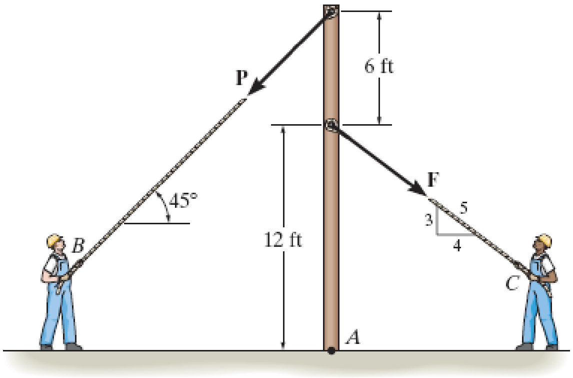
Problmes # 3-SOLUTION

Topics Force System Resultants (Chapter 4 in textbook).

Textbook: Engineering Mechanics, by R.C. Hibbeler, Pearson, 12th Edition.

*4–4. Two men exert forces of F=80lb N and P=50lb on the ropes. Determine the moment of each force about A. Which way will the pole rotate, clockwise or counterclockwise?



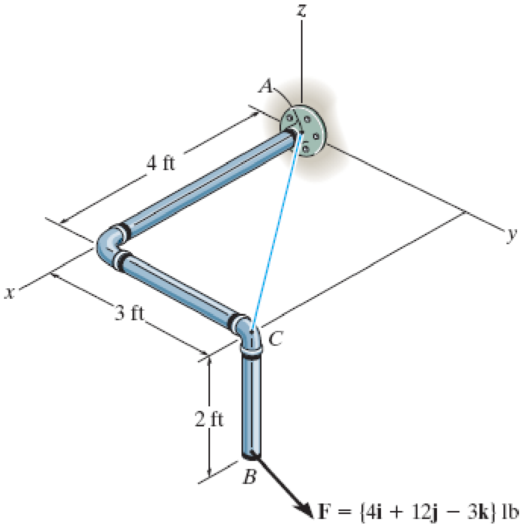
$$\curvearrowright + \ (M_A)_C = 80\left(\frac{4}{5}\right)(12) = 768 \text{ lb}\cdot\text{ft} \curvearrowright \quad \text{Ans}$$

$$\curvearrowleft + \ (M_A)_B = 50\,(\cos 45^\circ)\,(18) = 636 \text{ lb}\cdot\text{ft} \curvearrowleft \quad \text{Ans}$$

Since $(M_A)_C > (M_A)_B$

Clockwise Ans

4–54. Determine the magnitude of the moments of the force F about the x, y, and z axes. Solve the problem (a) using a Cartesian vector approach and (b) using a scalar approach.



a) *Vector Analysis*

Position Vector :

$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\} \text{ ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\} \text{ ft}$$

Moment of Force* F *About x, y and z Axes : The unit vectors al x, y and z axes are **i**, **j** and **k** respectively. Applying Eq. 4 – 7l, we have

$$M_x = \mathbf{i} \cdot (\mathbf{r}_{AB} \times \mathbf{F}) = \begin{vmatrix} 1 & 0 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 1[3(-3) - (12)(-2)] - 0 + 0 = 15.0 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$

$$M_y = \mathbf{j} \cdot (\mathbf{r}_{AB} \times \mathbf{F}) = \begin{vmatrix} 0 & 1 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 1[4(-3) - (4)(-2)] + 0 = 4.00 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$

$$M_z = \mathbf{k} \cdot (\mathbf{r}_{AB} \times \mathbf{F}) = \begin{vmatrix} 0 & 0 & 1 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 0 + 1[4(12) - 4(3)] = 36.0 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$

