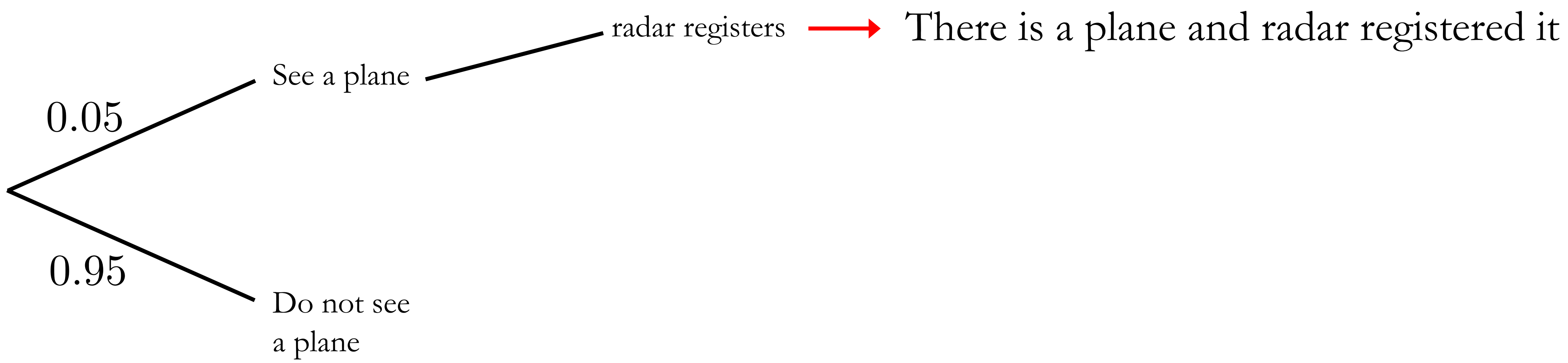

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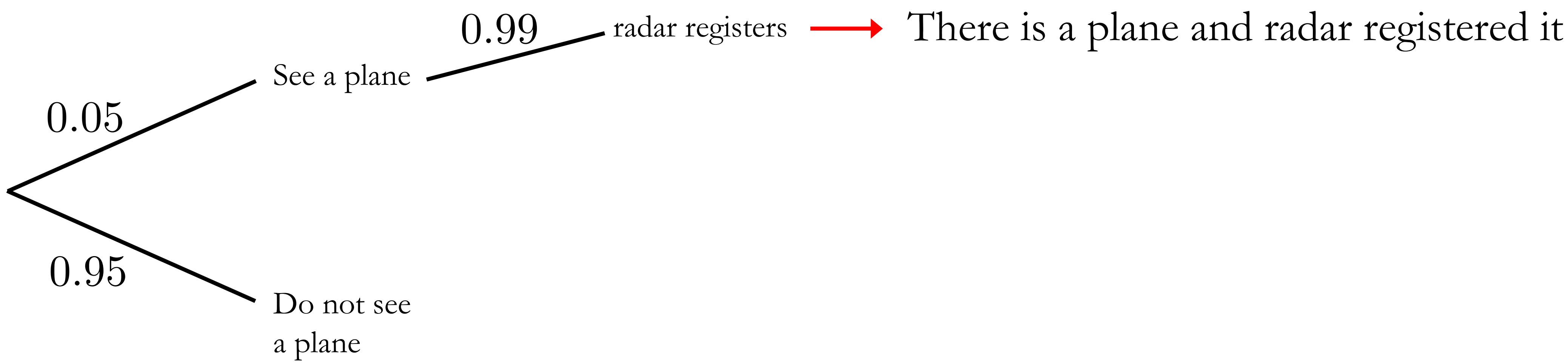

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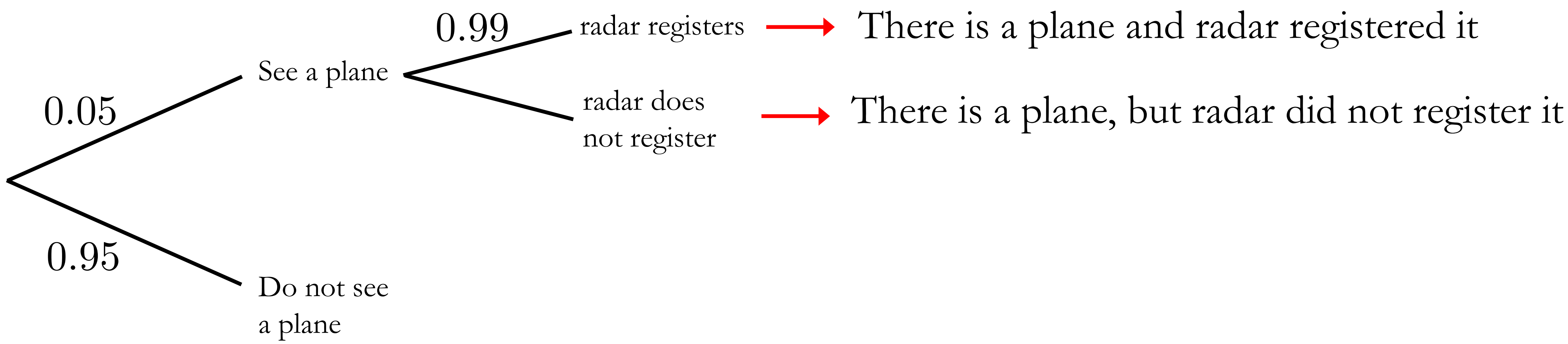

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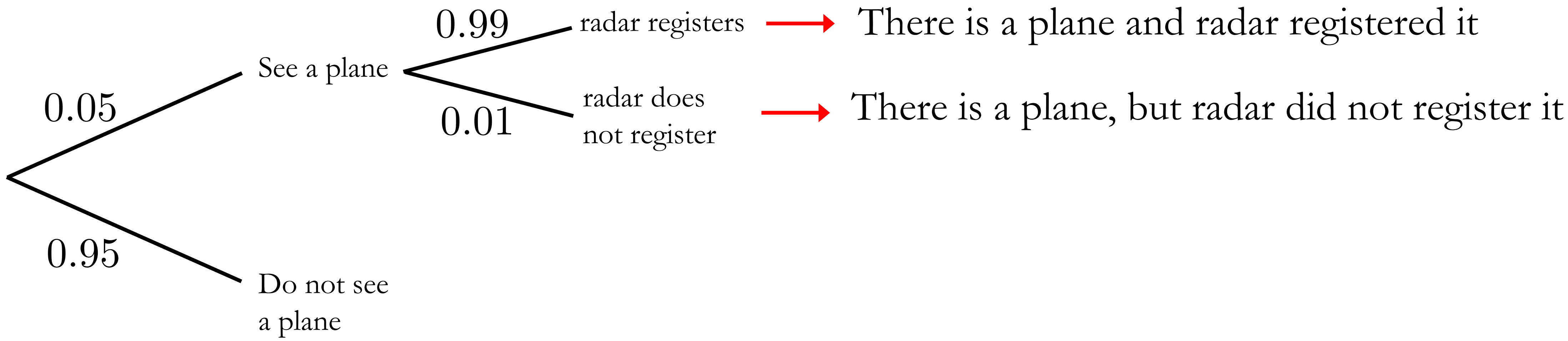

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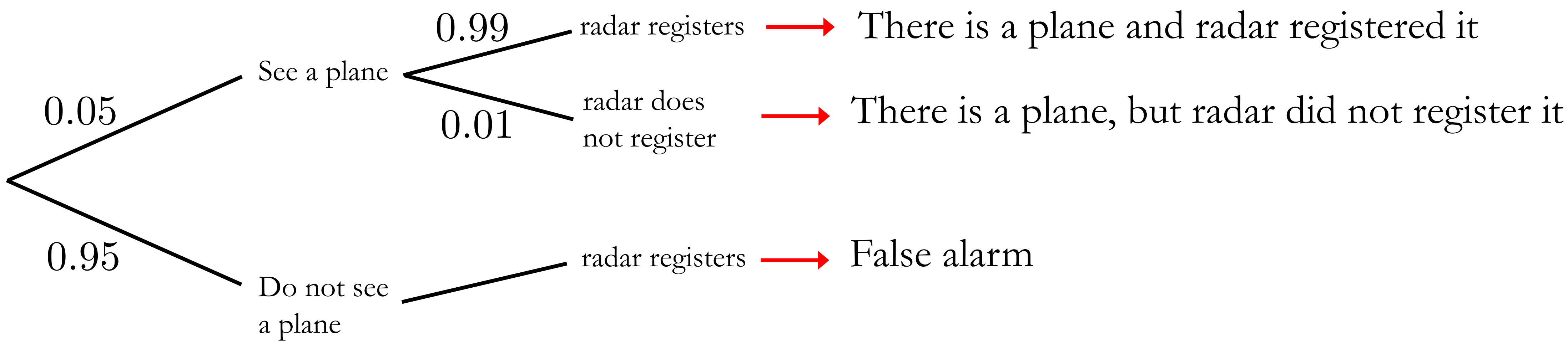

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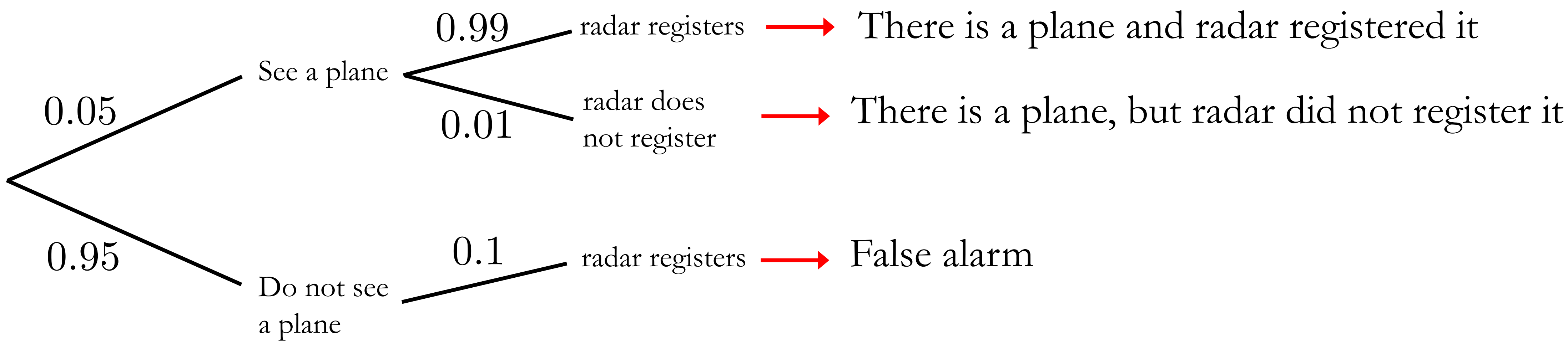

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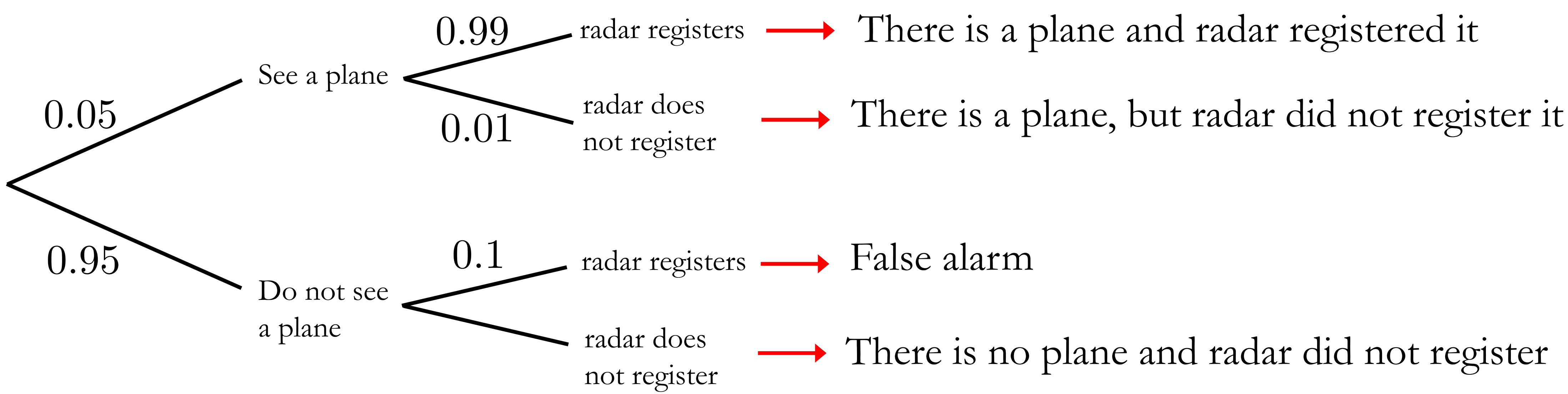

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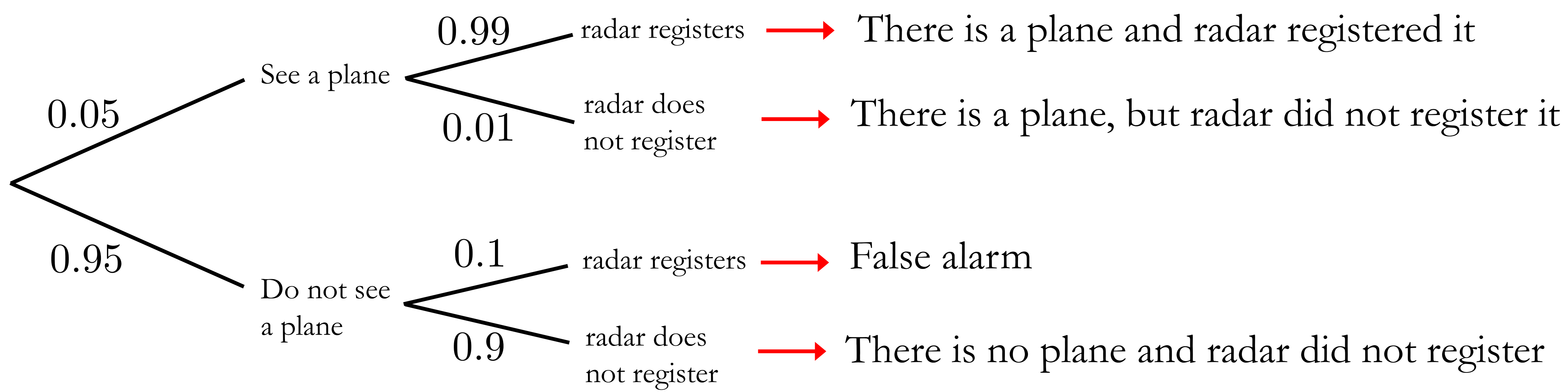

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We could rename the events for simplicity:

- A = event that we see a plane
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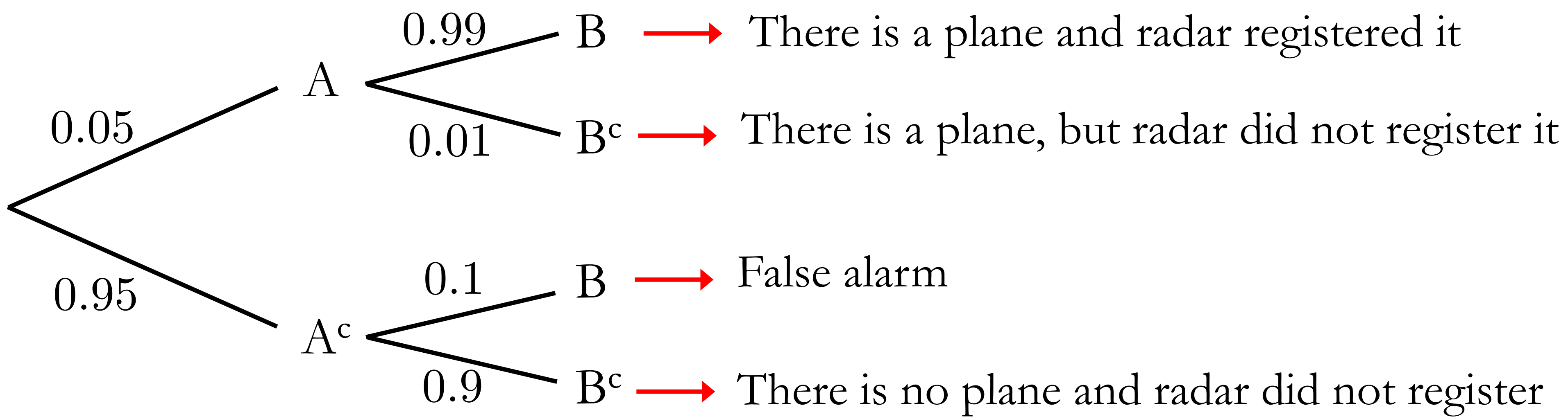

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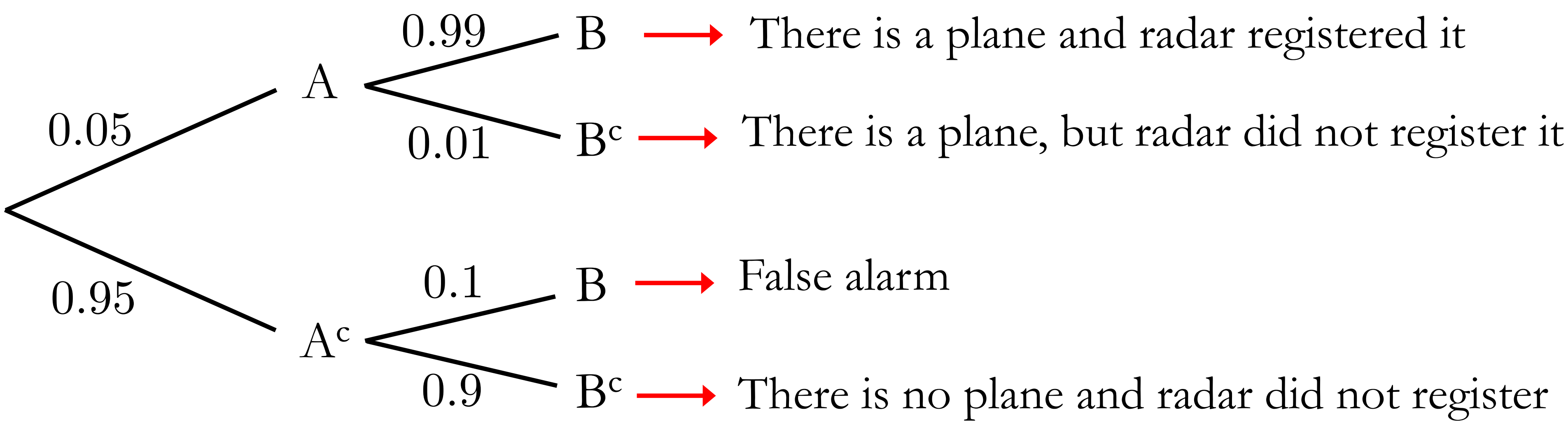

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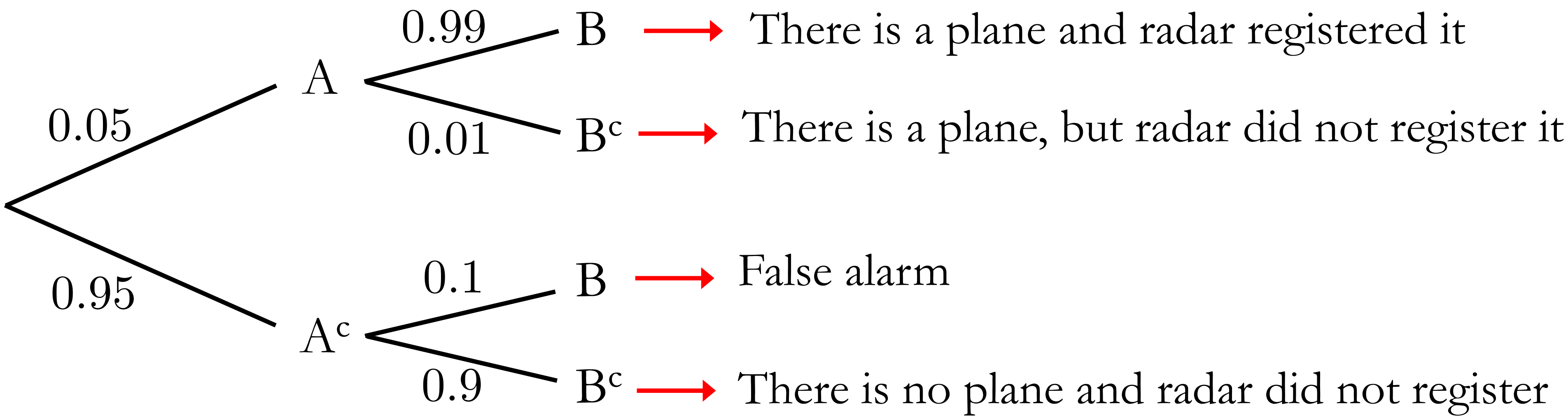

What is the probability that the radar registers something?

