1. (56%) Determine if the series below converge or diverge. In case of an alternating series, check whether the series converges absolutely, conditionally, or diverges:

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\,(\ln n)^3}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{3}{n}\right)^n$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(0.8)^n}{n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n^{1/n}+8)^n}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n+5)}{n^2+1}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{|\sin n|}{n(n^2 + \sqrt{n} + 1)}$

2. (8%) Use series to write the number $0.\overline{213}$ as a ratio $\dfrac{m}{n}$ of two nonzero integers. Show your work

3. (6%)Find two series: $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ with positive terms such that:

$$a_n \to 0, b_n \to 0, \sum_{n=1}^{\infty} a_n \text{ converges, } \sum_{n=1}^{\infty} b_n \text{ diverges}$$

4. Consider the series: $\sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^n$

   (a) (2%) Find the fourth partial sum s_4.

   (b) (3%) Estimate the magnitude of the error involved in using s_4 to approximate the sum of the entire series

   (c) (3%) Find the sum of the series:(that is, the limit to which it converges)

   (d) (2%) Deduce using parts a and c above **the exact error involved** in using s_4 to approximate the sum of the entire series.

5. (10%) Find the sum of the series:

$$\sum_{n=1}^{\infty} \frac{3}{(n+1)(n+2)}$$

6. (10%) For what values of x does the series $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$ converge?

7. (5%) *BONUS* : Give an example of a power series that converges for all $x \in \mathbb{R}$.