b) *Scalar Analysis*

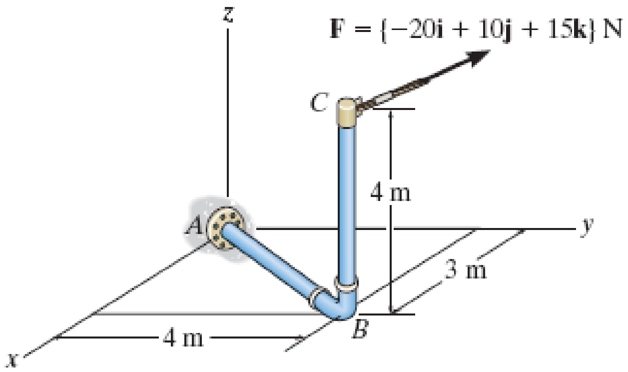
$$M_x = \Sigma M_x; \qquad M_x = 12(2) - 3(3) = 15.0 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$

$$M_y = \Sigma M_y; \qquad M_y = -4(2) + 3(4) = 4.00 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$

$$M_z = \Sigma M_z; \qquad M_z = -4(3) + 12(4) = 36.0 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$

z

***4–56.** Determine the moment produced by force **F** about segment *AB* of the pipe assembly.



Moment About Line AB: Either position vector $\mathbf{r}_{AC}$ or $\mathbf{r}_{BC}$ can be conveniently used to determine the moment of **F** about line *AB*.

$$\mathbf{r}_{AC} = (3-0)\mathbf{i} + (4-0)\mathbf{j} + (4-0)\mathbf{k} = [3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}]\ \text{m}$$

$$\mathbf{r}_{BC} = (3-3)\mathbf{i} + (4-4)\mathbf{j} + (4-0)\mathbf{k} = [4\mathbf{k}]\ \text{m}$$

The unit vector $\mathbf{u}_{AB}$, Fig. *a*, that specifies the direction of line *AB* is given by

$$\mathbf{u}_{AB} = \frac{(3-0)\mathbf{i} + (4-0)\mathbf{j} + (0-0)\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (0-0)^2}} = \frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$$

Thus, the magnitude of the moment of **F** about line *AB* is given by

$$M_{AB} = \mathbf{u}_{AB} \cdot \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \frac{3}{5} & \frac{4}{5} & 0 \\ 3 & 4 & 4 \\ -20 & 10 & 15 \end{vmatrix}$$

$$= \frac{3}{5}[4(15) - 10(4)] - \frac{4}{5}[3(15) - (-20)(4)] + 0$$

$$= -88\ \text{N}\cdot\text{m}$$

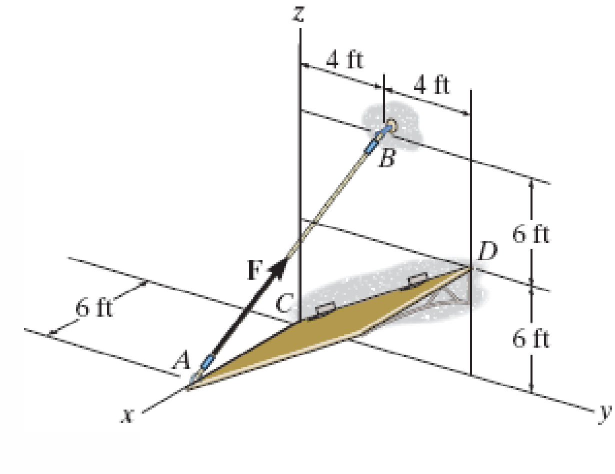
or

$$M_{AB} = \mathbf{u}_{AB} \cdot \mathbf{r}_{BC} \times \mathbf{F} = \begin{vmatrix} \frac{3}{5} & \frac{4}{5} & 0 \\ 0 & 0 & 4 \\ -20 & 10 & 15 \end{vmatrix}$$

$$= \frac{3}{5}[0(15) - 10(4)] - \frac{4}{5}[0(15) - (-20)(4)] + 0$$

$$= -88\ \text{N}\cdot\text{m}$$

•**4–61**. If the tension in the cable is F=140lb , determine the magnitude of the moment produced by this force about the hinged axis, CD, of the panel.



