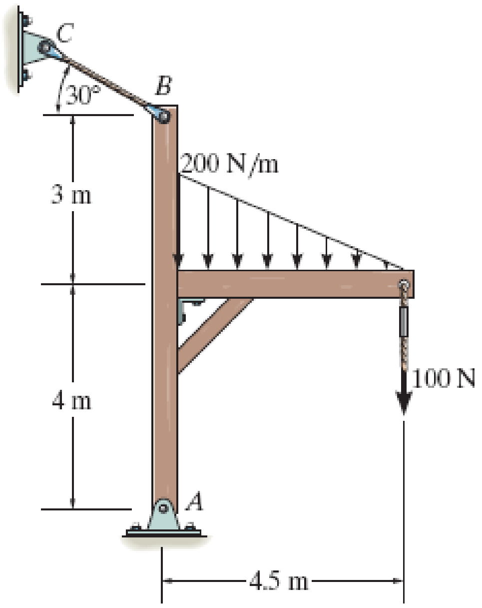
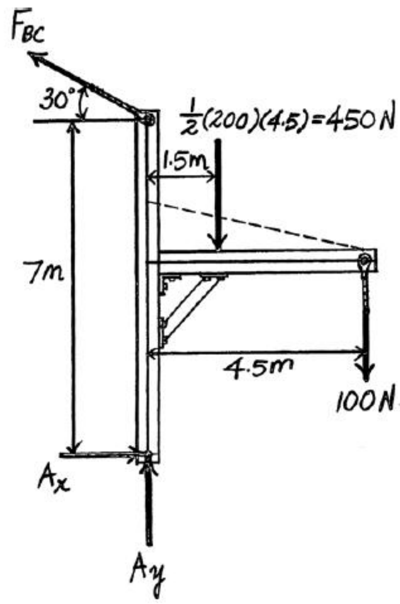
PROBLEMS # 4-SOLUTION

Topics Equilibrium of a rigid body (Chapter 5 in textbook).

Textbook: Engineering Mechanics, by R.C. Hibbeler, Pearson, 12th Edition.

•**5–88.** Determine the horizontal and vertical components of reaction at pin A and force in the cable BC. Neglect the thickness of the members.





$$\circlearrowleft + \Sigma M_A = 0: \quad F_{BC}\cos 30°(7) - 450(1.5) - 100(4.5) = 0$$

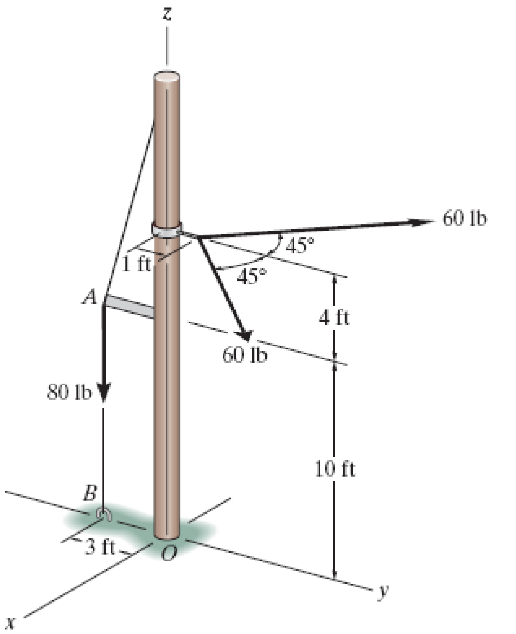
$$F_{BC} = 185.58 \text{ N} = 186 \text{ N}$$

