

Problem I: (25 points)

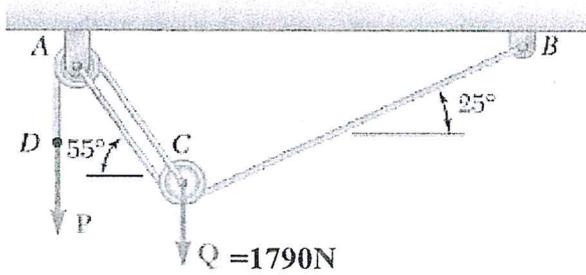


Figure I

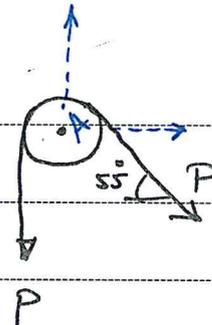
An 1790-N load Q is applied to the pulley C , which can roll on the cable ACB as shown in **Figure I**. The pulley is held in the position shown by a second cable CAD , which passes over the pulley A and supports a load P .

- Determine the tension in cable ACB , and the magnitude of load P .

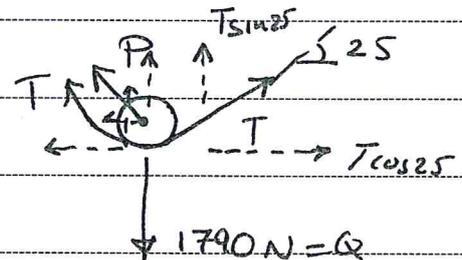
Note: FBD must be included

Calculations and/or Diagrams:

① at 'A'



