Homework # 1-SOLUTION

Topics Vectors and Forces (Chapter 2 in textbook).

Textbook: Engineering Mechanics, by R.C. Hibbeler, Pearson, 12th Edition.

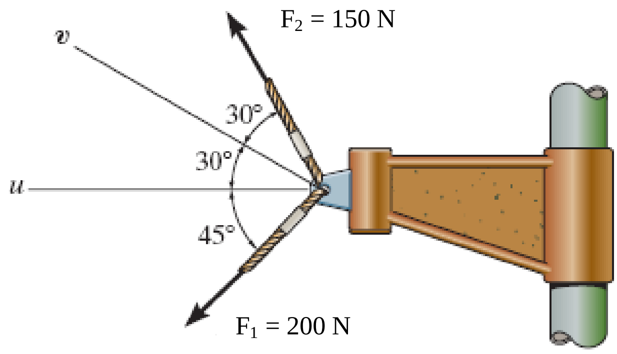
Problems:

Use Parallelogram Law only to solve:
Problems: 2-6 (page 28), and 2-15(page 29).

Use 2-D Cartesian Vector Notation to solve:
Problems: 2-32(page 39).

Use 3-D Cartesian Vector Notation and Dot Product to solve:
Problems: 2-77(page 54), 2-87(page 64), 2-104(page 67), 2-112(page 75), and 2-121 (page 76).

2–6. Resolve F2 into components along the u and *v* axes, and determine the magnitudes of these components.

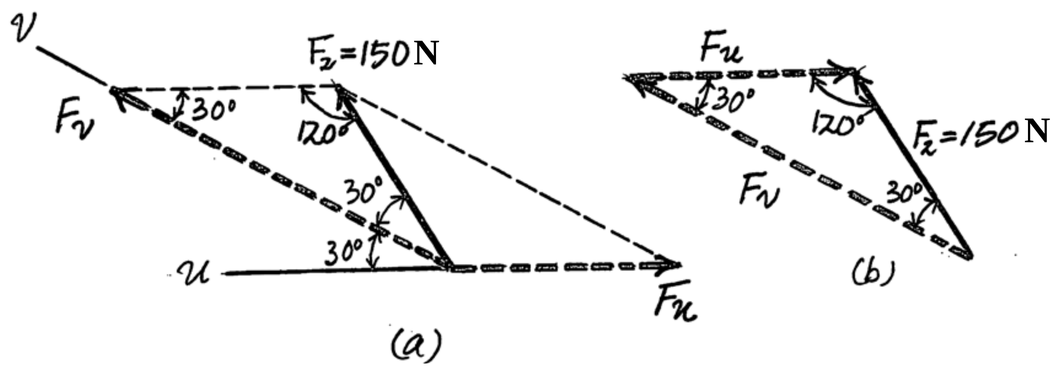


The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

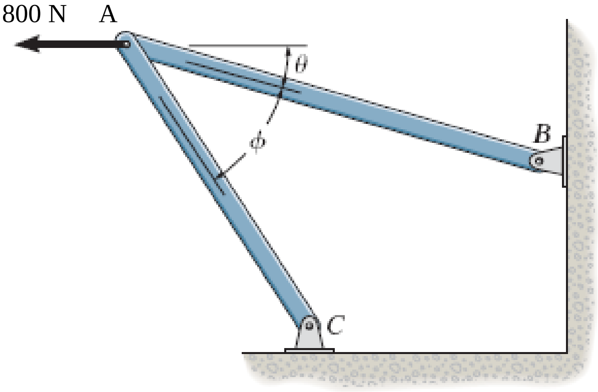
Applying the law of sines to Fig. *b*,

$$\frac{F_u}{\sin 30^\circ} = \frac{150}{\sin 30^\circ} \qquad F_u = 150\text{ N} \qquad \textbf{Ans.}$$

$$\frac{F_v}{\sin 120^\circ} = \frac{150}{\sin 30^\circ} \qquad F_v = 260\text{ N} \qquad \textbf{Ans.}$$



2–15. Determine the design angle between struts AB and AC so that the 800-N horizontal force has a component of 1200-N which acts up to the left, in the same direction as from B towards A. Take θ = 30°.