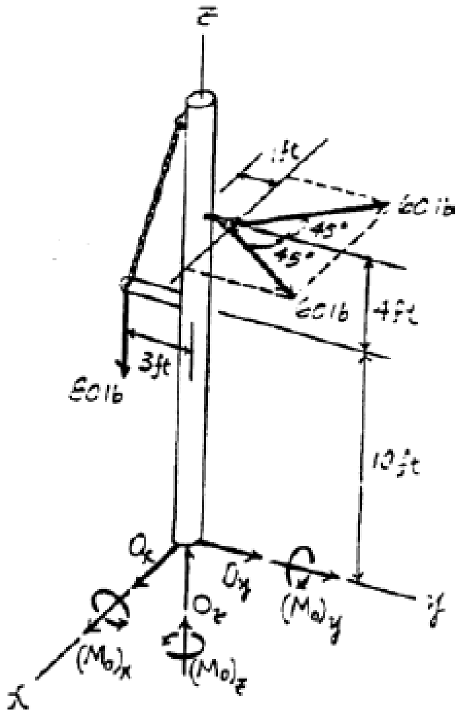
$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad A_x - 185.58 \cos 30° = 0 \qquad A_x = 161 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 185.58 \sin 30° - 450 - 100 = 0$$

$$A_y = 457 \text{ N}$$

*__5–64.__ The pole for a power line is subjected to the two cable forces of 60 lb, each force lying in a plane parallel to the plane. If the tension in the guy wire *AB* is 80 lb, determine the *x*, *y*, *z* components of reaction at the fixed base of the pole, *O*.





Equations of Equilibrium :

$\Sigma F_x = 0;\quad O_x + 60\sin 45° - 60\sin 45° = 0$

$O_x = 0$ **Ans**

$\Sigma F_y = 0;\quad O_y + 60\cos 45° + 60\cos 45° = 0$

$O_y = -84.9\ \text{lb}$ **Ans**

$\Sigma F_z = 0;\quad O_z - 80 = 0 \qquad O_z = 80.0\ \text{lb}$ **Ans**

$\Sigma M_x = 0;\quad (M_O)_x + 80(3) - 2[60\cos 45°(14)] = 0$

$(M_O)_x = 948\ \text{lb}\cdot\text{ft}$ **Ans**

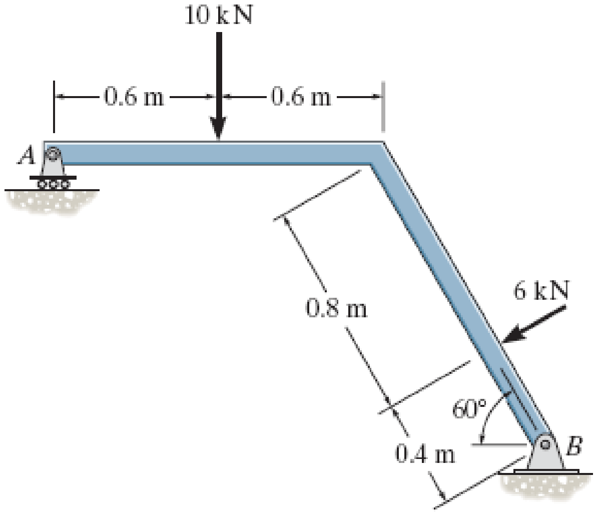
$\Sigma M_y = 0;\quad (M_O)_y + 60\sin 45°(14) - 60\sin 45°(14) = 0$

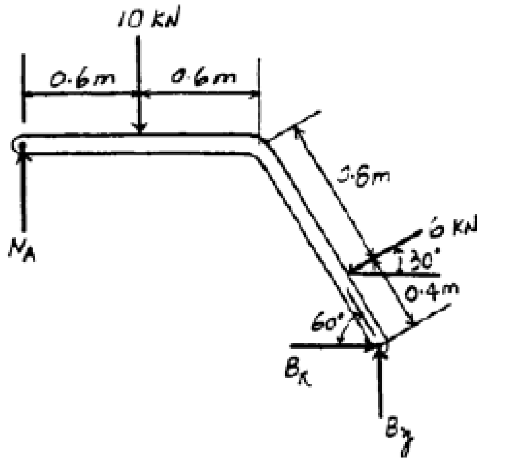
$(M_O)_y = 0$ **Ans**

$\Sigma M_z = 0;\quad (M_O)_y + 60\sin 45°(1) - 60\sin 45°(1) = 0$

$(M_O), = 0$ **Ans**

5–91. Determine the normal reaction at the roller *A* and horizontal and vertical components at pin *B* for equilibrium of the member.