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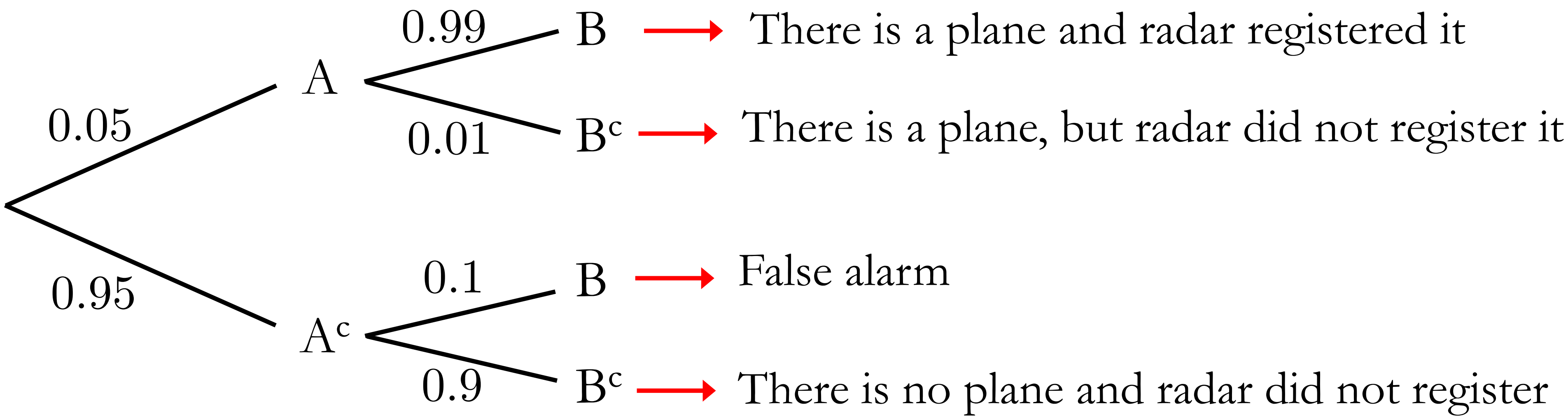

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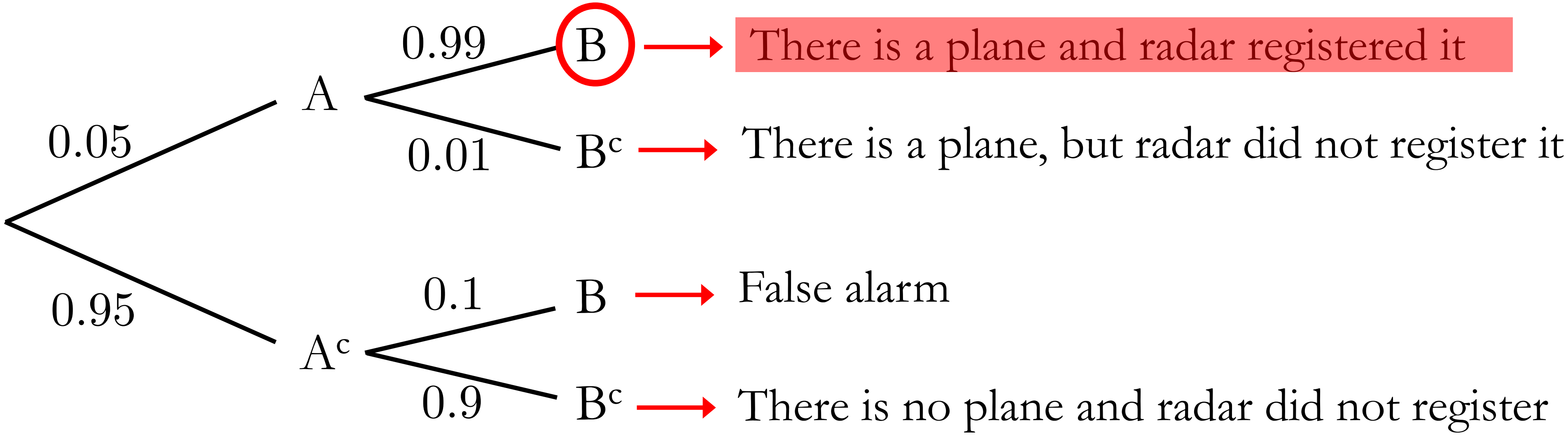

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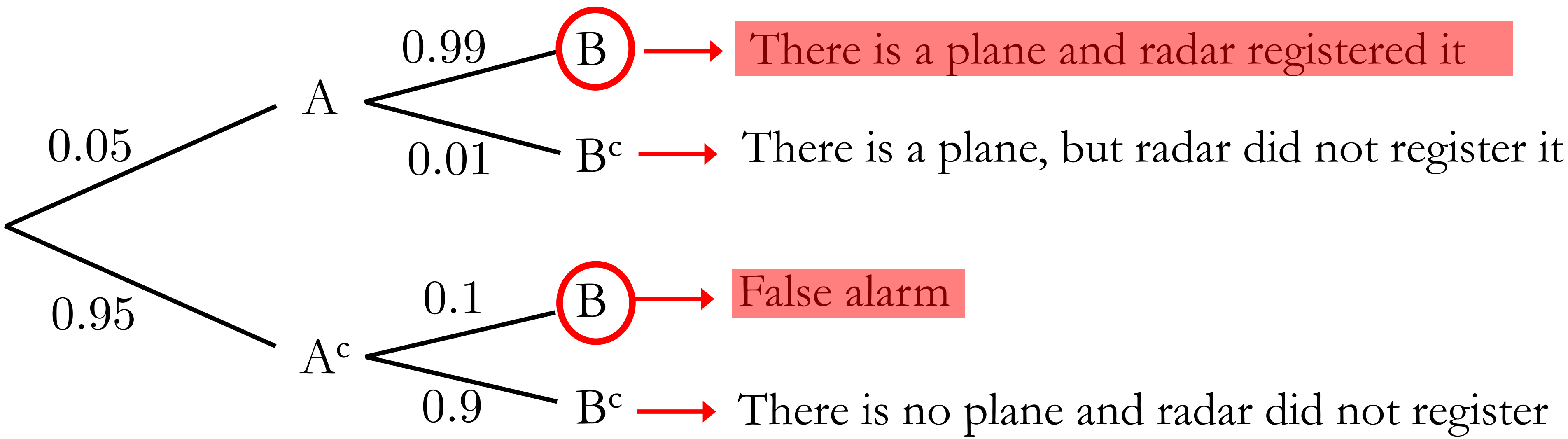

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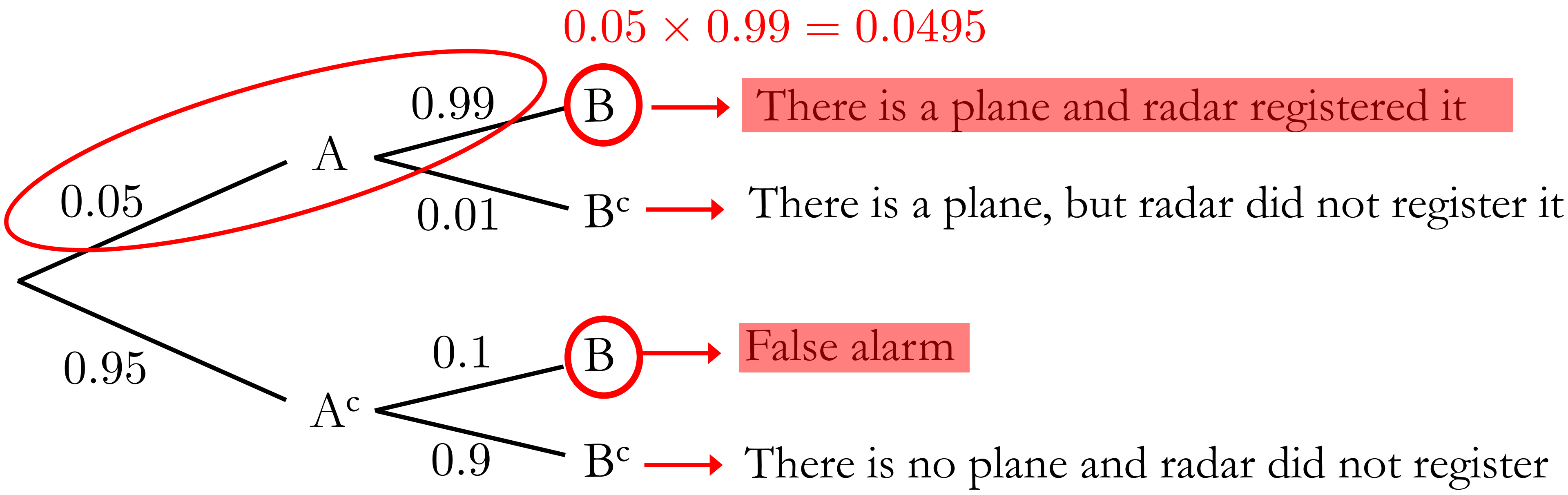

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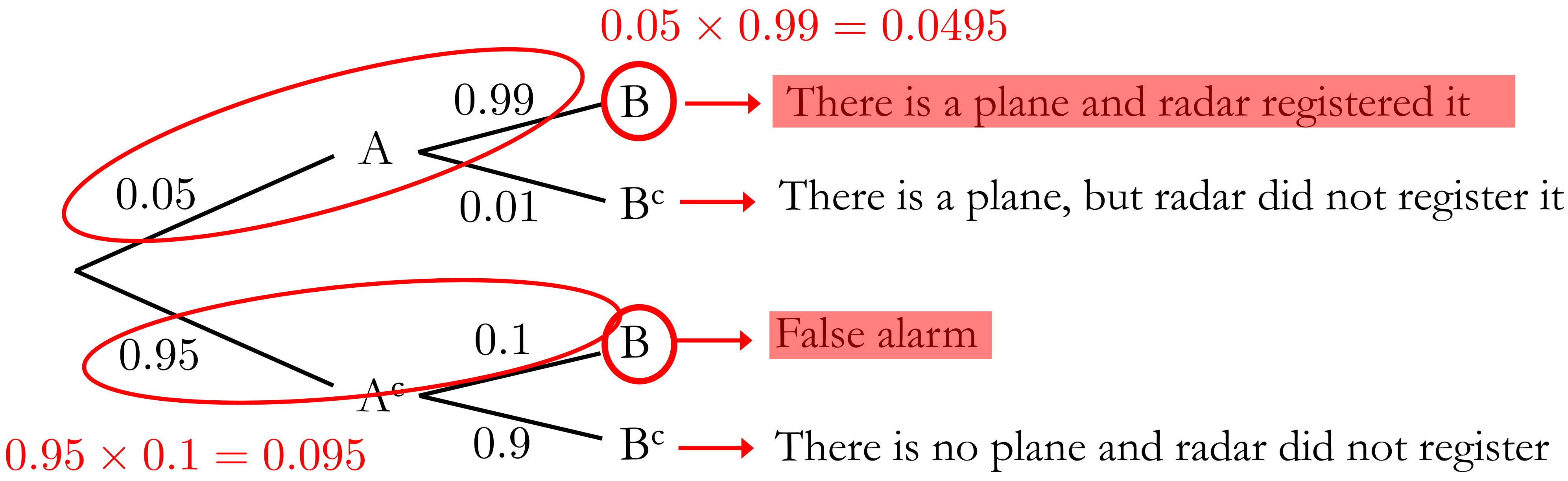

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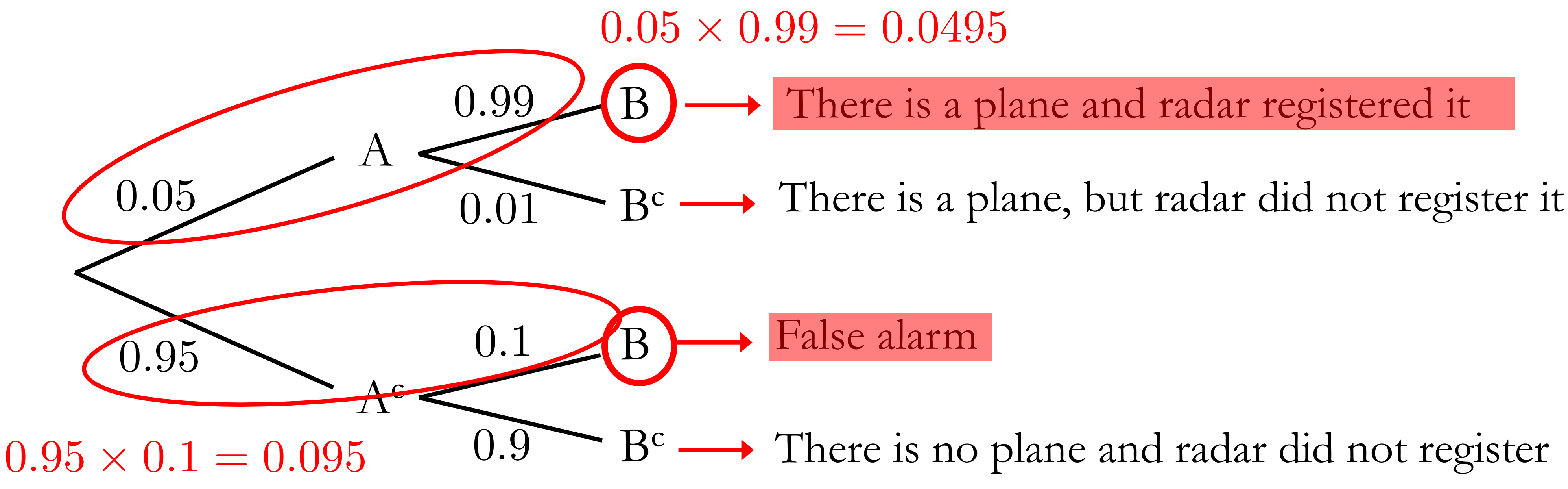
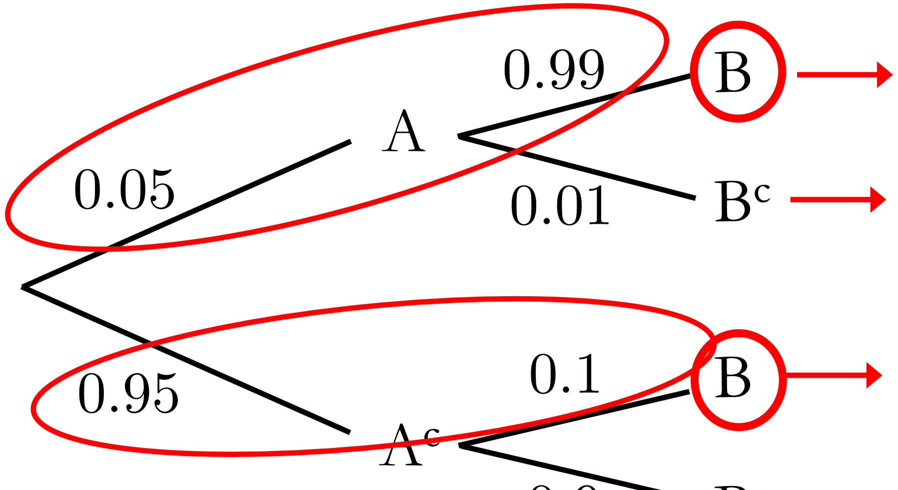


What is the probability that the radar registers something?

$$P(B) = P(A \text{ and } B) + P(A^c \text{ and } B)$$

Radar Example

$0.05 \times 0.99 = 0.0495$



There is a plane and radar registered it

There is a plane, but radar did not register it

False alarm

There is no plane and radar did not register

$0.95 \times 0.1 = 0.095$

What is the probability that the radar registers something?

$$P(B) = P(A \text{ and } B) + P(A^c \text{ and } B)$$
$$= 0.0495 + 0.095 = 0.1445$$

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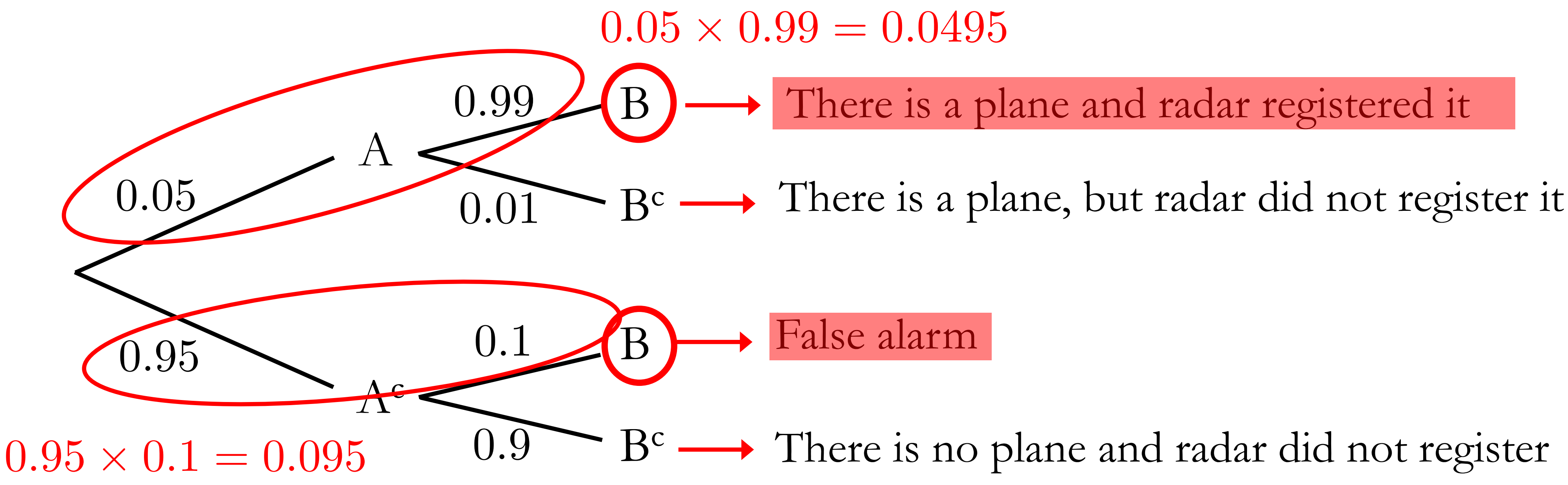