② at C



$$\rightarrow \sum F_x = 0$$

$$T \cos 25^\circ - P \cos 55^\circ - T \cos 55^\circ = 0 \quad \text{--- (1)}$$

$$0.332 T = P \cos 55^\circ$$

$$T = 1.723 P \quad \text{--- (1) ✓}$$

$$\uparrow \sum F_y = 0$$

$$-1790 + T \sin 25^\circ + P \sin 55^\circ + T \sin 55^\circ = 0$$

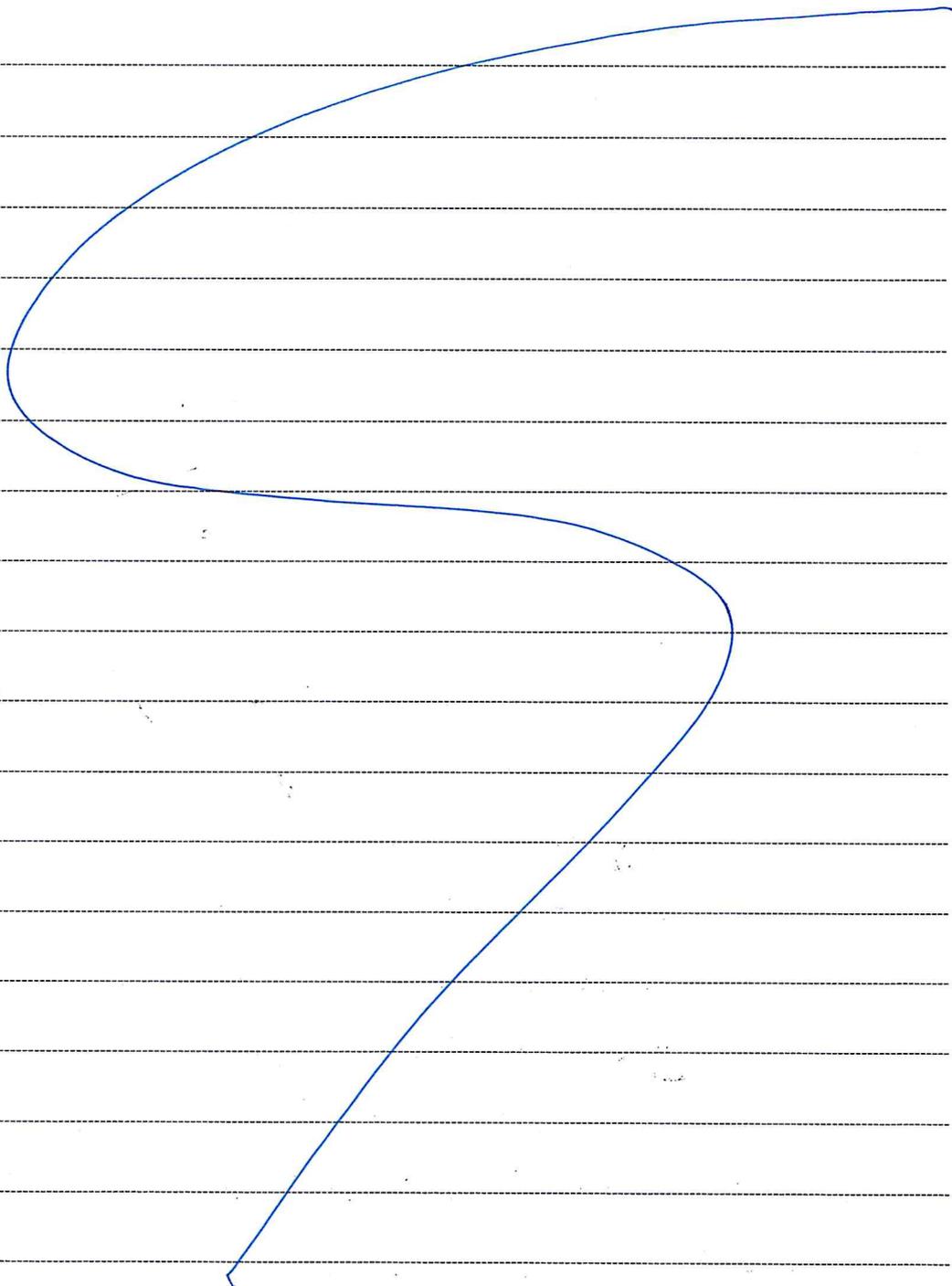
$$-1790 + 0.73 P + 0.82 P + 1.41 P = 0$$

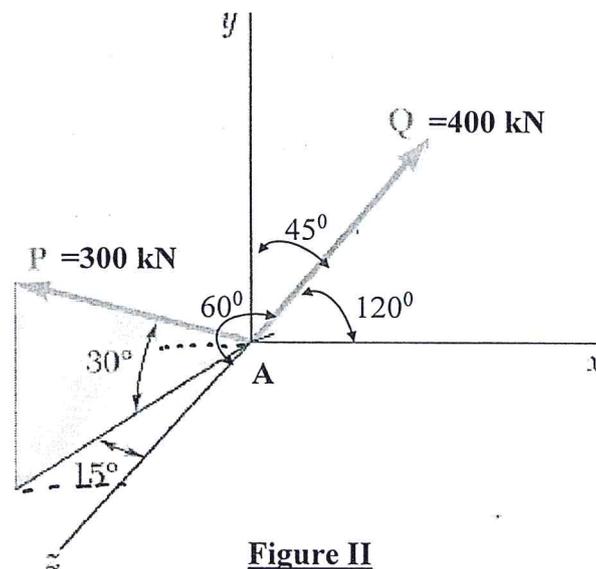
Calculations and/or Diagrams:

$$2.96 P = 1790$$

$$P = 604.4 \text{ N.}$$

$$T = 1042 \text{ N.}$$



Problem II: (35 points)**Figure II**

For the two forces shown in **Figure II**:

1. Determine the magnitude and direction angles of the resultant force acting at A.
Express your result as Cartesian vector. (15 points)
2. Determine the angles between **P** and **Q**. (5 points).
3. Determine the projected components of the force **P** along and perpendicular to **Q**.
Express the results as a Cartesian vector. (15 points).