Equations of Equilibrium : The normal reaction N_A can be obtained directly by summing moments about point *B*.

$$\curvearrowleft + \Sigma M_A = 0; \quad 10(0.6 + 1.2\cos 60°) + 6(0.4) - N_A(1.2 + 1.2\cos 60°) = 0$$

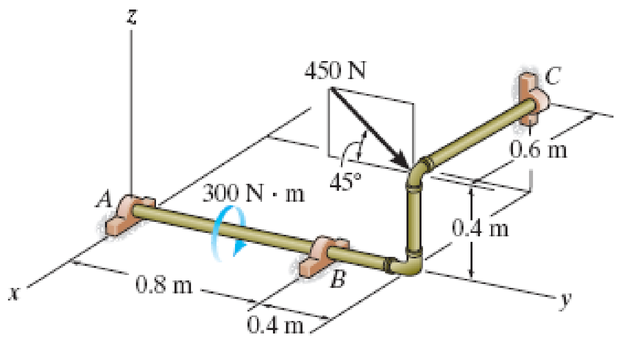
$$N_A = 8.00 \text{ kN} \qquad \textbf{Ans}$$

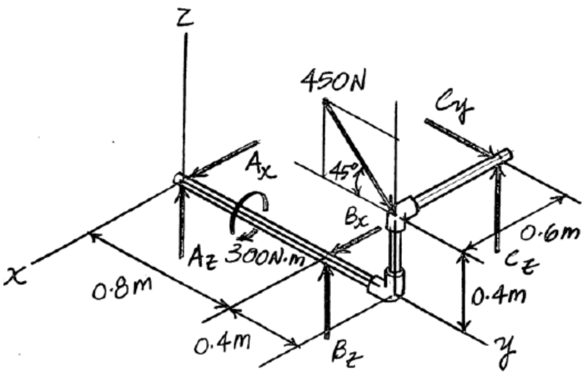
$$\xrightarrow{+} \Sigma F_x = 0; \quad B_x - 6\cos 30° = 0 \quad B_x = 5.20 \text{ kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad B_y + 8.00 - 6\sin 30° - 10 = 0$$

$$B_y = 5.00 \text{ kN} \qquad \textbf{Ans}$$

***5–72.** Determine the components of reaction acting at the smooth journal bearings A, B, C.





Equations of Equilibrium: From the free - body diagram of the shaft, Fig. *a*, C_y and C_z can be obtained by writing the force equation of equilibrium along the *y* axis and the moment equation of equilibrium about the *y* axis.

$\Sigma F_y = 0;\quad 450\cos 45° + C_y = 0$

$$C_y = -318.20\text{ N} = -318\text{ N} \qquad \textbf{Ans.}$$

$\Sigma M_y = 0;\quad C_z(0.6) - 300 = 0$

$$C_z = 500\text{ N} \qquad \textbf{Ans.}$$

Using the above results and writing the moment equations of equilibrium about the *x* and *z* axes,

$\Sigma M_x = 0;\quad B_z(0.8) - 450\cos 45°(0.4) - 450\sin 45°(0.8 + 0.4) + (318.20)(0.4) + 500(0.8 + 0.4) = 0$

$$B_z = -272.70\text{ N} = -273\text{ N} \qquad \textbf{Ans.}$$

$\Sigma M_z = 0;\quad -B_x(0.8) - (-318.20)(0.6) = 0$

$$B_x = 238.65\text{ N} = 239\text{ N} \qquad \textbf{Ans.}$$

Finally, using the above results and writing the force equations of equilibrium along the *x* and *y* axes,