Parallelogram Law : **The parallelogram law of addition is shown in Fig. (a).**

Trigonometry : **Using law of cosines [Fig. (b)], we have**

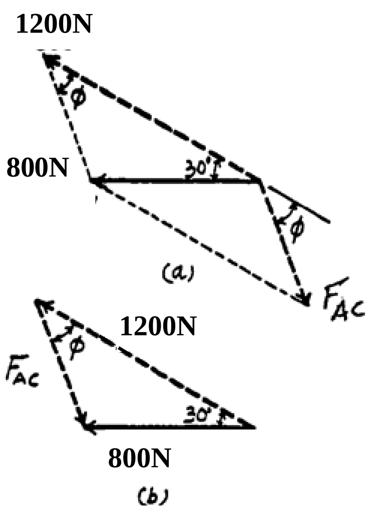
$$F_{AC} = \sqrt{800^2 + 1200^2 - 2(800)(1200)\cos 30} = 645.94N$$

The angle ϕ can be determined using law of sines [Fig. (b)].

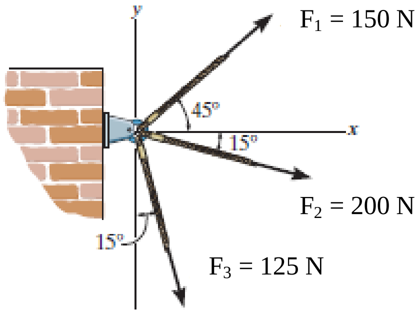
$$\frac{\sin\phi}{800} = \frac{\sin 30}{645.94}$$

$$\sin\phi = 0.6193$$

$$\phi = 38.3° \qquad \textbf{Ans}$$



2–32. Determine the magnitude of the resultant force acting on the pin and its direction measured clockwise from the positive *x* axis.



Rectangular Components: By referring to Fig. *a* , the *x* and *y* components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 150\cos 45 = 106.05N \qquad (F_1)_y = 150\sin 45 = 106.05N$$

$$(F_2)_x = 200\cos 15 = 193.2N \qquad (F_2)_y = 200\sin 15 = 51.75N$$

$$(F_3)_x = 125\sin 15 = 32.35N \qquad (F_3)_y = 125\cos 15 = 120.75N$$

Resultant Force: Summing the force components algebraically along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)x = \Sigma F_x; \qquad (F_R)_x = 106.05 + 193.2 + 32.35 = 331.6N \rightarrow$$

$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 106.05 + 51.75 + 120.75 = -66.45N = 66.45N \downarrow$$

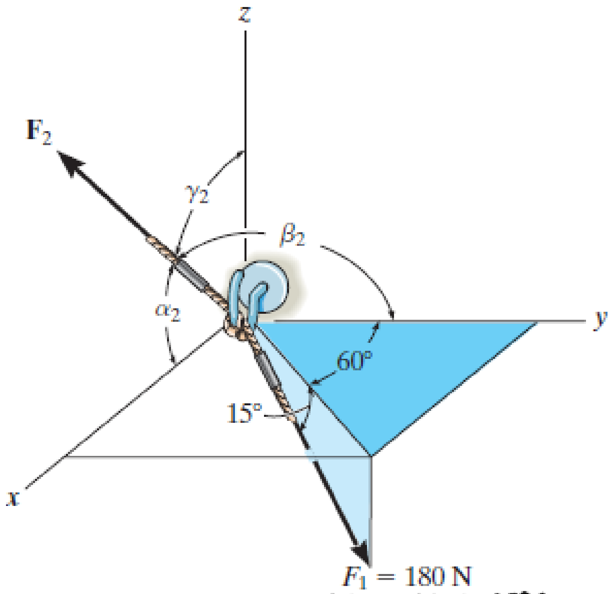
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{331.6^2 + 66.45^2} = 338.19N$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive *x* axis, is

$$\theta = tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = tan^{-1}\left[\frac{66.45}{331.6}\right] = 11.3^o \qquad \textbf{Ans.}$$

2–77. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$ so that the resultant of the two forces is zero.