Moment About the CD axis: Either position vector $\mathbf{r}_{CA}$ or $\mathbf{r}_{DB}$, Fig. *a*, can be used to determine the moment of **F** about the *CD* axis.

$\mathbf{r}_{CA} = (6-0)\mathbf{i}+(0-0)\mathbf{j}+(0-0)\mathbf{k} = [6\mathbf{i}]\text{ft}$

$\mathbf{r}_{DB} = (0-0)\mathbf{i}+(4-8)\mathbf{j}+(12-6)\mathbf{k} = [-4\mathbf{j}+6\mathbf{k}]\text{ft}$

Referring to Fig. *a*, the force vector **F** can be written as

$$\mathbf{F} = F\mathbf{u}_{AB} = 140\left[\frac{(0-6)\mathbf{i}+(4-0)\mathbf{j}+(12-0)\mathbf{k}}{\sqrt{(0-6)^2+(4-0)^2+(12-0)^2}}\right] = [-60\mathbf{i}+40\mathbf{j}+120\mathbf{k}]\text{ lb}$$

The unit vector $\mathbf{u}_{CD}$, Fig. *a*, that specifies the direction of the *CD* axis is given by

$$\mathbf{u}_{CD} = \frac{(0-0)\mathbf{i}+(8-0)\mathbf{j}+(6-0)\mathbf{k}}{\sqrt{(0-0)^2+(8-0)^2+(6-0)^2}} = \frac{4}{5}\mathbf{j}+\frac{3}{5}\mathbf{k}$$

Thus, the magnitude of the moment of **F** about the *CD* axis is given by

$$M_{CD} = \mathbf{u}_{CD}\cdot\mathbf{r}_{CA}\times\mathbf{F} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 6 & 0 & 0 \\ -60 & 40 & 120 \end{vmatrix}$$

$$= 0-\frac{4}{5}[6(120)-(-60)(0)]+\frac{3}{5}[6(40)-(-60)(0)]$$

$$= -432\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

or

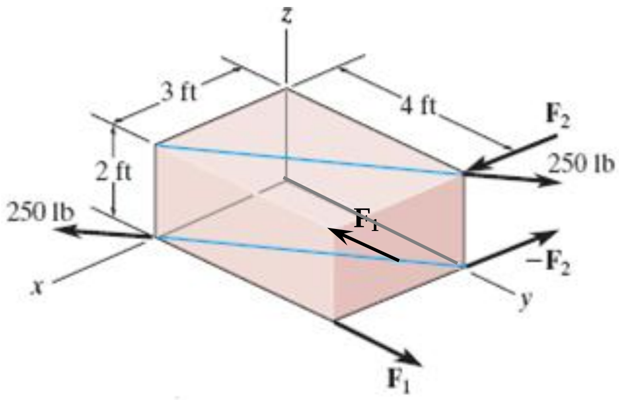
$$M_{CD} = \mathbf{u}_{CD}\cdot\mathbf{r}_{DB}\times\mathbf{F} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 0 & -4 & 6 \\ -60 & 40 & 120 \end{vmatrix}$$

$$= 0-\frac{4}{5}[0(120)-(-60)(6)]+\frac{3}{5}[0(40)-(-60)(-4)]$$

$$= -432\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{M}_{CD}$ acts in the opposite sense to that of $\mathbf{u}_{CD}$.

4–103. Determine the magnitude of couple forces $\mathbf{F_1}$ and $\mathbf{F_2}$ so that the resultant couple moment acting on the block is zero.



Scalar Approach:

Resultant Moment =0, $\sum M_R = 0 \Rightarrow \sum M = 0, M_x = 0 \quad M_y = 0 \quad and \quad M_z = 0$

$$M_x = -250\left(\frac{4}{5}\right)(2) + F_1(2) = 0 \Rightarrow F_1 = 200lb$$

$$M_y = -250\left(\frac{3}{5}\right)(2) + F_2(2) = 0 \Rightarrow F_2 = 150lb$$

$$M_y = -250\left(\frac{4}{5}\right)(3) + 250\left(\frac{3}{5}\right)(4) = 0$$

Vector Approach:

Couple Moment: The position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$, Fig. *a*, must be determined first.