$\Sigma F_x = 0;\quad A_x + 238.5 = 0$

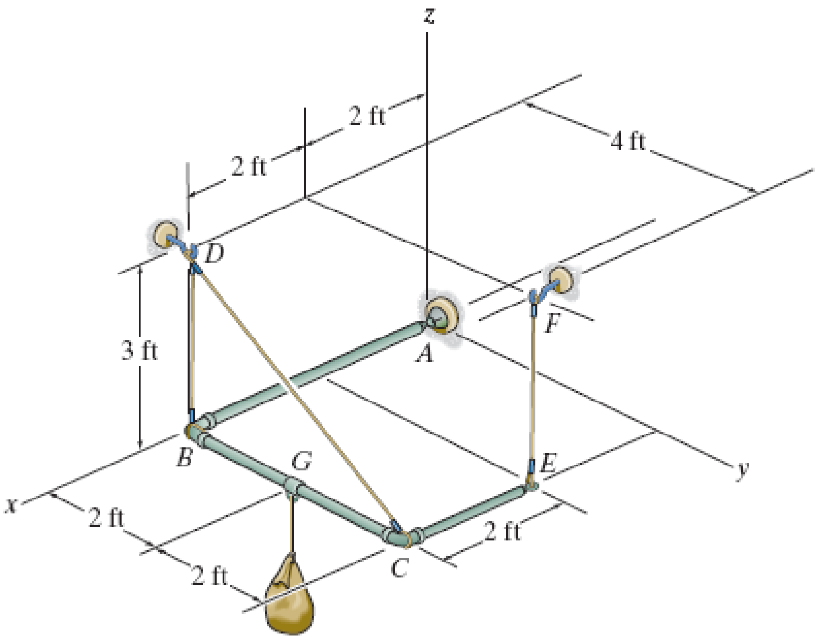
$$A_x = -238.65\text{ N} = -239\text{ N} \qquad \textbf{Ans.}$$

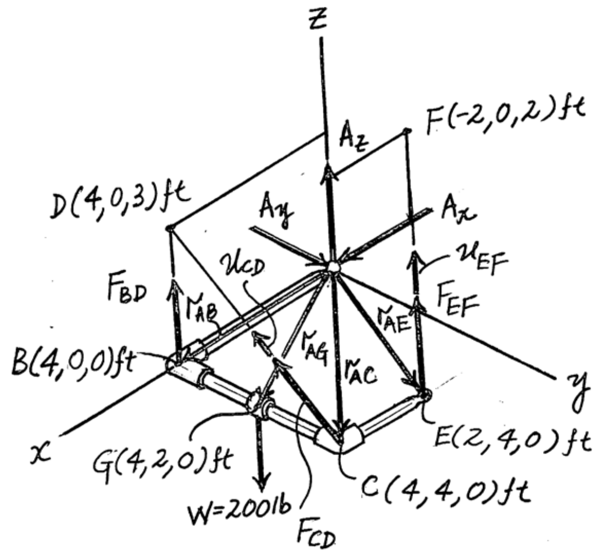
$\Sigma F_z = 0;\quad A_z - (-272.70) + 500 - 450\sin 45° = 0$

$$A_z = 90.90\text{ N} = 90.9\text{ N} \qquad \textbf{Ans.}$$

The negative signs indicate that $\mathbf{C}_y$, $\mathbf{B}_z$ and $\mathbf{A}_x$ act in the opposite sense of that shown on the free - body diagram.

5–74. If the load has a weight of 200 lb, determine the x, y, z components of reaction at the ball-and-socket joint A (*pin*) and the tension in each of the wires.