$$\mathbf{F}_1 = (180 \cos 15^\circ) \sin 60^\circ \, \mathbf{i} + (180 \cos 15^\circ) \cos 60^\circ \, \mathbf{j} - 180 \sin 15^\circ \, \mathbf{k}$$

$$= 150.57 \, \mathbf{i} + 86.93 \, \mathbf{j} - 46.59 \, \mathbf{k}$$

$$\mathbf{F}_2 = F_2 \cos \alpha_2 \, \mathbf{i} + F_2 \cos \beta_2 \, \mathbf{j} + F_2 \cos \gamma_2 \, \mathbf{k}$$

$$\mathbf{F}_R = \mathbf{0}$$

i components :

$$0 = 150.57 + F_2 \cos \alpha_2$$

$$F_2 \cos \alpha_2 = -150.57$$

j components :

$$0 = 86.93 + F_2 \cos \beta_2$$

$$F_2 \cos \beta_2 = -86.93$$

k components :

$$0 = -46.59 + F_2 \cos \gamma_2$$

$$F_2 \cos \gamma_2 = 46.59$$

$$F_2 = \sqrt{(-150.57)^2 + (-86.93)^2 + (46.59)^2}$$

Solving,

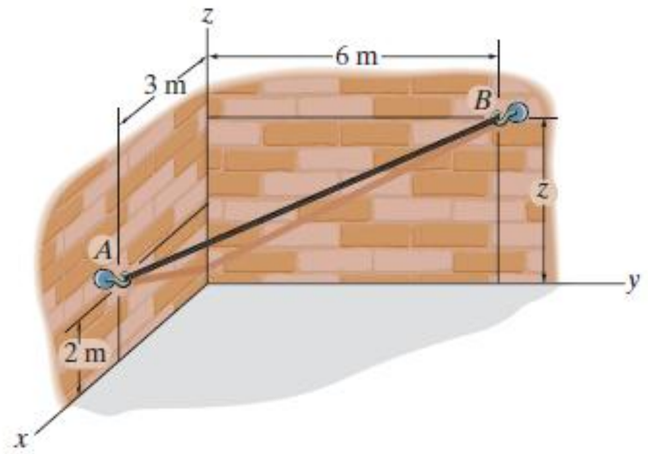
$F_2 = 180 \text{ N}$ **Ans**

$\alpha_2 = 147^\circ$ **Ans**

$\beta_2 = 119^\circ$ **Ans**

$\gamma_2 = 75.0^\circ$ **Ans**

2–87. If the cord *AB* is 7.5 m long, determine the coordinate position +*z* of point *B*.



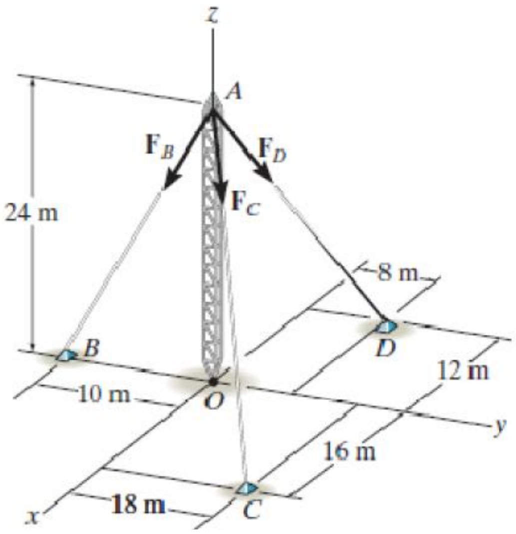
Position Vector: The coordinates for points *A* and *B* are *A*(3, 0, 2) m and *B*(0, 6, *z*) m, respectively. Thus,

$$\mathbf{r}_{AB} = (0-3)\mathbf{i} + (6-0)\mathbf{j} + (z-2)\mathbf{k}$$
$$= \{-3\mathbf{i} + 6\mathbf{j} + (z-2)\mathbf{k}\}\ \text{m}$$

Since the length of cord is equal to the magnitude of $\mathbf{r}_{AB}$, then

$$r_{AB} = 7.5 = \sqrt{(-3)^2 + 6^2 + (z-2)^2}$$
$$56.25 = 45 + (z-2)^2$$
$$z - 2 = \pm 3.354$$
$$z = 5.35\ \text{m}$$ **Ans.**

2–104. The antenna tower is supported by three cables. If the forces of these cables acting on the antenna are $\mathbf{F_B}$ = 520 N, $\mathbf{F_C}$ = 680 N, and $\mathbf{F_D}$ = 560 N, determine the magnitude and coordinate direction angles of the resultant force acting at *A*.