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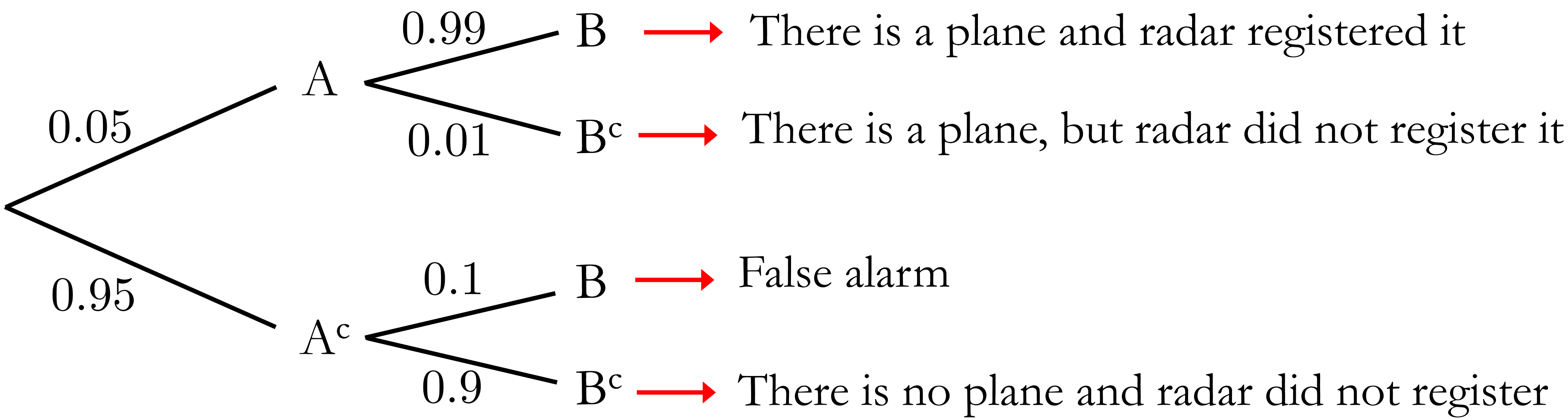
$$P(B) = P(A \text{ and } B) + P(A^c \text{ and } B)$$
$$= 0.0495 + 0.095 = 0.1445$$

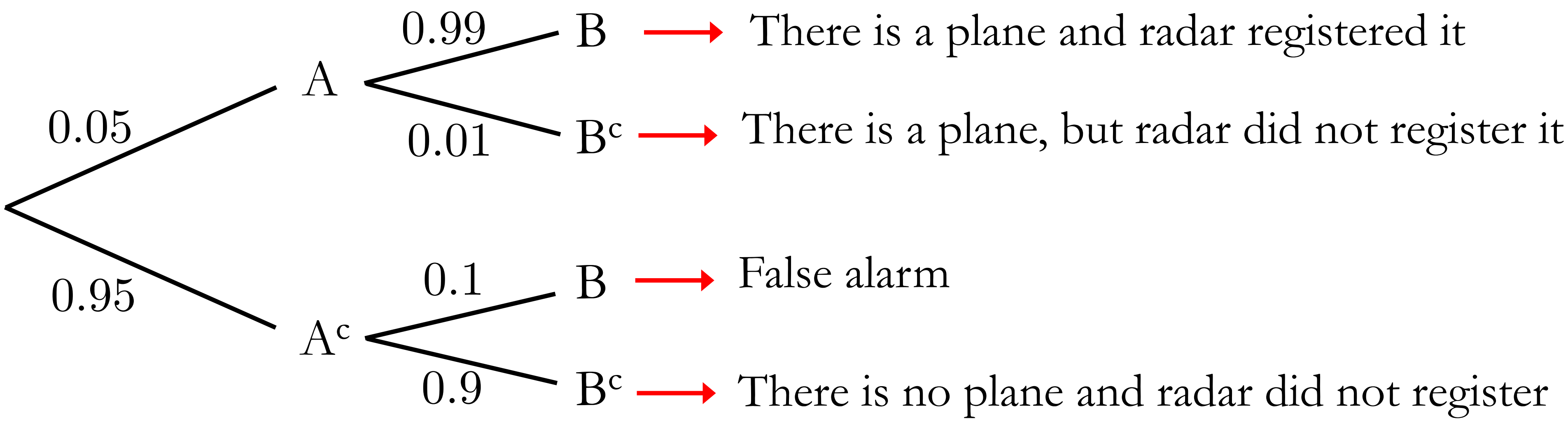
There is a 14.45% chance that the radar registers something!

Radar Example



What is the probability that **there is a plane** given that **the radar registered something**?

Radar Example



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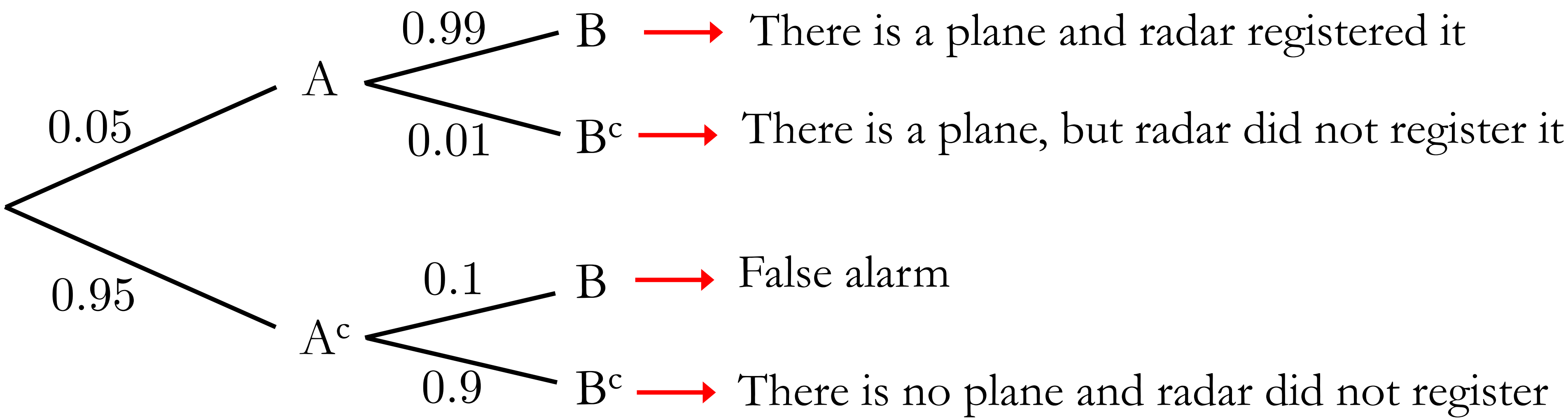
$$P(A|B)$$

Radar Example



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$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

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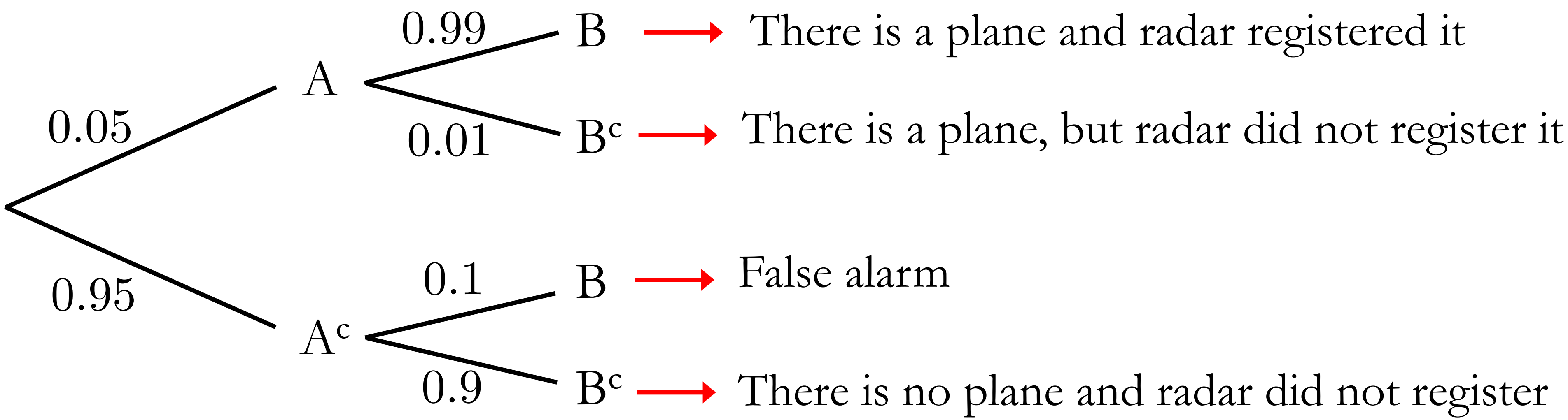

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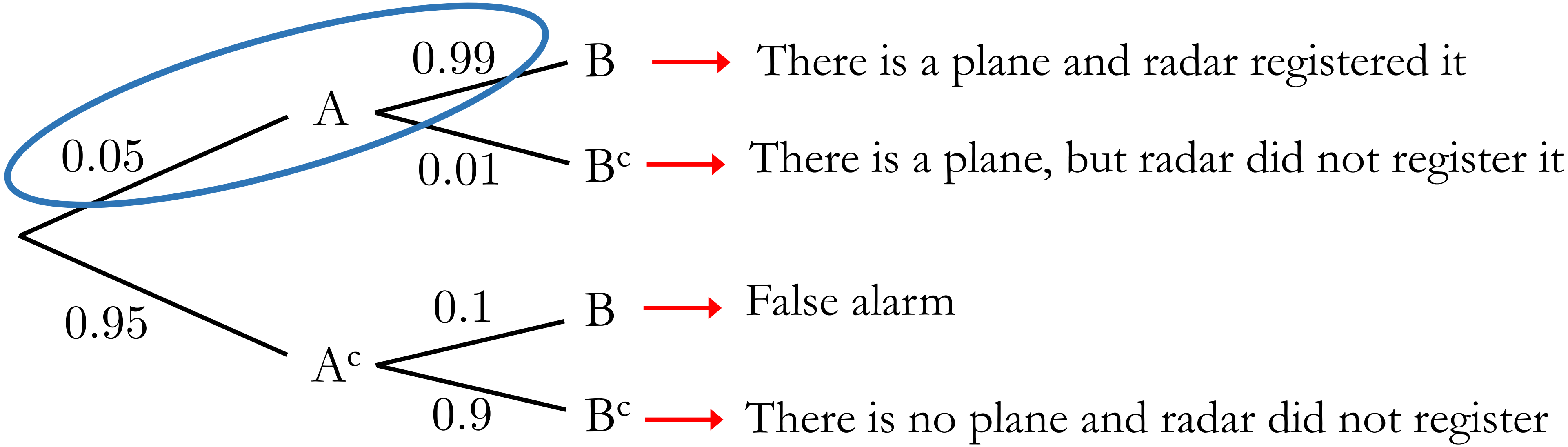

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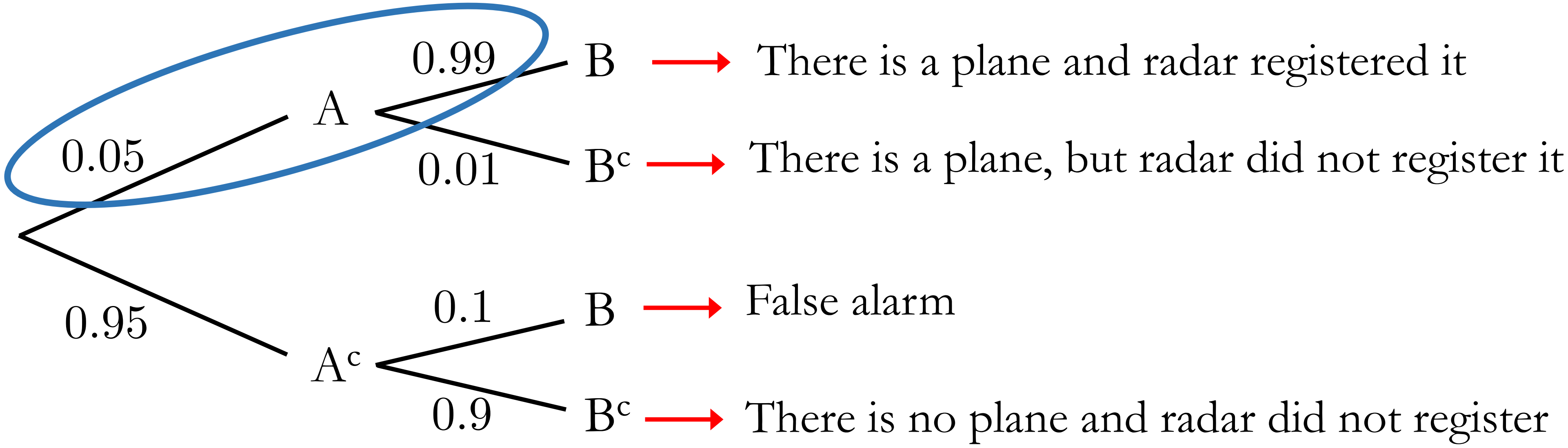

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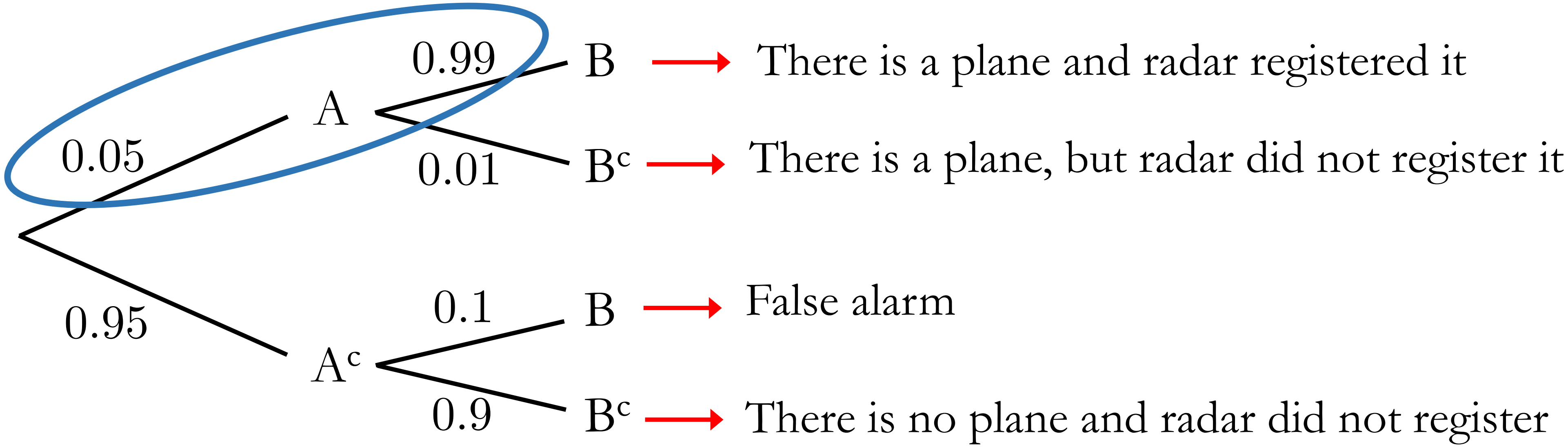

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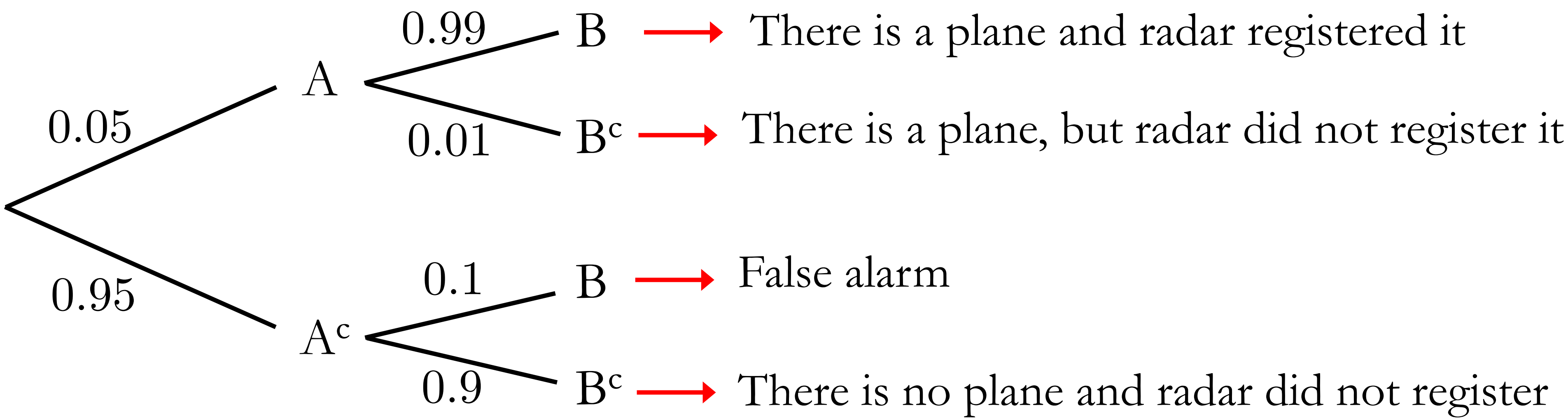

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If the radar registers something, there's only a 34% chance that there's actually a plane!

HIV Testing

HIV Example

As of 2009, Swaziland had the highest HIV prevalence in the world. 25.9% of this country's population is infected with HIV. The ELISA test is one of the first and most accurate tests for HIV. For those who carry HIV, the ELISA test is 99.7% accurate. For those who do not carry HIV, the test is 92.6% accurate. If an individual from Swaziland has tested positive, what is the probability that they carry HIV?