Calculations and/or Diagrams:

$$\vec{Q} = 400 \cos 60^\circ \vec{k} + 400 \cos 120^\circ \vec{i} + 400 \cos 45^\circ \vec{j}$$

$$\vec{Q} = 200 \vec{k} - 200 \vec{i} + 282.84 \vec{j} \text{ (kN)}$$

$$\vec{P} = ? \quad P_y = 300 \sin 30 = 150$$

$$P_{x-y} = 300 \cos 30 = 260$$

$$P_x = 260 \sin 15 = 67.3$$

$$P_z = 260 \cos 15 = 251.14$$

$$\vec{P} = -67.3 \vec{i} + 150 \vec{j} + 251.14 \vec{k}$$

$$\vec{F}_R = -267.3 \vec{i} + 432.4 \vec{j} + 451.14 \vec{k}$$

$$F_R = \sqrt{267.3^2 + 432.4^2 + 451.14^2} = 679.85 \text{ kN}$$

$$\downarrow 461946.35$$

Calculations and/or Diagrams (cont'd):

$$\alpha = \cos^{-1} \frac{-267.3}{679.85} = 113.15^\circ$$

$$\beta = \cos^{-1} \frac{432.4}{679.85} = 50.50^\circ$$

$$\gamma = \cos^{-1} \frac{451.44}{679.85} = 48.40^\circ$$

(2) θ between P & Q

dot product

$$\vec{P} \cdot \vec{Q} = P \cdot Q \cos \theta$$

$$-67.3 \times -200 + 282.84 \times 150 + 251.14 \times 300 = P \cdot Q \cos \theta$$

$$106114 = 400 \times 300 \cos \theta$$

$$\cos \theta = 0.884 \quad \checkmark$$

$$\theta = 27.83^\circ \quad \checkmark$$

dot product.

(3) $P_Q = \frac{\vec{P} \cdot \vec{L}_Q}{L_Q}$

$$\vec{L}_Q = \cos 120^\circ \vec{i} + \cos 45^\circ \vec{j} + \cos 60^\circ \vec{k}$$

$$\vec{L}_Q = -0.5 \vec{i} + 0.707 \vec{j} + 0.5 \vec{k}$$

$$P_Q = -67.3(-0.5) + 0.707(150) + 0.5(251.44)$$

$$P_Q = 265.42 \text{ kN}$$

$$\vec{P}_{\parallel Q} = 265.42 / (-0.5 \vec{i} + 0.707 \vec{j} + 0.5 \vec{k})$$

