American University of Beirut MATH 201 Calculus and Analytic Geometry III

Spring 2003

quiz # 1

Name:

ID #:

- **1.** (8 points) Find the limit of the following sequence $a_n = (e^n 1)^{\frac{1}{n}}$.
- 2. Determine if the following series converge or diverge. Justify your answers

a. (8 points)
$$\sum_{n=2}^{+\infty} \frac{1}{n\sqrt{n}}$$

b. (8 points) $\sum_{n=1}^{+\infty} \left(1 - \frac{2}{n}\right)^{3n}$
c. (8 points) $\sum_{n=1}^{+\infty} (-1)^n \frac{\sin n}{n^2}$
d. (10 points) $\sum_{n=2}^{+\infty} \frac{1}{n(\ln n)^{3/2}}$

3. (16 points) Find the sum of the series $\sum_{n=0}^{+\infty} \left[(-1)^n \frac{(\pi)^n}{4^n} - \frac{1}{(n+1)(n+2)} \right].$

4. (22 points) What is the interval of convergence of the power series $\sum_{n=0}^{+\infty} \frac{(\frac{1}{2})^n}{3n+1} (x+2)^n.$

(be sure to check convergence at the endpoints).

5. The Maclaurin series for $\tan x$ is given by:

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots$$

- a. (8 points) Using the series, find the first three nonzero terms in the Maclaurin series for f(x) = ln(cos x).
 (hint: what is f'(x)?).
- b. (6 points) For what values of x can we replace $\sin x$ by $x \frac{x^3}{6}$ with an error of magnitude no greater than 10^{-3} .
- 6. (6 points) Use power series to evaluate the limit: $\lim_{x\to 0} \frac{e^{3x^2} 1}{x^2}$.