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- HIV: the event that a person has HIV
- no HIV: the event that a person does not have HIV
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all possible events

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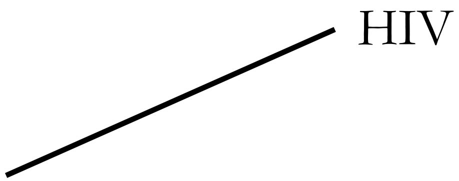
all possible events



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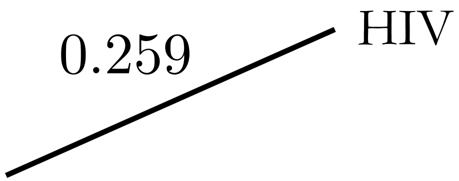

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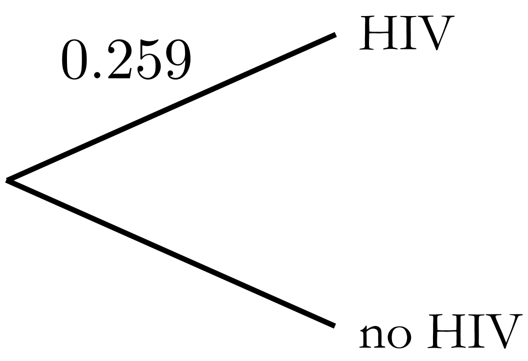

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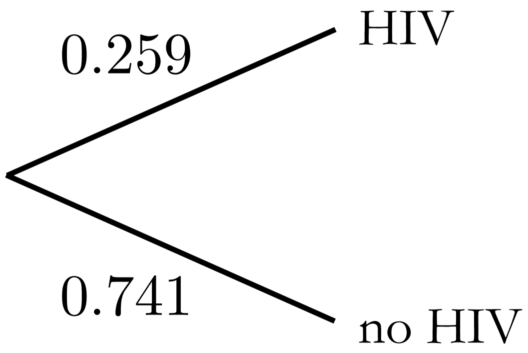

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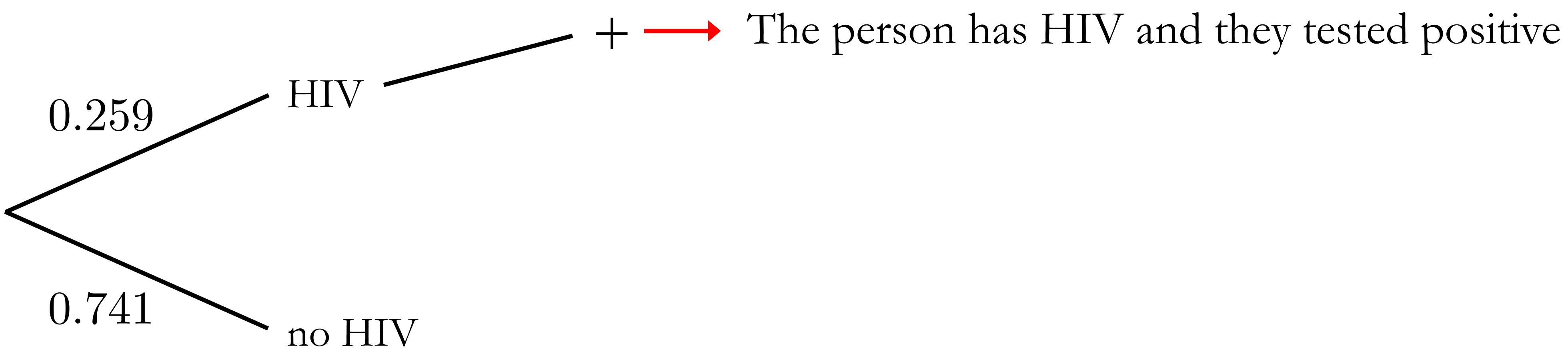

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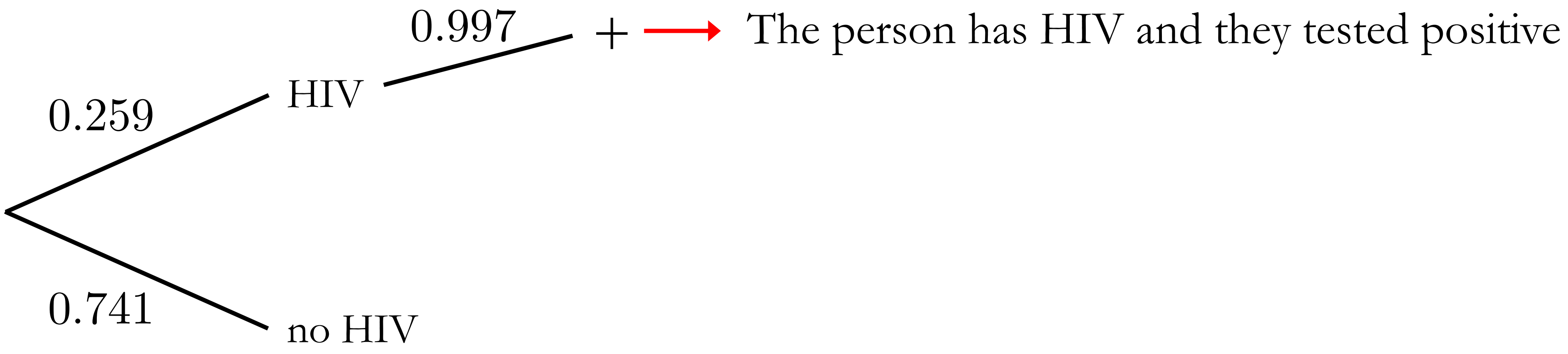

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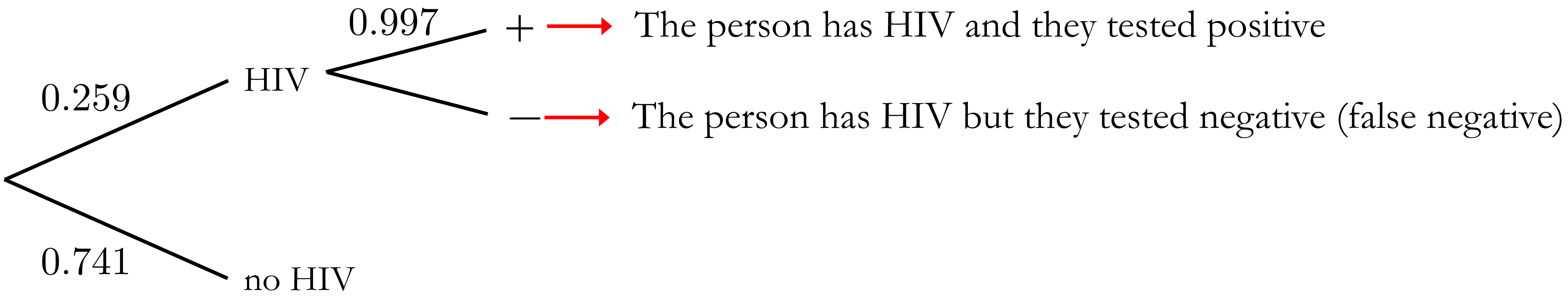

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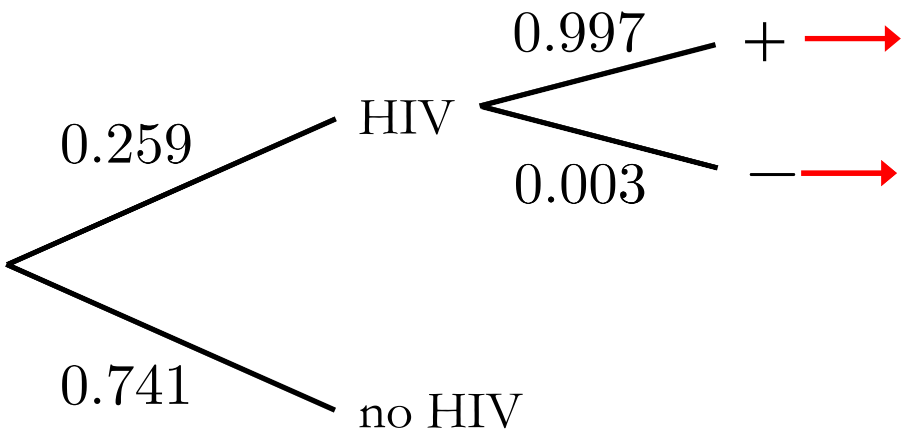

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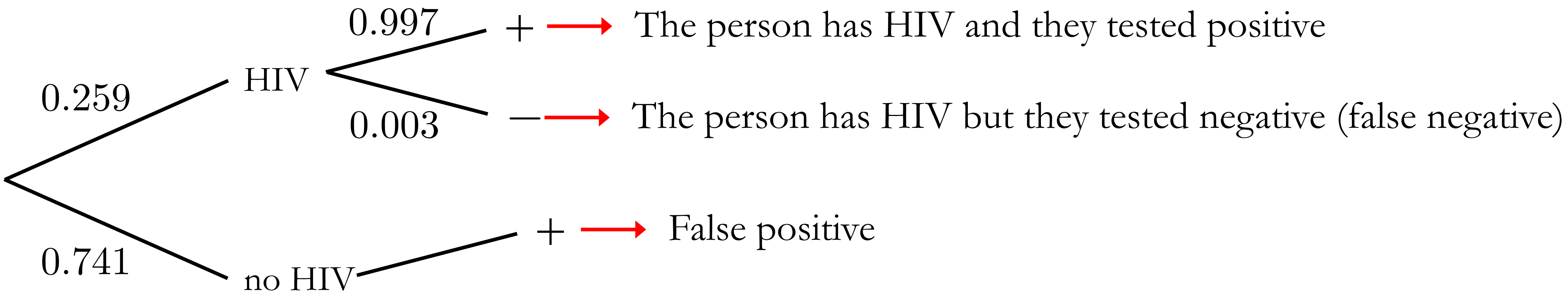

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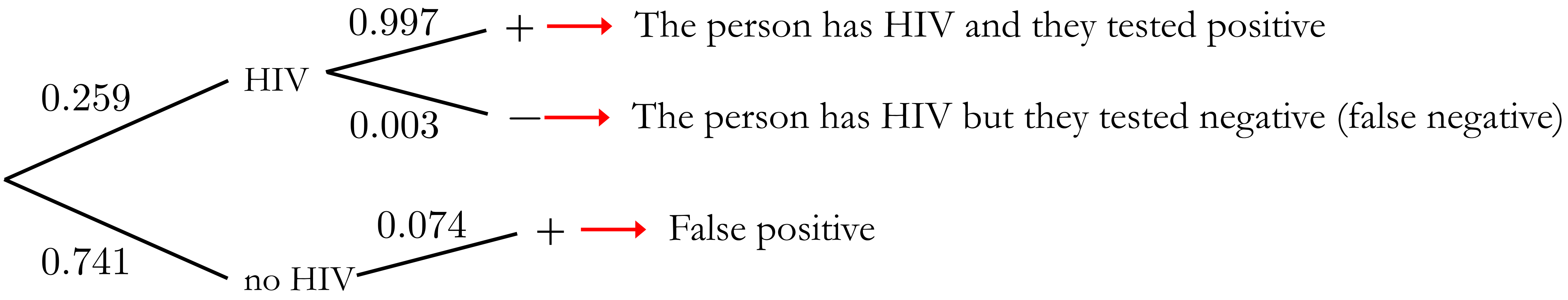

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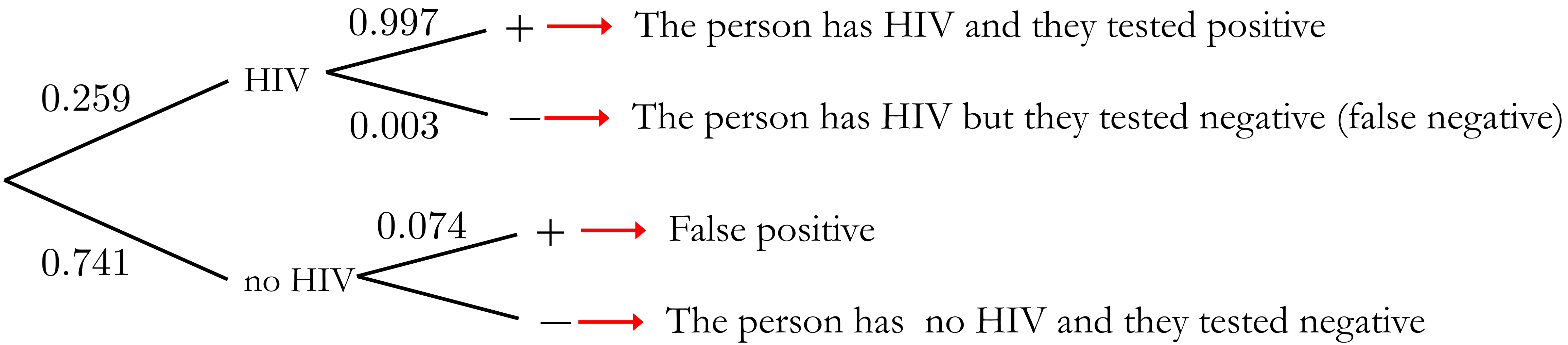

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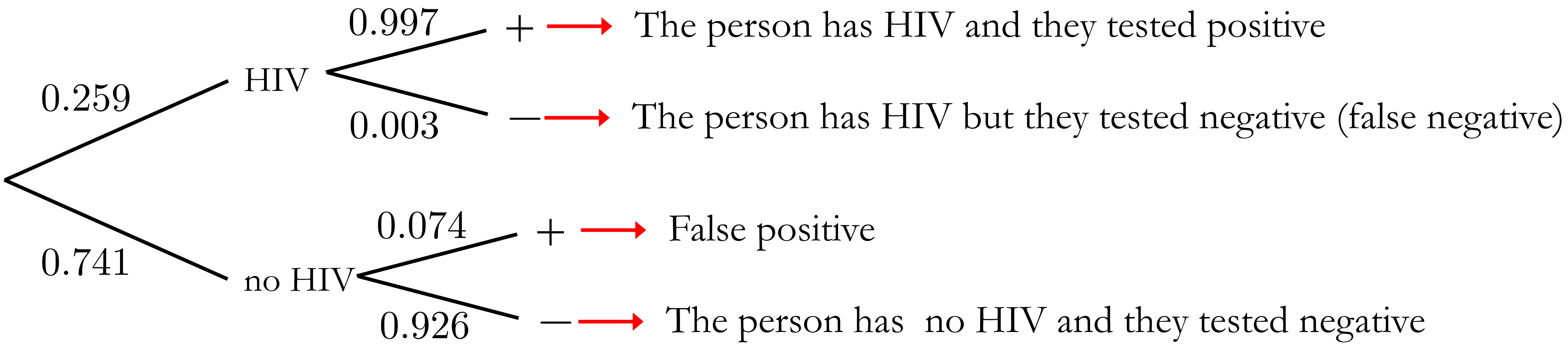
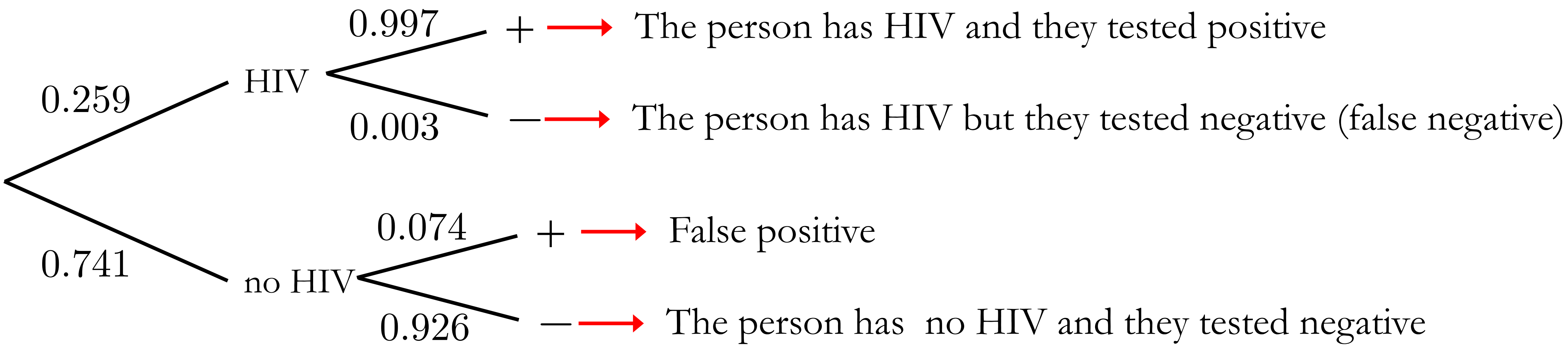
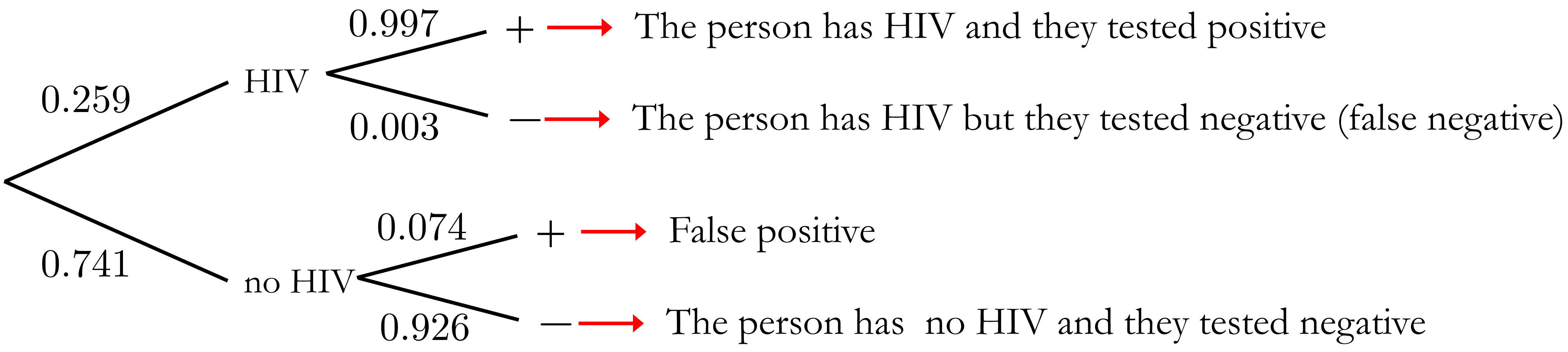


HIV Example


0.259
HIV
0.997
+
The person has HIV and they tested positive
0.003
−
The person has HIV but they tested negative (false negative)
0.741
no HIV
0.074
+
False positive
0.926
−
The person has no HIV and they tested negative

HIV Example



If the patient from Swaziland takes the ELISA test and it yields a positive result, what is the probability that they have HIV?

