# American University of Beirut <br> MATH 201 

Calculus and Analytic Geometry III
Spring 2003
quiz \# 1

Name: $\qquad$ ID \#:

1. (8 points) Find the limit of the following sequence $a_{n}=\left(e^{n}-1\right)^{\frac{1}{n}}$.
2. Determine if the following series converge or diverge. Justify your answers
a. $(8$ points $) \sum_{n=2}^{+\infty} \frac{1}{n \sqrt{n}}$
b. $(8$ points $) \sum_{n=1}^{+\infty}\left(1-\frac{2}{n}\right)^{3 n}$
c. (8 points) $\sum_{n=1}^{+\infty}(-1)^{n} \frac{\sin n}{n^{2}}$
d. $(10$ points $) \sum_{n=2}^{+\infty} \frac{1}{n(\ln n)^{3 / 2}}$
3. (16 points) Find the sum of the series $\sum_{n=0}^{+\infty}\left[(-1)^{n} \frac{(\pi)^{n}}{4^{n}}-\frac{1}{(n+1)(n+2)}\right]$.
4. (22 points) What is the interval of convergence of the power series $\sum_{n=0}^{+\infty} \frac{\left(\frac{1}{2}\right)^{n}}{3 n+1}(x+2)^{n}$. (be sure to check convergence at the endpoints).
5. The Maclaurin series for $\tan x$ is given by:

$$
\tan x=x+\frac{x^{3}}{3}+\frac{2 x^{5}}{15}+\ldots
$$

a. (8 points) Using the series, find the first three nonzero terms in the Maclaurin series for $f(x)=\ln (\cos x)$.
(hint: what is $f^{\prime}(x)$ ?).
b. (6 points) For what values of $x$ can we replace $\sin x$ by $x-\frac{x^{3}}{6}$ with an error of magnitude no greater than $10^{-3}$.
6. (6 points) Use power series to evaluate the limit: $\lim _{x \rightarrow 0} \frac{e^{3 x^{2}}-1}{x^{2}}$.