(a)

Vector Approach:

Equations of Equilibrium: Expressing the forces indicated on the free - body diagram, Fig. *a*, in Cartesian vector form,

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$
$$\mathbf{W} = [-200\mathbf{k}]\text{ lb}$$
$$\mathbf{F}_{BD} = F_{BD}\mathbf{k}$$
$$\mathbf{F}_{CD} = F_{CD}\mathbf{u}_{CD} = F_{CD}\left[\frac{(4-4)\mathbf{i}+(0-4)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(4-4)^2+(0-4)^2+(3-0)^2}}\right] = \left(-\frac{4}{5}F_{CD}\mathbf{j}+\frac{3}{5}F_{CD}\mathbf{k}\right)$$
$$\mathbf{F}_{EF} = F_{EF}\mathbf{k}$$

Applying the force equation of equilibrium,

$$\Sigma\mathbf{F} = \mathbf{0}; \quad \mathbf{F}_A + \mathbf{F}_{BD} + \mathbf{F}_{CD} + \mathbf{F}_{EF} + \mathbf{W} = \mathbf{0}$$
$$(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) + F_{BD}\mathbf{k} + \left(-\frac{4}{5}F_{CD}\mathbf{j} + \frac{3}{5}F_{CD}\mathbf{k}\right) + F_{EF}\mathbf{k} + (-200\mathbf{k}) = \mathbf{0}$$
$$(A_x)\mathbf{i} + \left(A_y - \frac{4}{5}F_{CD}\right)\mathbf{j} + \left(A_z + F_{BD} + \frac{3}{5}F_{CD} + F_{EF} - 200\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components,

$$A_x = 0 \qquad (1)$$
$$A_y - \frac{4}{5}F_{CD} = 0 \qquad (2)$$
$$A_z + F_{BD} + \frac{3}{5}F_{CD} + F_{EF} - 200 = 0 \qquad (3)$$

In order to write the moment equation of equilibrium about point *A*, the position vectors $\mathbf{r}_{AB}$, $\mathbf{r}_{AG}$, $\mathbf{r}_{AC}$, and $\mathbf{r}_{AE}$ must be determined first.

$$\mathbf{r}_{AB} = [4\mathbf{i}]\text{ft}$$
$$\mathbf{r}_{AG} = [4\mathbf{i}+2\mathbf{j}]\text{ft}$$
$$\mathbf{r}_{AC} = [4\mathbf{i}+4\mathbf{j}]\text{ft}$$
$$\mathbf{r}_{AE} = [2\mathbf{i}+4\mathbf{j}]\text{ft}$$

Thus,

$$\Sigma\mathbf{M}_A = \mathbf{0}; \ (\mathbf{r}_{AB} \times \mathbf{F}_{BD}) + (\mathbf{r}_{AC} \times \mathbf{F}_{CD}) + (\mathbf{r}_{AE} \times \mathbf{F}_{EF}) + (\mathbf{r}_{AG} \times \mathbf{W}) = \mathbf{0}$$
$$(4\mathbf{i}) \times (F_{BD}\mathbf{k}) + (4\mathbf{i}+4\mathbf{j}) \times \left(-\frac{4}{5}F_{CD}\mathbf{j} + \frac{3}{5}F_{CD}\mathbf{k}\right) + (2\mathbf{i}+4\mathbf{j}) \times (F_{EF}\mathbf{k}) + (4\mathbf{i}+2\mathbf{j}) \times (-200\mathbf{k})$$
$$\left(\frac{12}{5}F_{CD} + 4F_{EF} - 400\right)\mathbf{i} + \left(-4F_{BD} - \frac{12}{5}F_{CD} - 2F_{EF} + 800\right)\mathbf{j} + \left(-\frac{16}{5}F_{CD}\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components,

$$\frac{12}{5}F_{CD} + 4F_{EF} - 400 = 0 \qquad (4)$$

$$-4F_{BD} - \frac{12}{15}F_{CD} - 2F_{EF} + 800 = 0 \qquad (5)$$

$$-\frac{16}{5}F_{CD} = 0 \qquad (6)$$

Solving Eqs. (1) through (6),

$F_{CD} = 0$ **Ans.**

$F_{EF} = 100\text{ lb}$ **Ans.**

$F_{BD} = 150\text{ lb}$ **Ans.**

$A_x = 0$ **Ans.**

$A_y = 0$ **Ans.**

A_z **50** lb **lb** **Ans.**

The negative signs indicate that $\mathbf{A}_z$ acts in the opposite sense to that on the free - body diagram.