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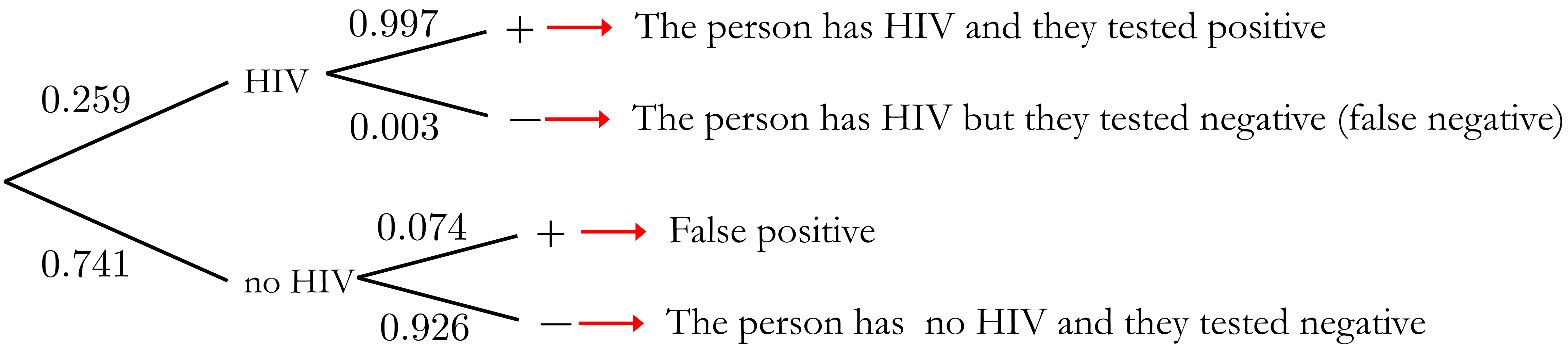

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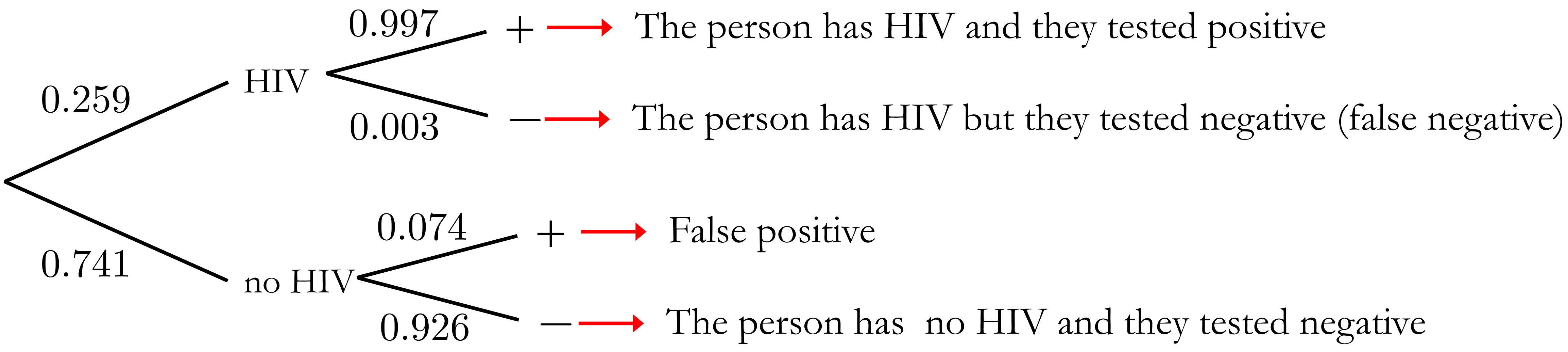
want to find $P(HIV|+)$

HIV Example



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$$\text{want to find } P(HIV|+) = \frac{P(HIV \text{ and } +)}{P(+)}$$

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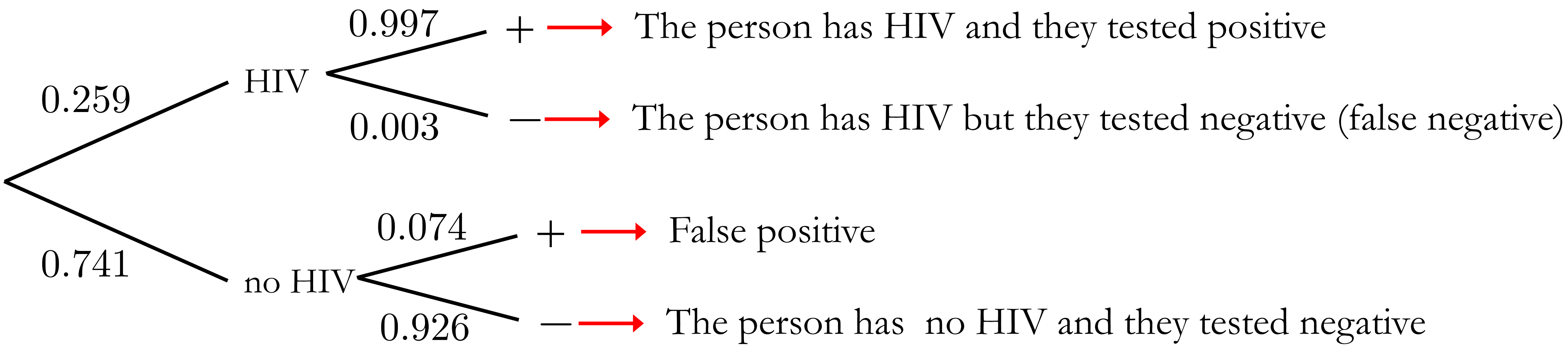

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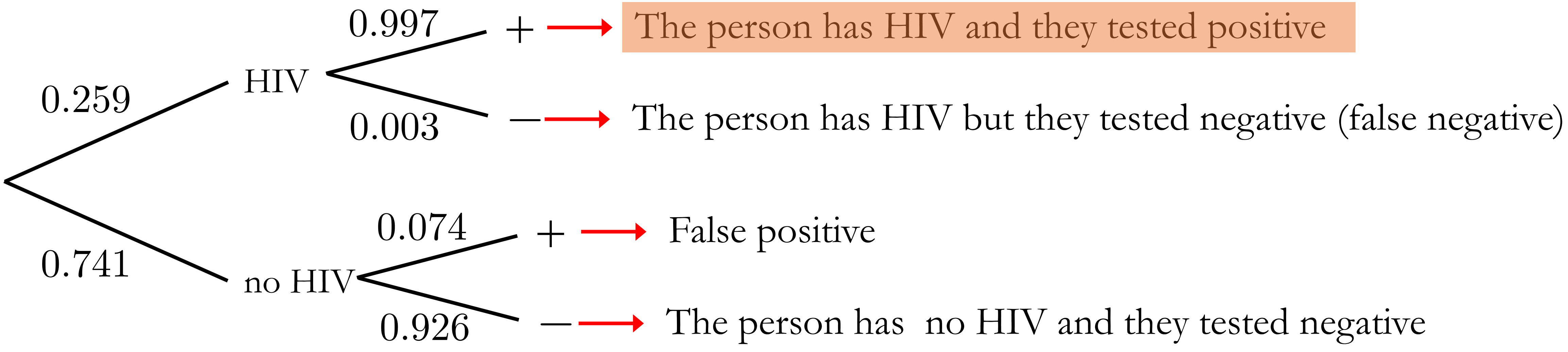

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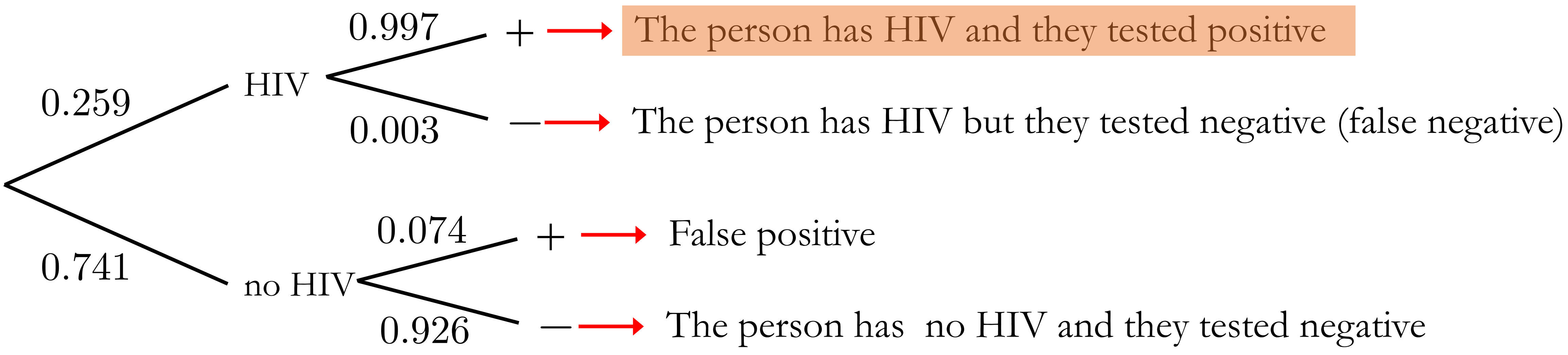
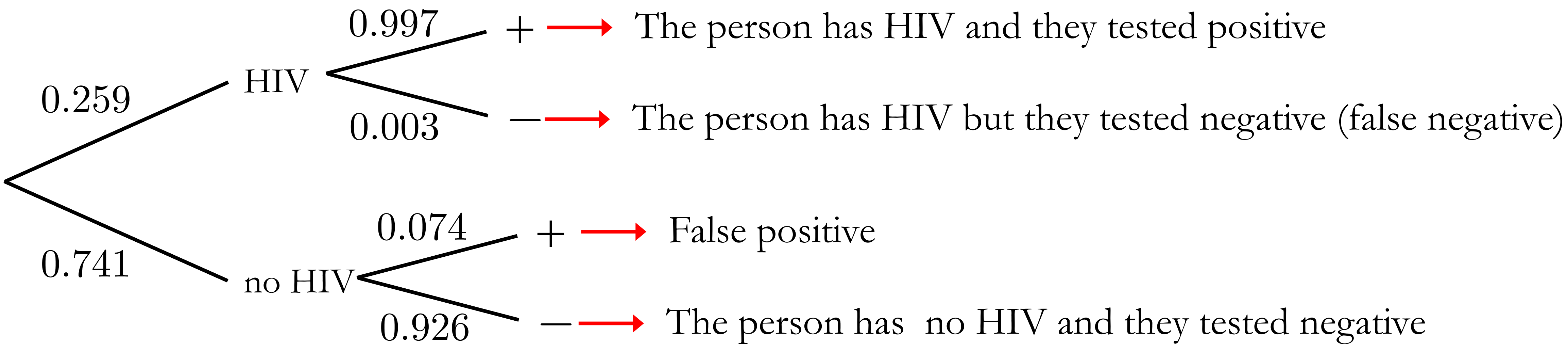


$$P(HIV|+) = \frac{P(HIV \text{ and } +)}{P(+)} = \frac{0.259 \times 0.997}{P(+)}$$

HIV Example



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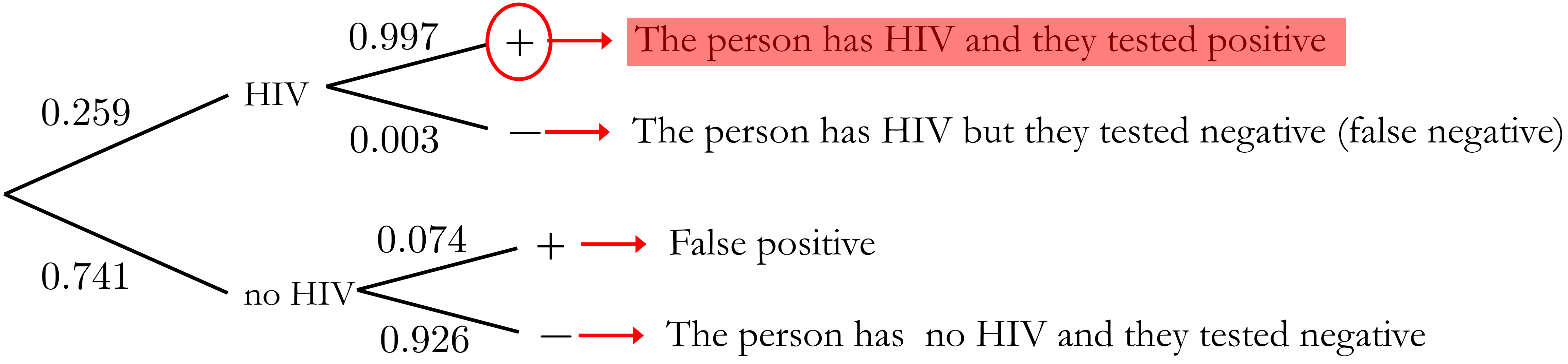

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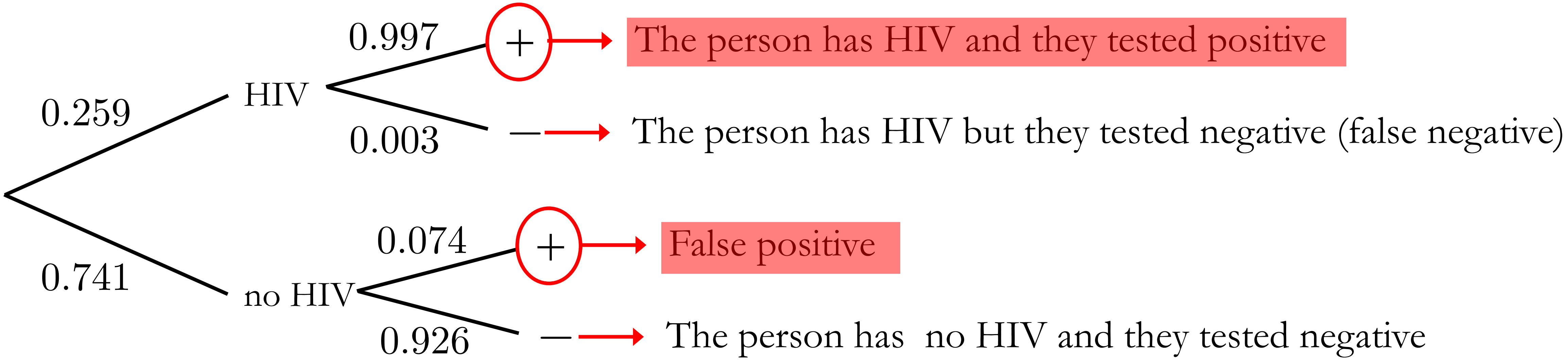

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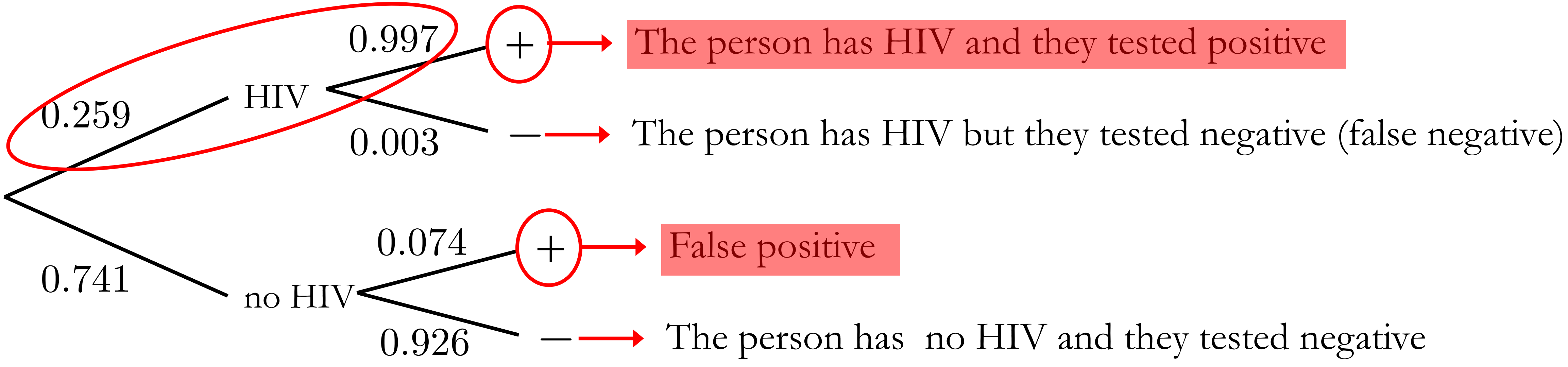

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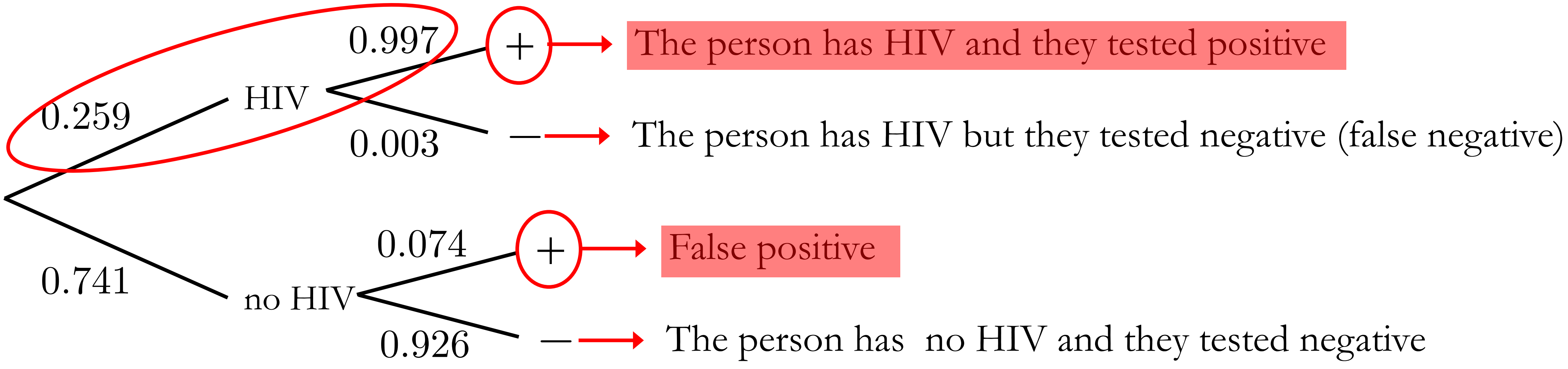
HIV Example

$0.259 \times 0.997 = 0.258$



$$P(HIV|+) = \frac{P(HIV \text{ and } +)}{P(+)} = \frac{0.259 \times 0.997}{P(+)}$$

$$= \frac{0.259 \times 0.997}{0.258 + \ldots}$$

HIV Example

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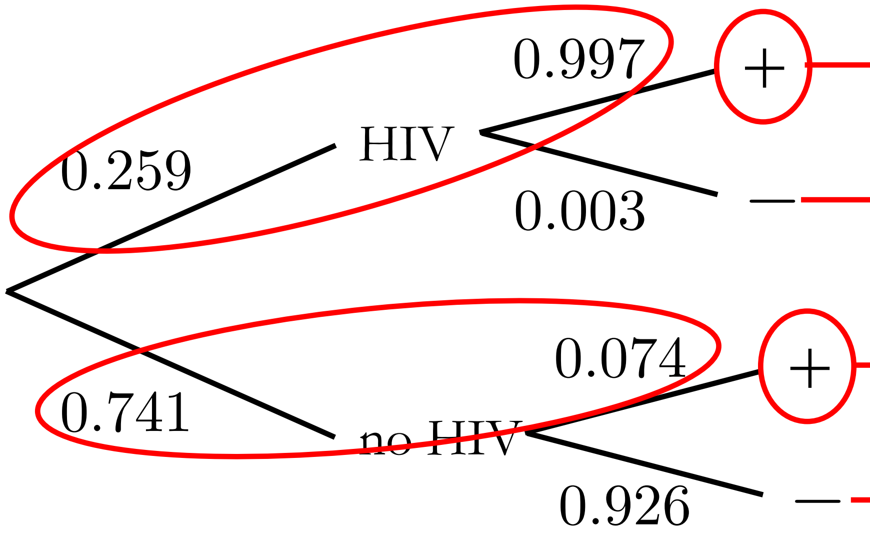