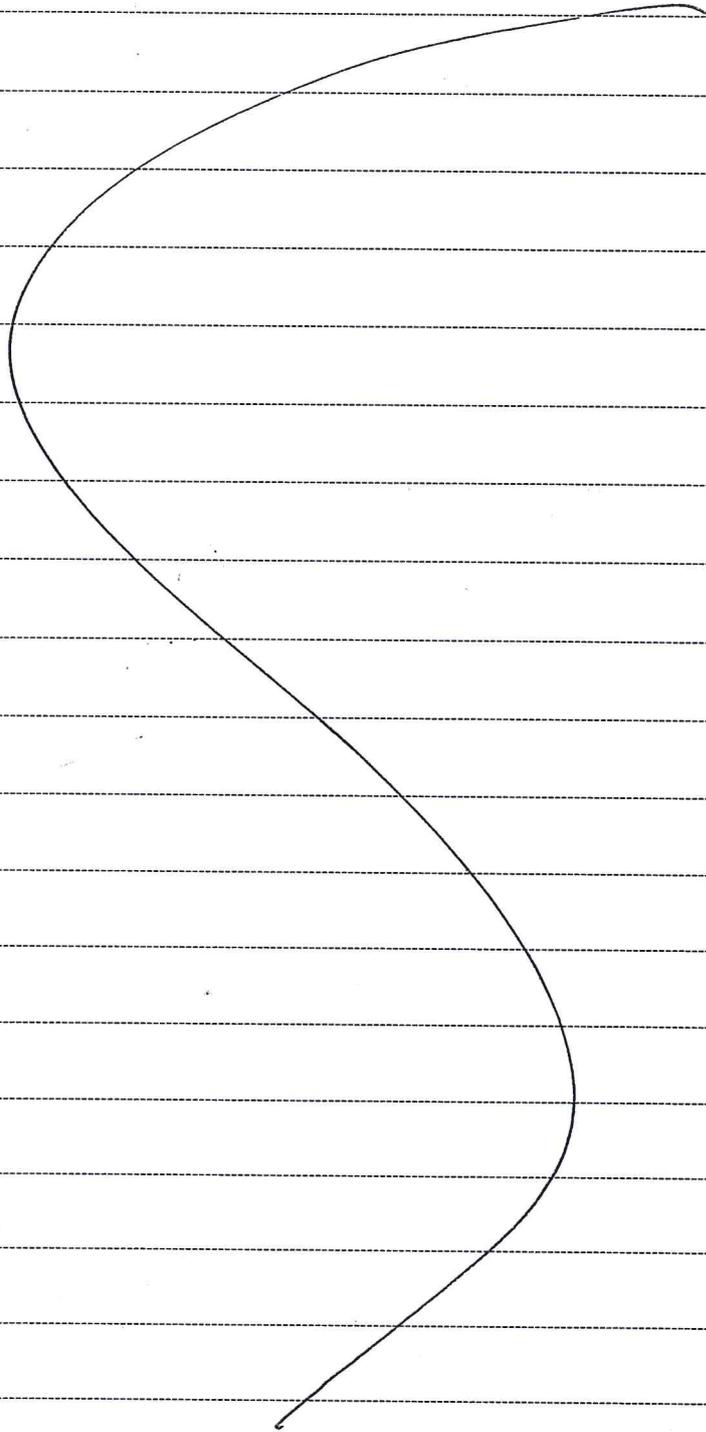
$$= -132.71 \vec{i} + 187.65 \vec{j} + 132.71 \vec{k}$$

$$\vec{P}_{\perp Q} = \vec{P} - \vec{P}_{\parallel Q} = (-67.3 + 132.71) \vec{i} + (150 - 187.65) \vec{j} + (251.14 - 132.71) \vec{k}$$

$$= 65.41 \vec{i} - 37.65 \vec{j} + 118.43 \vec{k}$$

Calculations and/or Diagrams (cont'd):

$$P_h \text{ to } Q = 140.43 \text{ kN}$$



Problem III: (40 points)