$\mathbf{r}_1 = [-2\mathbf{k}]$ ft $\qquad \mathbf{r}_2 = [2\mathbf{k}]$ ft $\qquad \mathbf{r}_3 = [2\mathbf{k}]$ ft

The force vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are given by

$\mathbf{F}_1 = F_1\mathbf{j} \qquad \mathbf{F}_2 = F_2\mathbf{i}$

$$\mathbf{F}_3 = F_3\mathbf{u} = 250\left[\frac{(0-3)\mathbf{i}+(4-0)\mathbf{j}+(2-2)\mathbf{k}}{\sqrt{(0-3)^2+(4-0)^2+(2-2)^2}}\right] = [-150\mathbf{i}+200\mathbf{j}]\text{ lb}$$

Thus,

$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = (-2\mathbf{k})\times(F_1\mathbf{j}) = 2F_1\mathbf{i}$$
$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = (2\mathbf{k})\times(F_2\mathbf{i}) = 2F_2\mathbf{j}$$
$$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = (2\mathbf{k})\times(-150\mathbf{i}+200\mathbf{j}) = [-400\mathbf{i}-300\mathbf{j}]\text{ lb}\cdot\text{ft}$$

Resultant Moment: Since the resultant couple moment is required to be equal to zero,

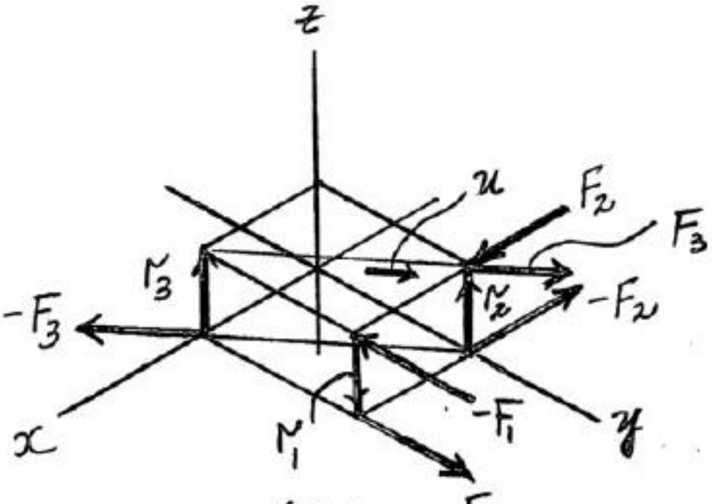
$(\mathbf{M}_c)_R = \Sigma\mathbf{M};$

$$\mathbf{0} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$$
$$\mathbf{0} = (2F_1\mathbf{i}) + (2F_2\mathbf{j}) + (-400\mathbf{i}-300\mathbf{j})$$
$$\mathbf{0} = (2F_1-400)\mathbf{i} + (2F_2-300)\mathbf{j}$$

Equating the **i**, **j**, and **k** components yields

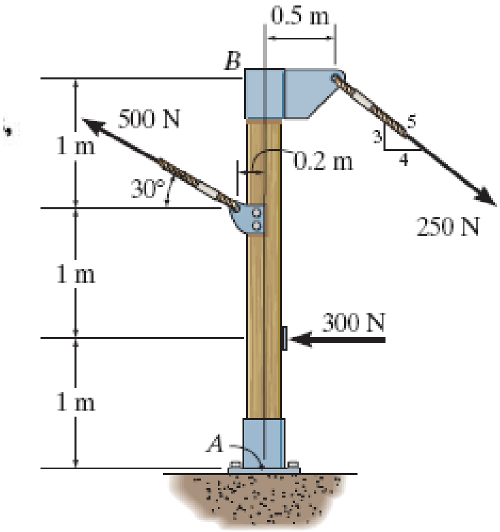
$0 = 2F_1 - 400 \qquad F_1 = 200$ lb **Ans.**