The person has HIV and they tested positive

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False positive

The person has no HIV and they tested negative

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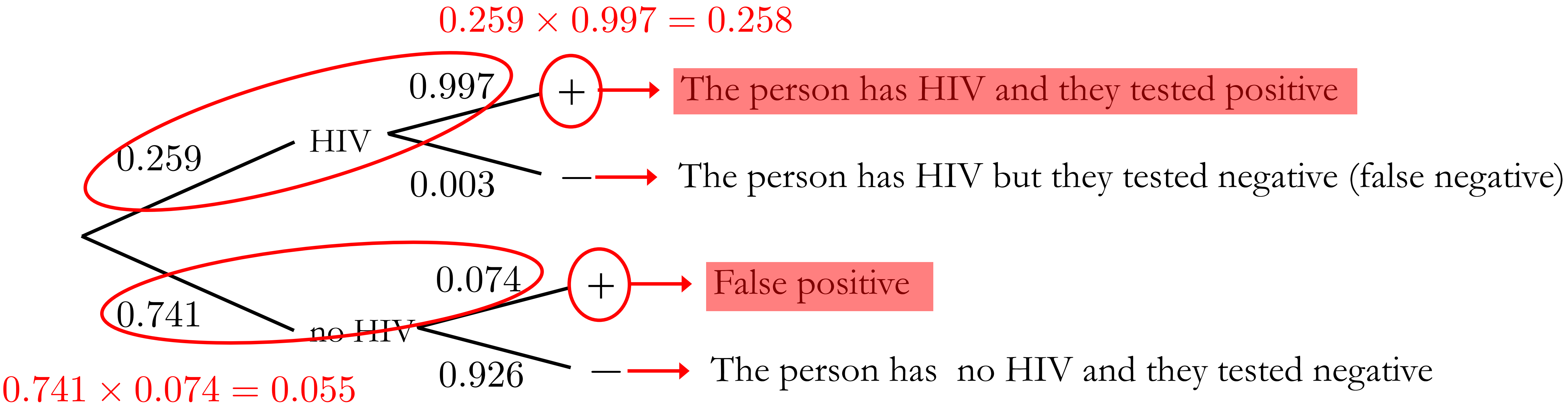

$$P(HIV|+) = \frac{P(HIV \text{ and } +)}{P(+)} = \frac{0.259 \times 0.997}{P(+)}$$

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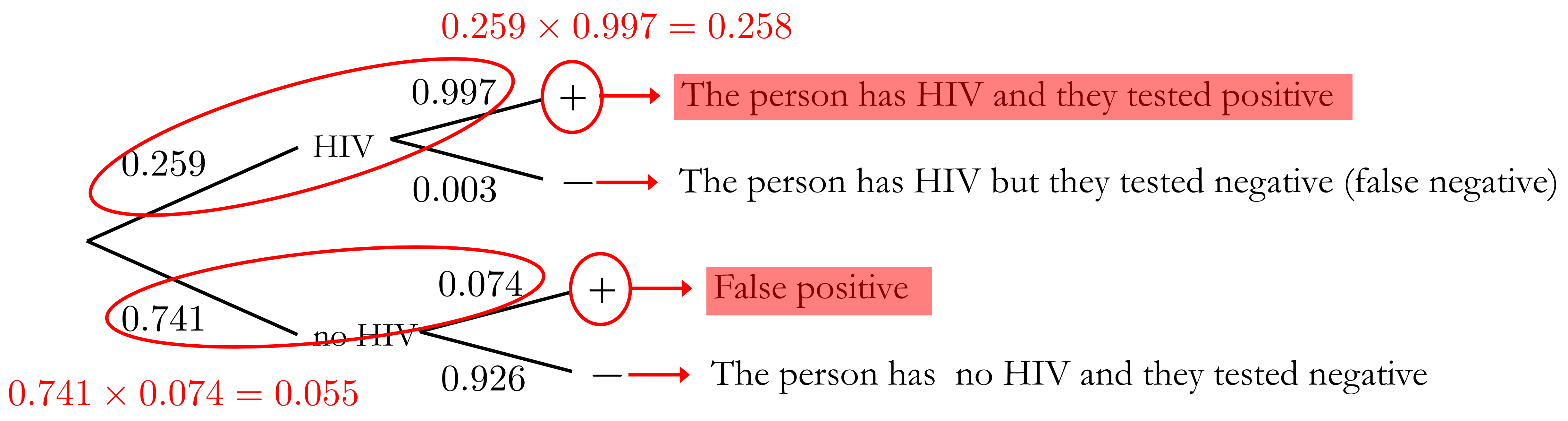
HIV Example



$$P(HIV|+) = \frac{P(HIV \text{ and } +)}{P(+)} = \frac{0.259 \times 0.997}{P(+)}$$

$$= \frac{0.259 \times 0.997}{0.258 + 0.055} \approx 0.825$$

HIV Example

Conclusion:

Therefore, if a person from Swaziland tests positive for HIV, there is an 82.5% chance that they have HIV (or a 17.5% chance that they do not have HIV). So, their chances of having the disease are pretty high.

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If I go and take the HIV test now and it turns out positive, should I freak out?

HIV Example

Suppose someone living in the US takes the ELISA test, where only 0.04% of the population has HIV. Then, if a person in the US tests positive for HIV, the chance that they actually have the disease is:

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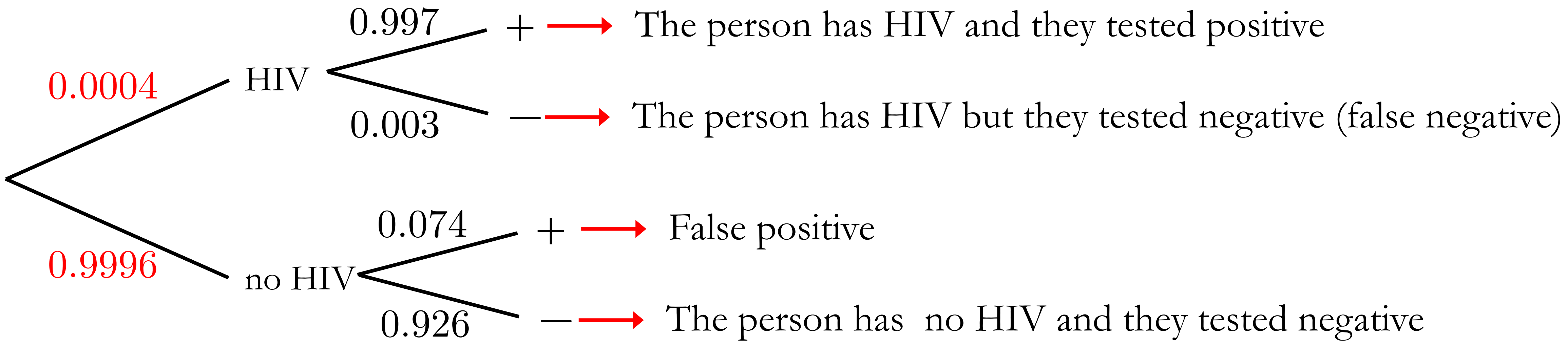

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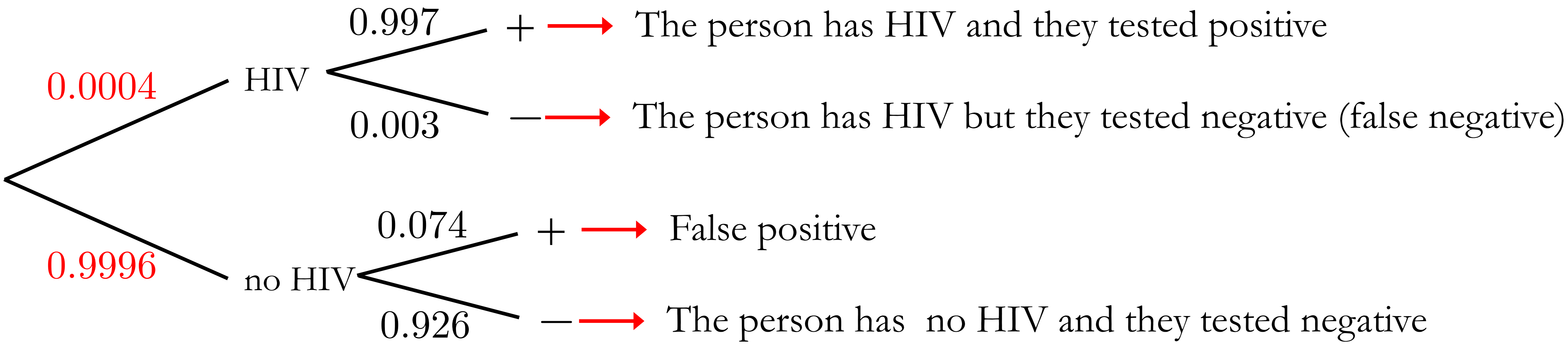

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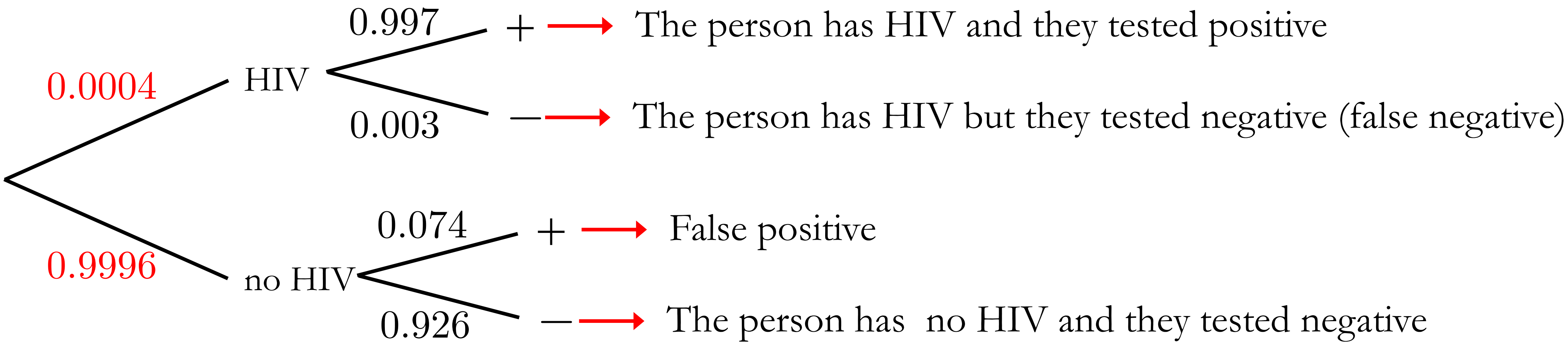

$$P(HIV|+) = \frac{P(HIV \text{ and } +)}{P(+)} \approx 0.0054$$

HIV Example

Conclusion:

Therefore, if a person from Swaziland tests positive for HIV, there is an 82.5% chance that they have HIV (or a 17.5% chance that they do not have HIV). So, their chances of having the disease are pretty high.

If I go and take the HIV test now and it turns out positive, should I freak out?

Not really, there's only 0.54% chance that I actually have HIV if the test turns out positive!

Breast Cancer Testing (Mammograms)

Breast Cancer Example

American Cancer Society estimates that about 2% of women have breast cancer. Susan G. Komen for The Cure Foundation states that "Overall, the sensitivity of mammography is about 84 percent. This means mammography correctly identifies about 84 percent of women who truly have breast cancer." A recent study published in 2015 by the journal Cancer Epidemiology, Biomarkers & Prevention suggests that about 8% of all mammograms are false positives.

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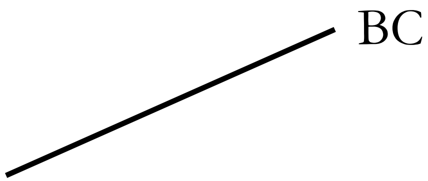

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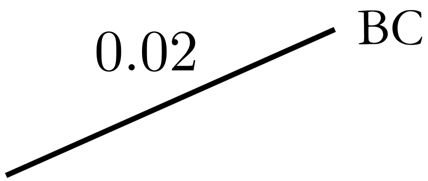

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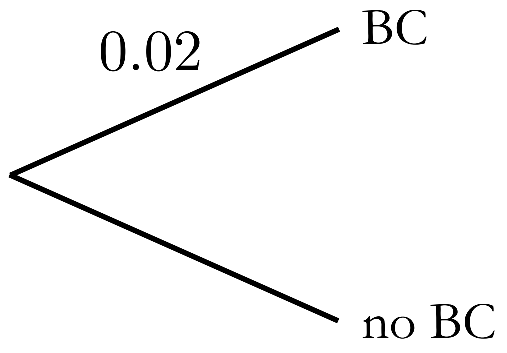

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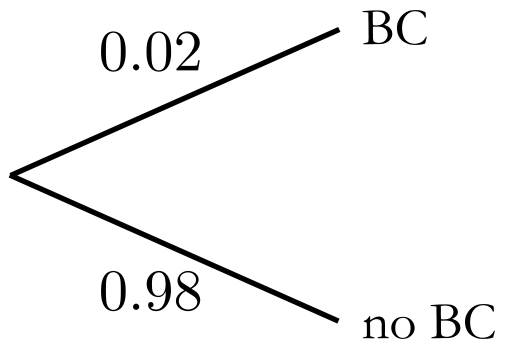

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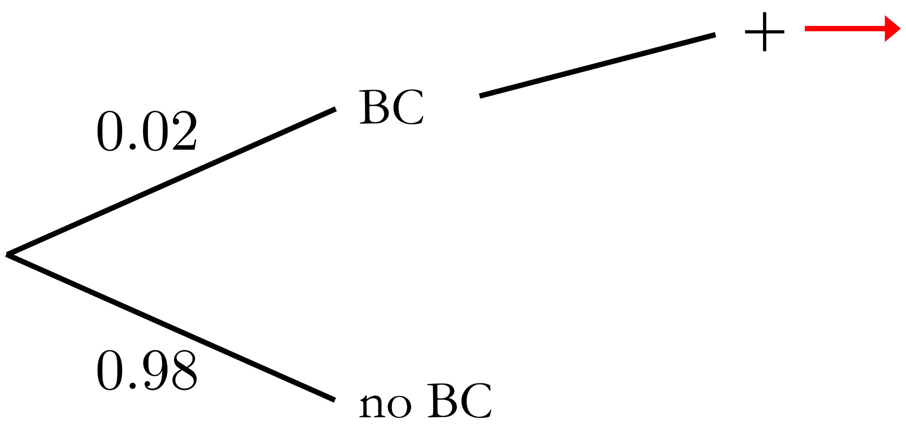

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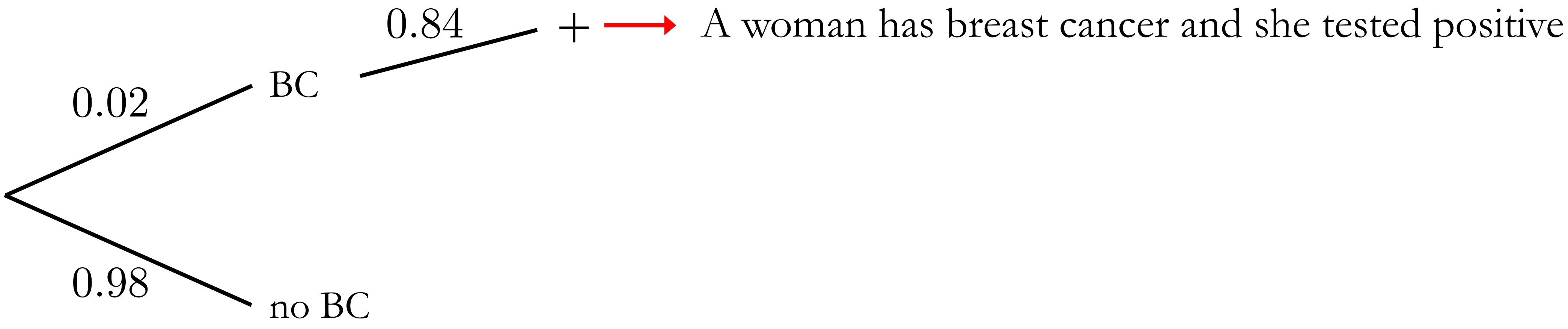

Breast Cancer Example

American Cancer Society estimates that about 2% of women have breast cancer. Susan G. Komen for The Cure Foundation states that "Overall, the sensitivity of mammography is about 84 percent. This means mammography correctly identifies about 84 percent of women who truly have breast cancer." A recent study published in 2015 by the journal Cancer Epidemiology, Biomarkers & Prevention suggests that about 8% of all mammograms are false positives.