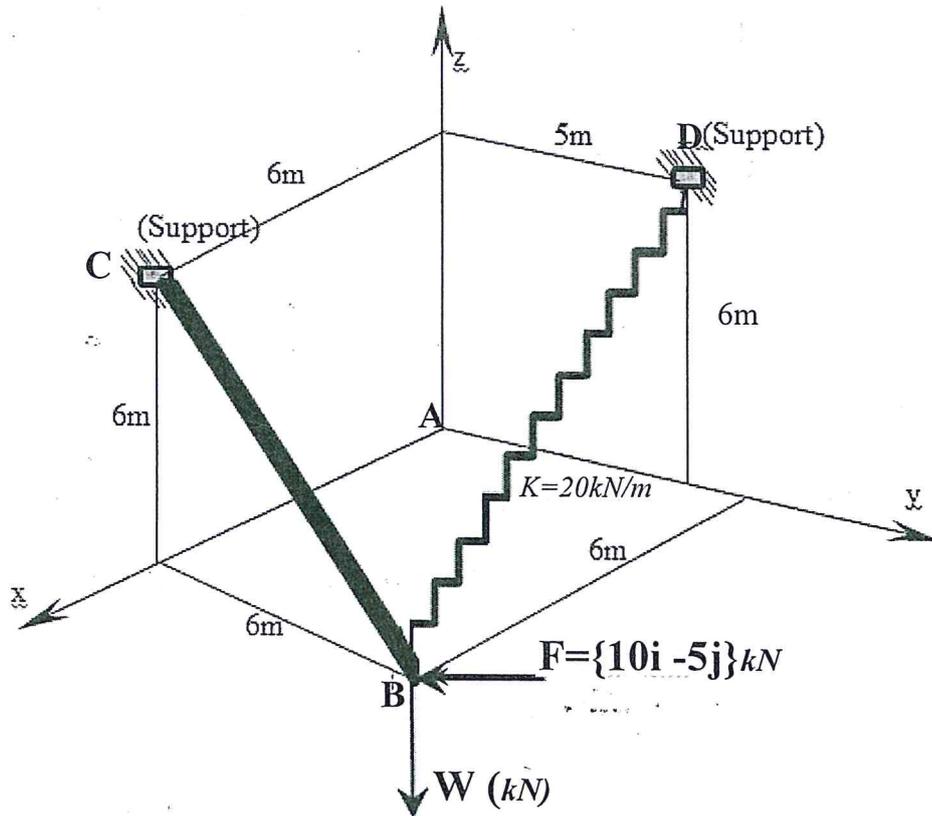


Figure III

The spring/chord system shown in Figure III is in equilibrium.

1. Determine the forces in spring **BD**, chord **BC** and the weight **W** suspended at **B**. (30 points).
2. What would be the un-stretched length of spring **BD**? (10 points).

Calculations and/or Diagrams:

$$A(0,0,0) \quad , \quad B(6,6,0) \quad C(6,0,6) \\ D(0,5,6)$$

$$\vec{W} = -W\vec{k}$$

$$\vec{F} = 10\vec{i} - 5\vec{j}$$

$$\vec{F}_{BD} = F_{BD} \cdot \vec{u}_{BD} = \left(\frac{-6\vec{i} - \vec{j} + 6\vec{k}}{\sqrt{36+1+36}} \right) F_{BD} \Rightarrow 8.54$$

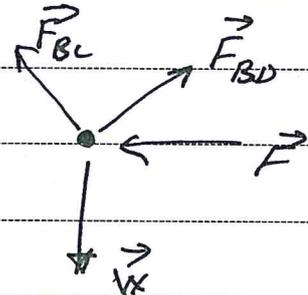
$$= -0.702 F_{BD} \vec{i} - 0.117 F_{BD} \vec{j} + 0.702 F_{BD} \vec{k}$$

Calculations and/or Diagrams (cont'd):

$$\vec{F}_{BC} = F_{BC} \cdot \vec{u}_{BC} = F_{BC} \left(\frac{0\vec{i} - 6\vec{j} + 6\vec{k}}{\sqrt{72}} \right)$$

$$\vec{F}_{BC} = -0.707 F_{BC} \vec{j} + 0.707 F_{BC} \vec{k}$$

FBD at B



$$\sum F_x = 0. \quad 10 - 0.707 F_{BD} = 0$$

$$F_{BD} = 14.24 \text{ kN (Tension)}$$

$$\sum F_y = 0$$

$$-5 - 0.707 F_{BD} - 0.707 F_{BC} = 0$$

$$F_{BC} = -9.43 \text{ kN (comp)}$$

$$\sum F_z = 0$$

$$0.707 F_{BD} + 0.707 F_{BC} - W = 0$$

$$W = +9.996 - 6.66 = 3.33 \text{ kN}$$

$$\text{Stretch in BD } \Delta = \frac{14.24}{20} = 0.712 \text{ m}$$

$$L_{BD \text{ unstretched}} = L_{BD} - 0.712 = 8.54 - 0.712$$

$$L_{BD \text{ unstretched}} = 7.83 \text{ m}$$