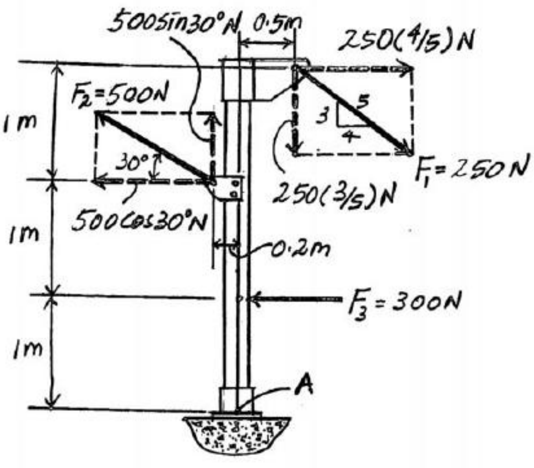
$0 = 2F_2 - 300 \qquad F_2 = 150$ lb **Ans.**



(a)

•**4–109.** Replace the force system acting on the post by a resultant force and couple moment at point *A*.





(a)

Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. *a*. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30^\circ - 300 = -533.01\text{ N} = 533.01\text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 500\sin 30^\circ - 250\left(\frac{3}{5}\right) = 100\text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{533.01^2 + 100^2} = 542.31\text{ N} = 542\text{ N}$$

The angle θ of $\mathbf{F}_R$ is

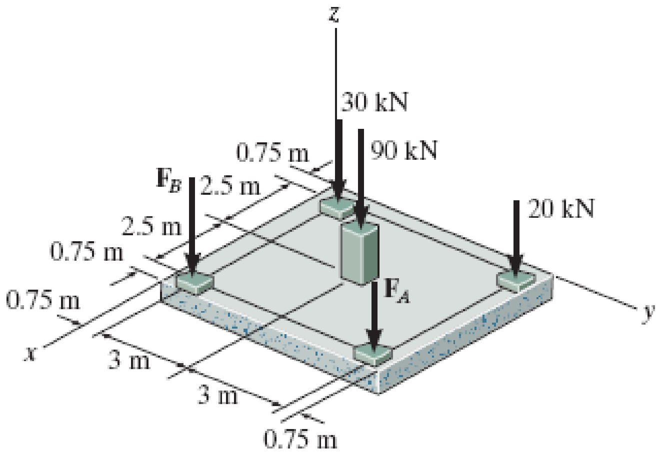
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63^\circ = 10.6^\circ$$

Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. *a*, and summing the moments of the force components algebraically about point *A*,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \qquad (M_R)_A = 500\cos 30^\circ(2) - 500\sin 30^\circ(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 250\left(\frac{4}{5}\right)(3) + 300(1)$$

$$= 441.02\text{ N}\cdot\text{m} = 441\text{ N}\cdot\text{m} \text{ (counterclockwise)} \quad \textbf{Ans.}$$

4–134. If , F_A =40 KN and F_B=35 kN, determine the magnitude of the resultant force and specify the location of its point of application (x, y) on the slab.



Equivalent Resultant Force: By equating the sum of the forces along the *z* axis to the resultant force $\mathbf{F}_R$, Fig. *b*,

$+\uparrow F_R = \Sigma F_z;$ $\quad -F_R = -30 - 20 - 90 - 35 - 40$

$F_R = 215 \text{ kN}$ **Ans.**

Point of Application: By equating the moment of the forces and $\mathbf{F}_R$, about the *x* and *y* axes,