- BC: the event that a woman has Breast Cancer
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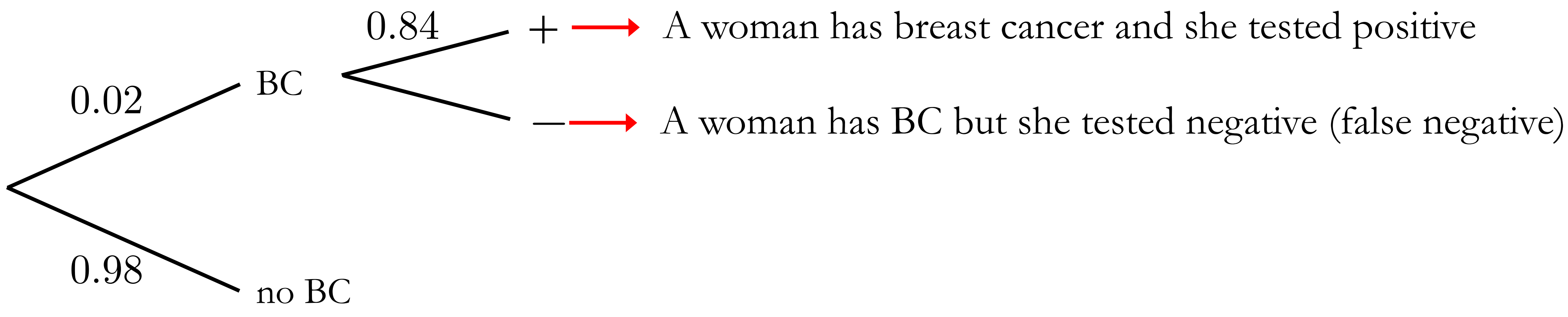

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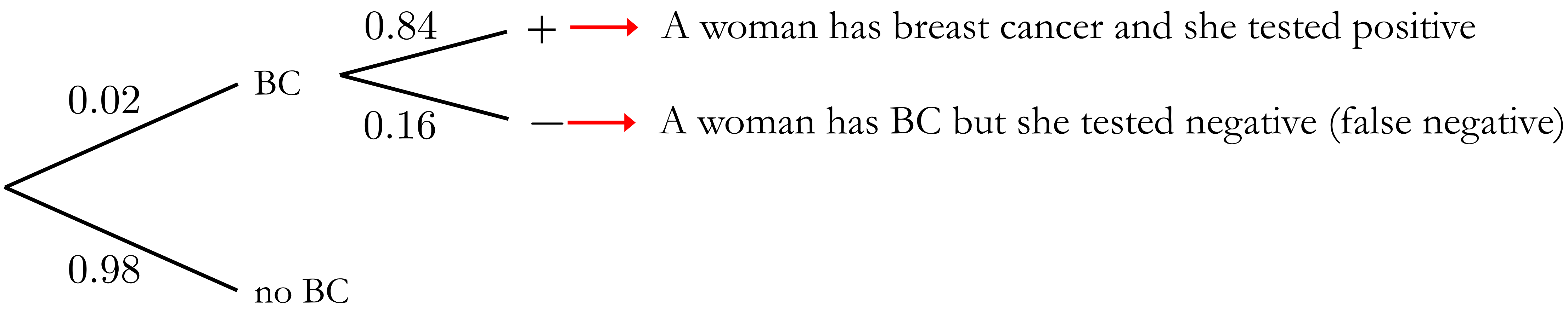

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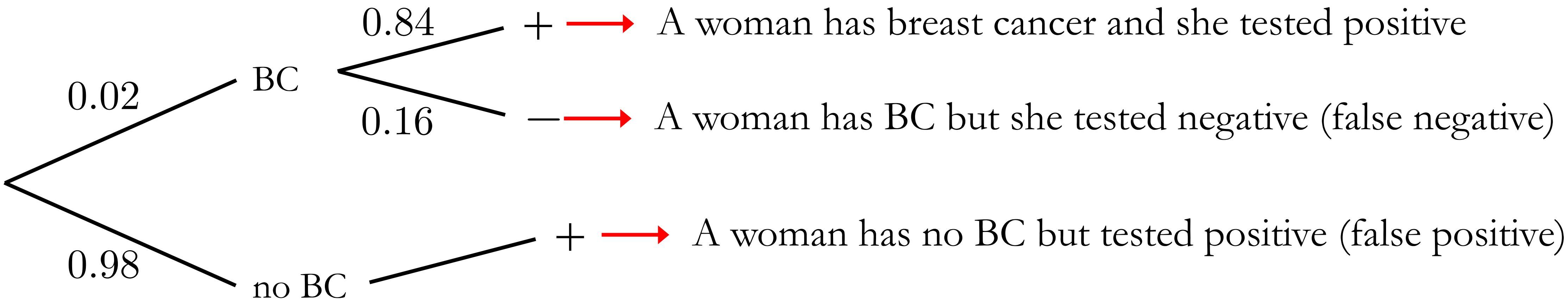

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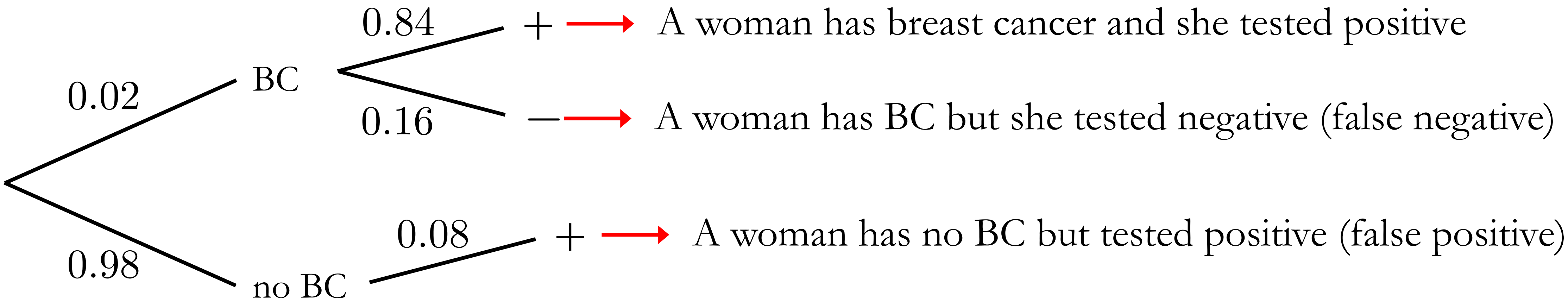

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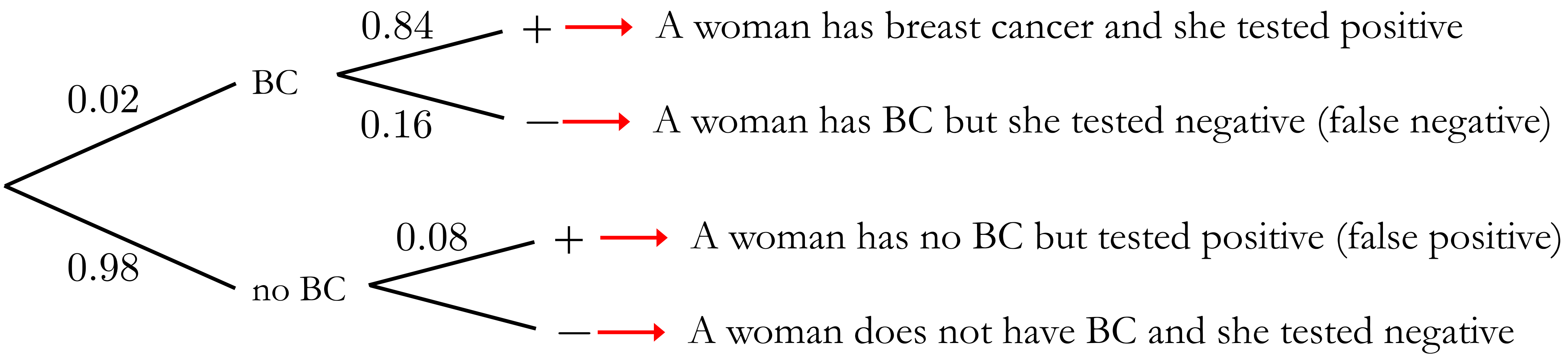

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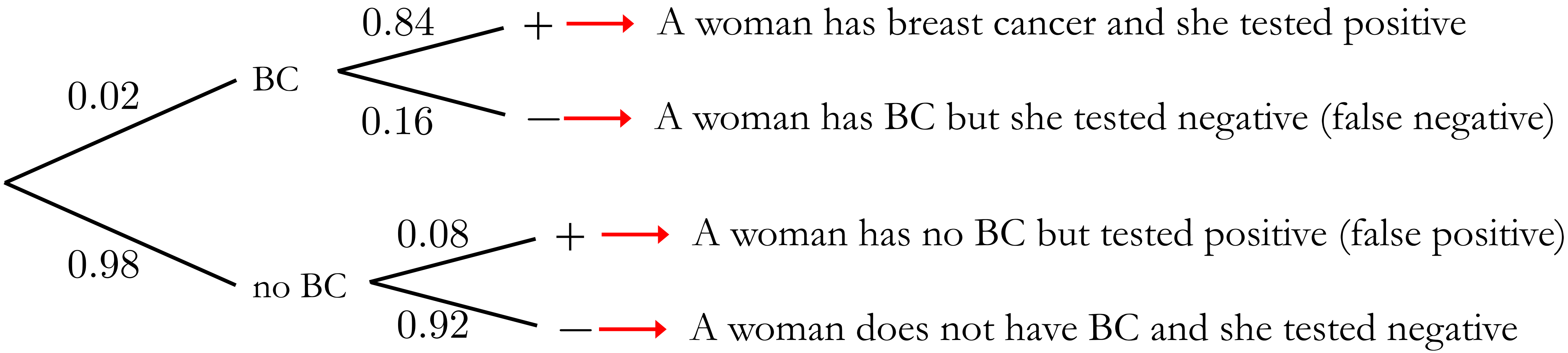

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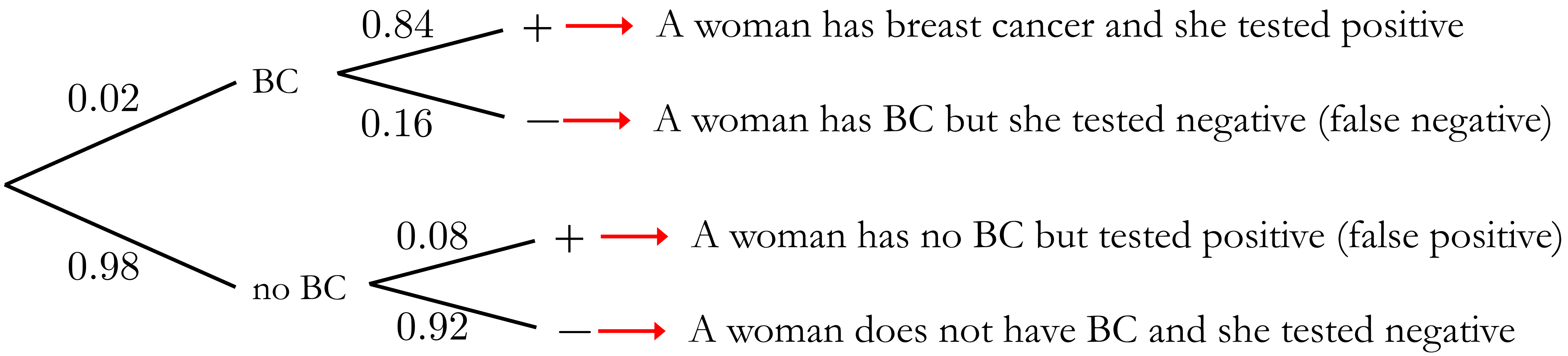
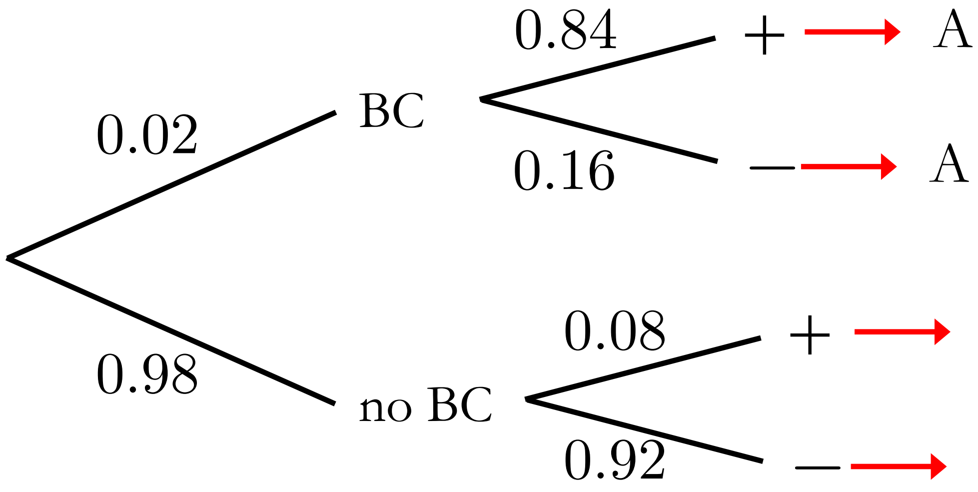


If the patient goes through breast cancer screening and her mammogram yields a positive result, what is the probability that she has cancer?

Breast Cancer Example



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want to find $P(BC|+)$

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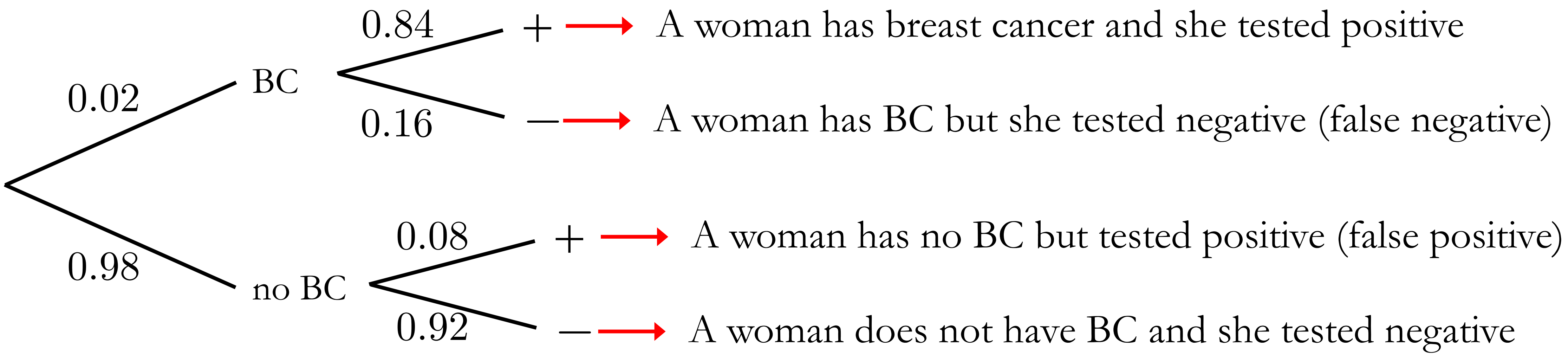

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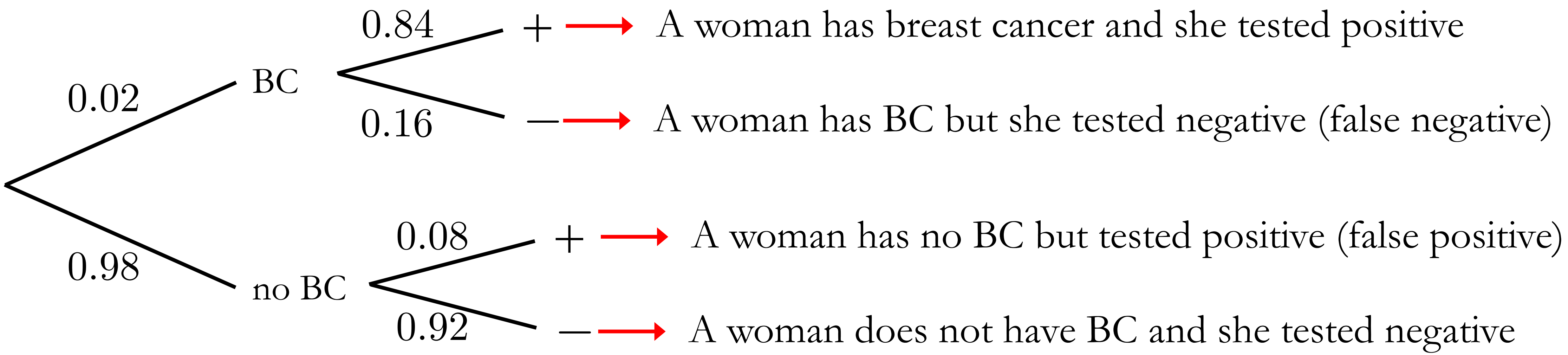

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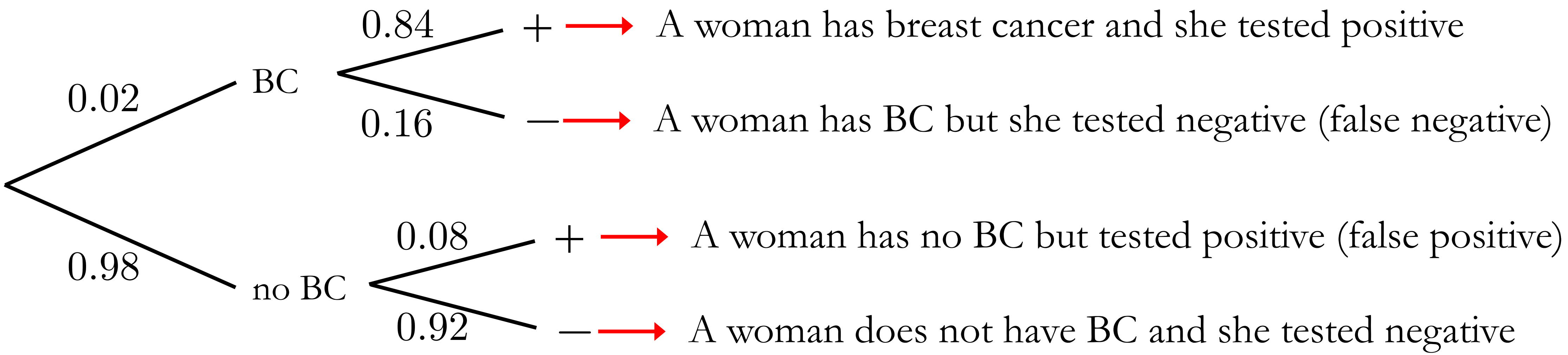

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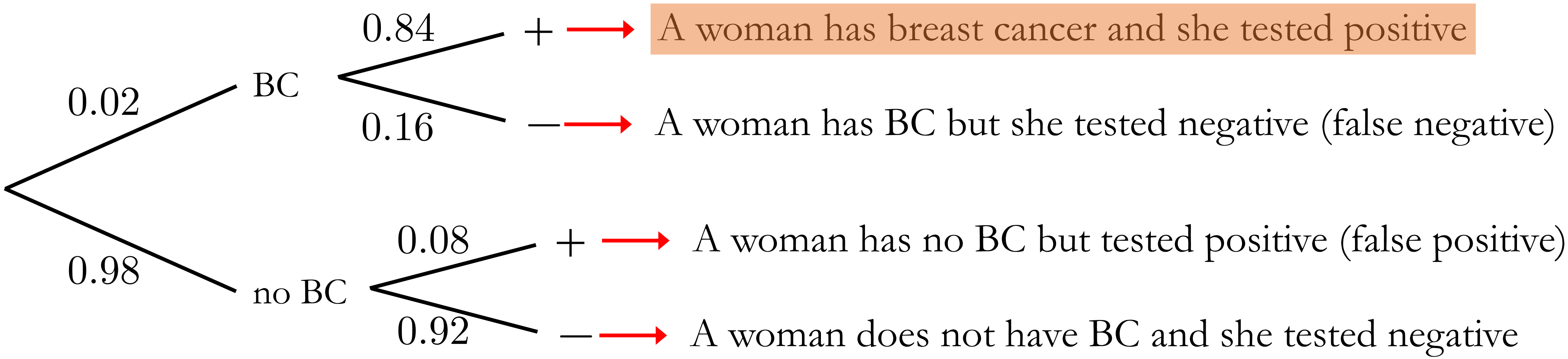