Scalar Approach:

$$\sum F_x = 0 \quad A_x = 0 \qquad \text{.......Eq.1}$$

$$\sum F_y = 0 \quad A_y - F_{CD}\left(\frac{4}{5}\right) = 0 \qquad \text{......Eq.2}$$

$$\sum F_z = 0 \quad A_z + F_{BD} + F_{CD}\left(\frac{3}{5}\right) + F_{EF} - 200 = 0 \qquad \text{......Eq.3}$$

$$\sum M_x = 0 \quad -200(2) + F_{CD}\left(\frac{3}{5}\right)(4) + F_{EF}(4) = 0 \qquad \text{......Eq.4}$$

$$\sum M_y = 0 \quad -F_{BD}(4) - F_{CD}\left(\frac{3}{5}\right)(4) - F_{EF}(2) + 200(4) = 0 \qquad \text{.....Eq.5}$$

$$\sum M_z = 0 \quad -F_{CD}\left(\frac{4}{5}\right)(4) = 0$$ …..Eq.6

$$Eq.1 \Rightarrow A_x = 0$$

$$Eq.6 \Rightarrow F_{CD} = 0$$

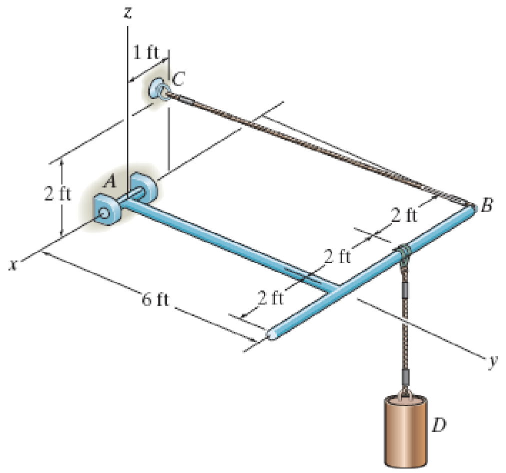
$$Eq.2 \Rightarrow A_y = 0$$

$$Eq.4 \Rightarrow F_{EF} = 100N$$

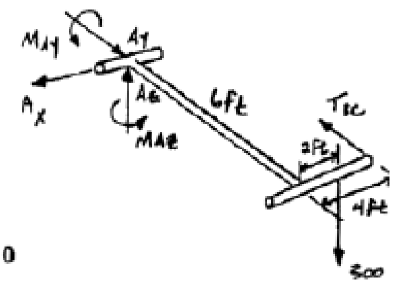
$$Eq.5 \Rightarrow F_{BD} = 150N$$

$$Eq.3 \Rightarrow A_z = -50N$$

*5–76. The member is supported by a pin at *A* and a cable *BC*. If the load at *D* is 300 lb, determine the *x, y, z* components of reaction at the support *A (fixed, but it allows rotation around x)* and the tension in *cable B C.*



$$\mathbf{T}_{BC} = T_{BC}\left\{\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right\} \text{ ft}$$



$\Sigma F_x = 0;\quad A_x + \left(\frac{3}{7}\right)T_{BC} = 0$

$\Sigma F_y = 0;\quad A_y - \left(\frac{6}{7}\right)T_{BC} = 0$

$\Sigma F_z = 0;\quad A_z - 300 + \left(\frac{2}{7}\right)T_{BC} = 0$

$\Sigma M_x = 0;\quad -300\,(6) + \left(\frac{2}{7}\right)T_{BC}\ (6) = 0$

$\Sigma M_y = 0;\quad M_{Ay} - 300\,(2) + \left(\frac{2}{7}\right)T_{BC}\ (4) = 0$