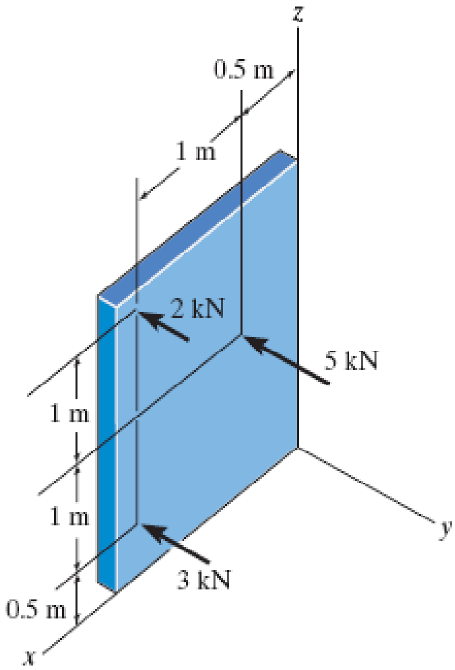
$(M_R)_x = \Sigma M_x;$ $\quad -215(y) = -35(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - 40(6.75$

$y = 3.68 \text{ m}$ **Ans.**

$(M_R)_y = \Sigma M_y;$ $\quad 215(x) = 30(0.75) + 20(0.75) + 90(3.25) + 35(5.75) + 40(5.75)$

$x = 3.54 \text{ m}$ **Ans.**

***4–136.** Replace the parallel force system acting on the plate by a resultant force and specify its location on the *x*–*z* plane.



Resultant Force: Summing the forces acting on the plate,

$(F_R)_y = \Sigma F_y;$ $\quad F_R = -5\text{ kN} - 2\text{ kN} - 3\text{ kN} = -10\text{ kN}$

The negative sign indicates that $\mathbf{F}_R$ acts along the negative *y* axis.

Resultant Moment: Using the right - hand rule, and equating the moment of $\mathbf{F}_R$ to the sum of the moments of the force system about the *x* and *z* axes,

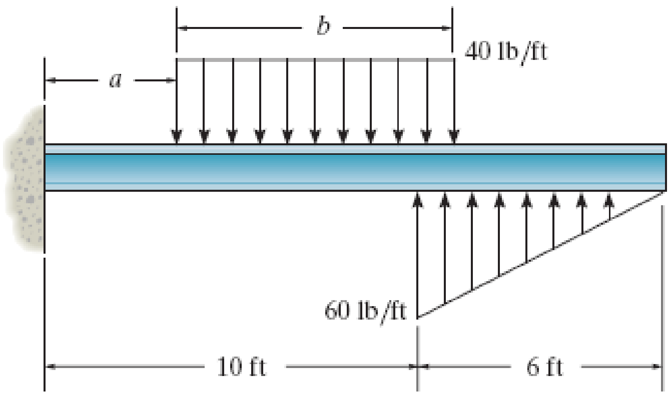
$(M_R)_x = \Sigma M_x;$ $\quad (10\text{ kN})(z) = (3\text{ kN})(0.5\text{ m}) + (5\text{ kN})(1.5\text{ m}) + 2\text{ kN}(2.5\text{ m})$

$z = 1.40\text{ m}$

$(M_R)_z = \Sigma M_z;$ $\quad -(10\text{ kN})(x) = -(5\text{ kN})(0.5\text{ m}) - (2\text{ kN})(1.5\text{ m}) - (3\text{ kN})(1.5\text{ m})$

$x = 1.00\text{ m}$

4–150. The beam is subjected to the distributed loading. Determine the length *b* of the uniform load and its position *a* on the beam such that the resultant force and couple moment acting on the eam are zero.



Require $F_R = 0$.

$+\uparrow F_R = \Sigma F_y$: $0 = 180 - 40b$

$$b = 4.50 \text{ ft}$$

Require $M_{R_A} = 0$. Using the result $b = 4.50$ ft, we have

$\circlearrowleft + M_{R_A} = \Sigma M_A$: $0 = 180(12) - 40(4.50)\left(a + \frac{4.50}{2}\right)$

$$a = 9.75 \text{ ft}$$