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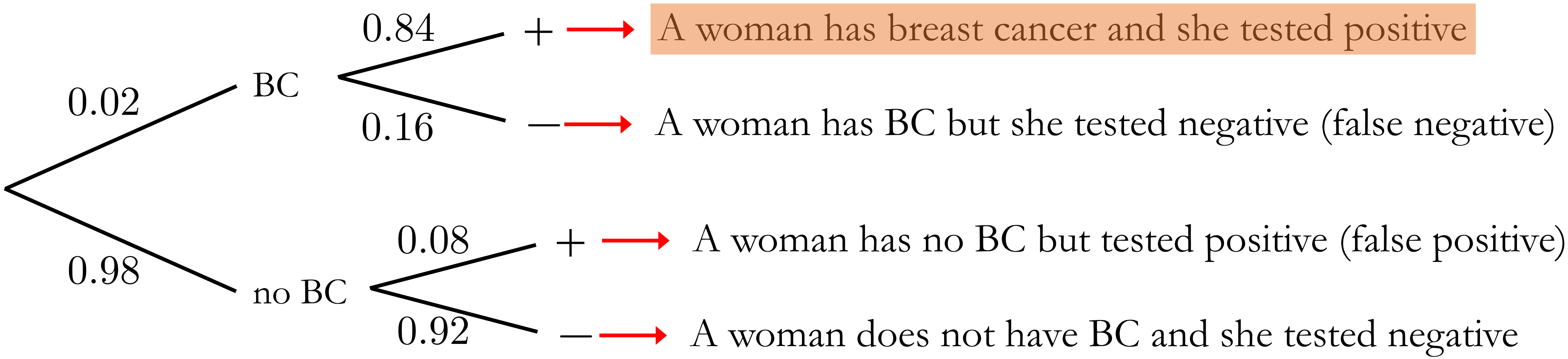

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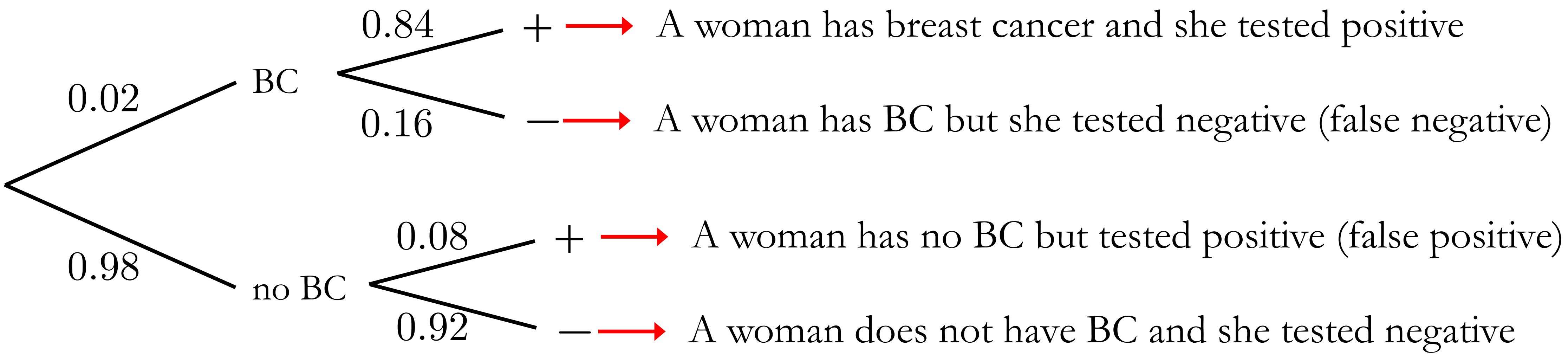

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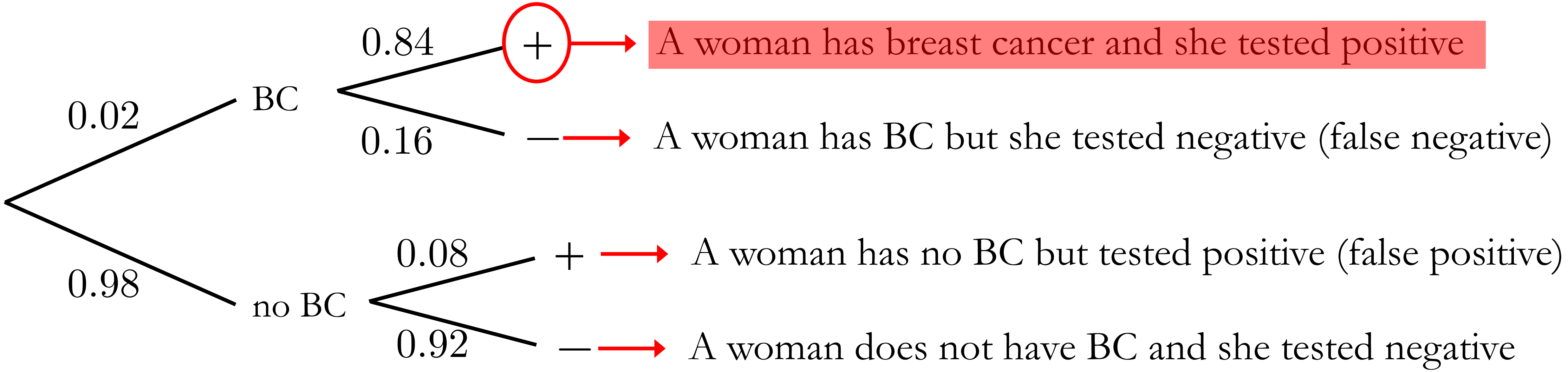

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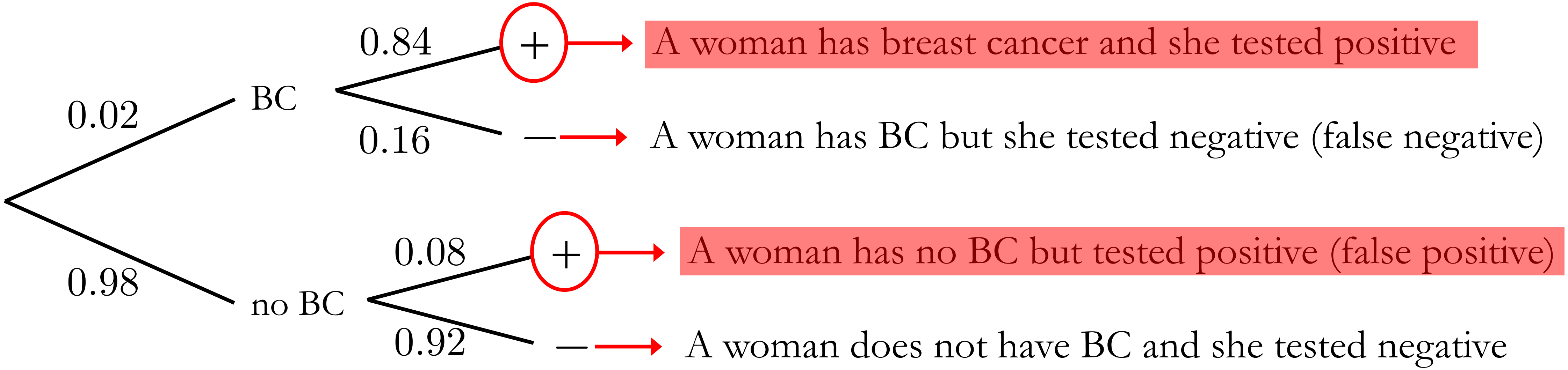

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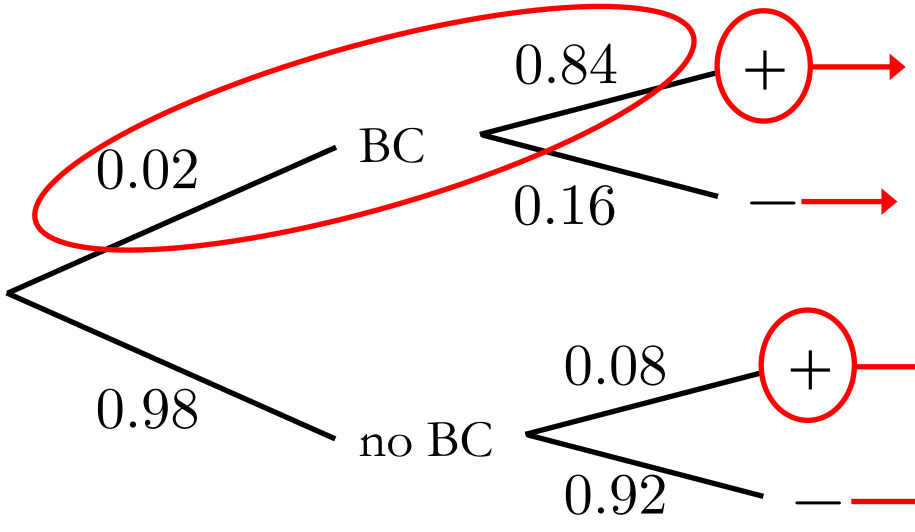

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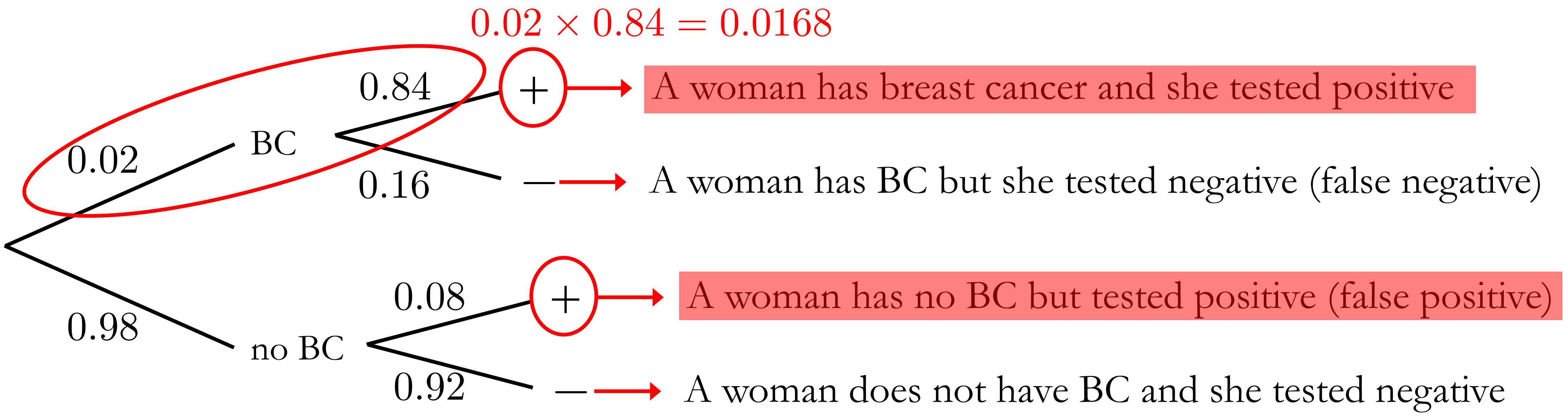

$$P(BC|+) = \frac{P(BC \text{ and } +)}{P(+)} = \frac{0.02 \times 0.84}{P(+)}$$

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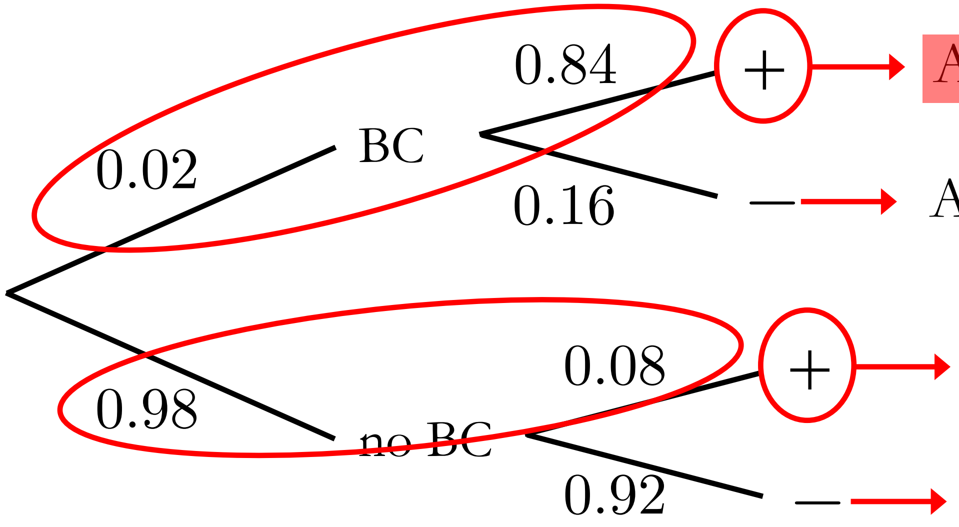
Breast Cancer Example

$0.02 \times 0.84 = 0.0168$



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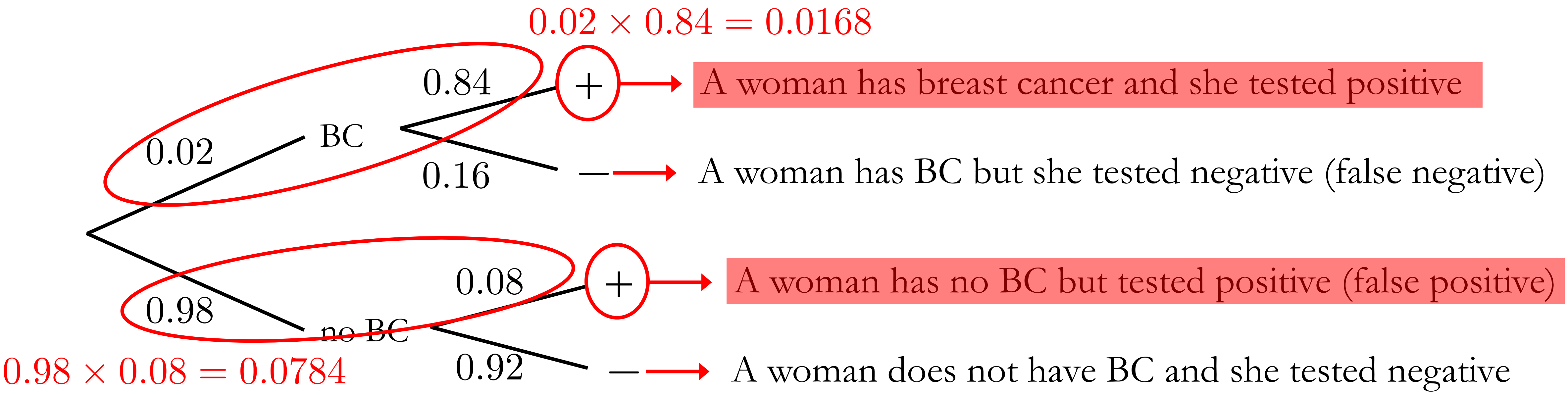

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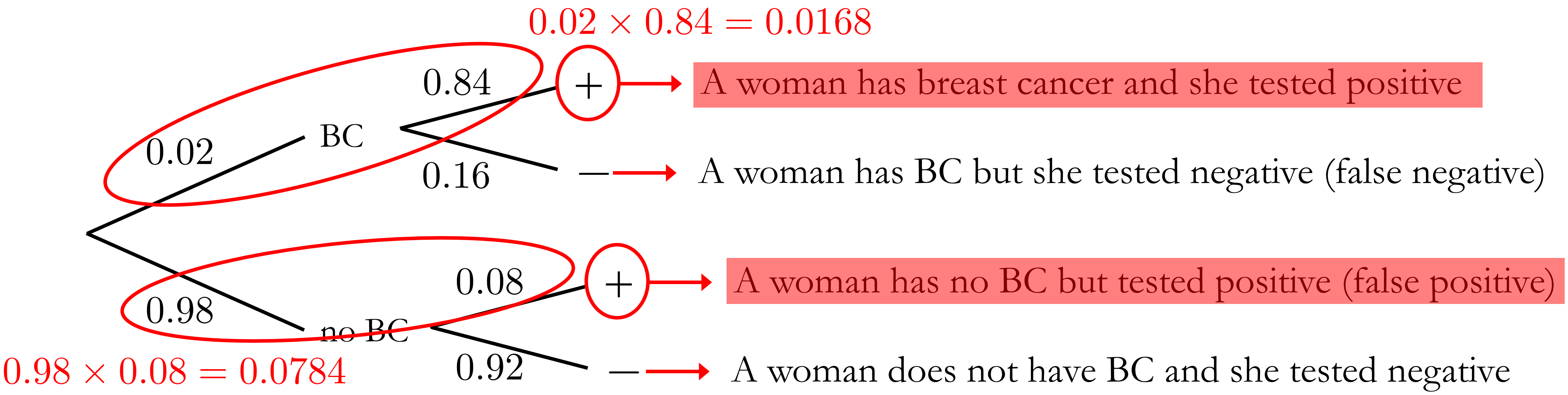

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$$= \frac{0.02 \times 0.84}{0.0168 + 0.0784} \approx 0.176$$

Breast Cancer Example

Conclusion:

If a woman tests positive for a mammogram, the probability that she has cancer is about 0.176, or 17.6%.

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If a woman tests positive for a mammogram, the probability that she has cancer is about 0.176, or 17.6%.

If she repeats the test and it turns out positive again, do you think her chances of actually having cancer are higher than 17.6%?

Breast Cancer Example

Conclusion:

If a woman tests positive for a mammogram, the probability that she has cancer is about 0.176, or 17.6%.

If she repeats the test and it turns out positive again, do you think her chances of actually having cancer are higher than 17.6%?

Your prior assumption now has to change: before taking the test, this woman's chances of having breast cancer were 2%; after she has tested positive, her chances increased to 17.6%

Breast Cancer Example

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If a woman tests positive for a mammogram, the probability that she has cancer is about 0.176, or 17.6%.

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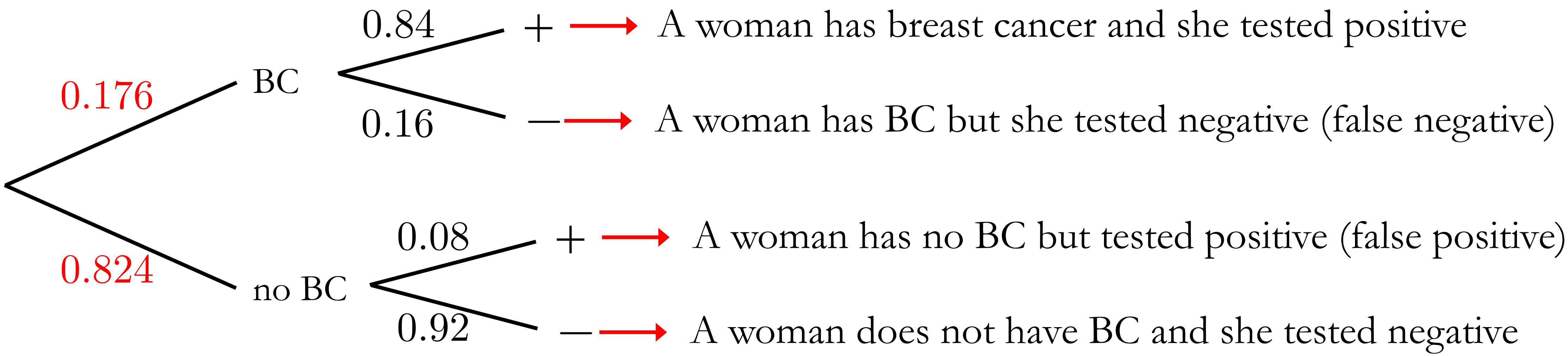

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Based on the new tree:

$$P(BC|+) = \frac{P(BC \text{ and } +)}{P(+)} = \frac{0.176 \times 0.84}{(0.176 \times 0.84) + (0.824 \times 0.08)}$$

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Therefore, if this woman's test turns out positive again, her chances of having breast cancer have gone up to 69% ☹