$\Sigma M_z = 0;\quad M_{Az} - \left(\frac{3}{7}\right)T_{BC}\ (6) + \left(\frac{6}{7}\right)T_{BC}\ (4) = 0$

Solving,

$T_{BC} = 1.05$ kip **Ans**

$A_x = -450$ lb **Ans**

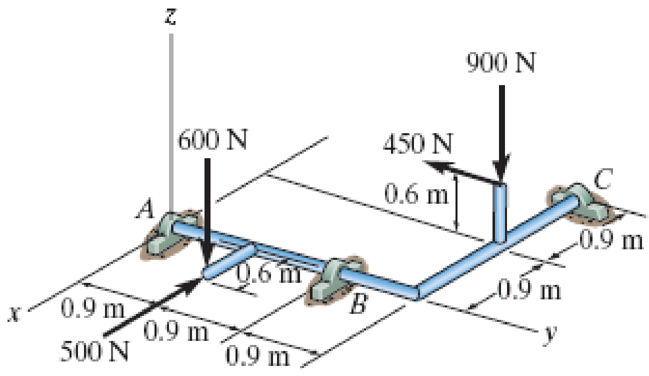
$A_y = 900$ lb **Ans**

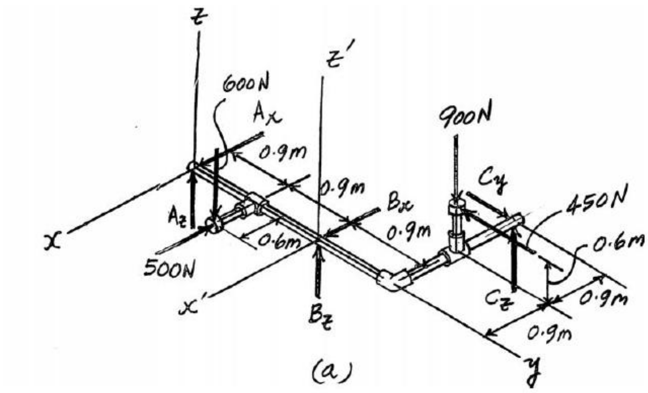
$A_z = 0$ **Ans**

$M_{Ay} = -600$ lb · ft **Ans**

$M_{Az} = -900$ lb · ft **Ans**

5-69. The shaft is supported by three smooth journal bearings at *A*, *B, and C.* Determine the components of reaction at these bearings.
Hint: A and B allow translation along y-direction, and C allows translation along x-direction.





Free-body Diagram

Equations of Equilibrium: From the free - body diagram, Fig. *a*, C_y and C_z can be obtained by writing the force equation of equilibrium along the *y* axis and the moment equation of equilibrium about the *y* axis.

$\Sigma F_y = 0;\quad C_y - 450 = 0$

$C_y = 450\,\text{N}$ **Ans.**

$\Sigma M_y = 0;\quad C_z(0.9 + 0.9) - 900(0.9) + 600(0.6) = 0$

$C_z = 250\,\text{N}$ **Ans.**

Using the above results

$\Sigma M_x = 0;\quad B_z(0.9 + 0.9) + 250(0.9 + 0.9 + 0.9) + 450(0.6) - 900(0.9 + 0.9 + 0.9) - 600(0.9) = 0$

$B_z = 1125\,\text{N} = 1.125\,\text{kN}$ **Ans.**

$\Sigma M_{x'} = 0;\quad 600(0.9) + 450(0.6) - 900(0.9) + 250(0.9) - A_z(0.9 + 0.9) = 0$

$A_z = 125\,\text{N}$ **Ans.**

$\Sigma M_z = 0;\quad -B_x(0.9 + 0.9) + 500(0.9) + 450(0.9) - 450(0.9 + 0.9) = 0$

$B_x = 25\,\text{N}$ **Ans.**

$\Sigma F_x = 0;\quad A_x + 25 - 500 = 0$

$A_x = 475\,\text{N}$ **Ans.**