$$\mathbf{F}_B = 520\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 520\left(-\frac{10}{26}\mathbf{j} - \frac{24}{26}\mathbf{k}\right) = -200\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_C = 680\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = 680\left(\frac{16}{34}\mathbf{i} + \frac{18}{34}\mathbf{j} - \frac{24}{34}\mathbf{k}\right) = 320\,\mathbf{i} + 360\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_D = 560\left(\frac{\mathbf{r}_{AD}}{r_{AD}}\right) = 560\left(-\frac{12}{28}\mathbf{i} + \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}\right) = -240\,\mathbf{i} + 160\,\mathbf{j} - 480\,\mathbf{k}$$

$$\mathbf{F}_R = \Sigma\mathbf{F} = \{80\,\mathbf{i} + 320\,\mathbf{j} - 1440\,\mathbf{k}\}\text{ N}$$

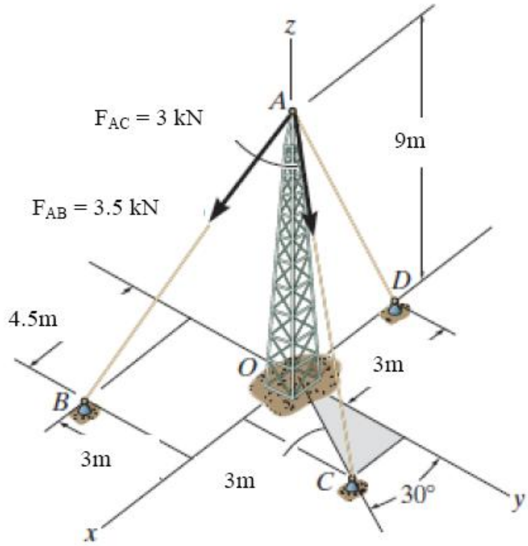
$$F_R = \sqrt{(80)^2 + (320)^2 + (-1440)^2} = 1477.3 = 1.48\text{ kN} \qquad \textbf{Ans}$$

$$\alpha = \cos^{-1}\left(\frac{80}{1477.3}\right) = 86.9° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{320}{1477.3}\right) = 77.5° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-1440}{1477.3}\right) = 167° \qquad \textbf{Ans}$$

2–121. Determine the magnitude of the projected component of force $\mathbf{F}_{AC}$ acting along the z axis.



Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

$$u_{AC} = \frac{r_{AC}}{r_{AC}} = \frac{(3\sin 30 - 0)i + (3\cos 30 - 0)j + (0-9)k}{\sqrt{(3\sin 30 - 0)^2 + (3\cos 30 - 0)^2 + (0-9)^2}} = 0.1581i + 0.2739j - 0.9487k$$

Thus, the force vector $\mathbf{F}_{AC}$ is given by

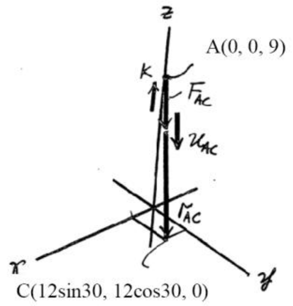
$$F_{AC} = F_{AC}u_{AC} = 3(0.1581i + 0.2739j - 0.9487k) = \{0.4743i + 0.8217j - 2.8461k\}kN$$

Vector Dot Product: The projected component of $\mathbf{F}_{AC}$ along the z axis is

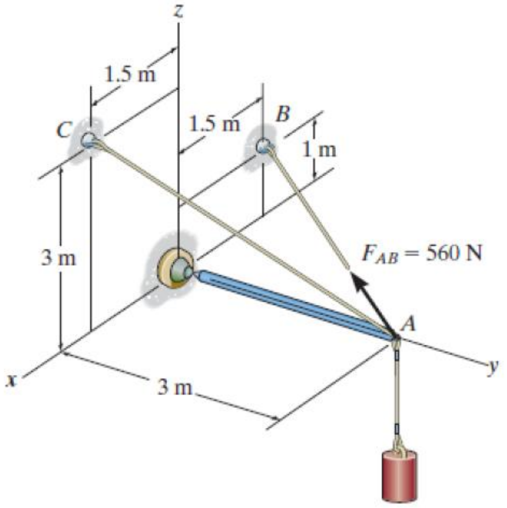
$$(F_{AC})_z = F_{AC}.K = \{0.4743i + 0.8217j - 2.8461k\}.K$$
$$= -2.8461kN$$

The negative sign indicates that $(\mathbf{F}_{AC})_z$ is directed towards the negative z axis. Thus

$$(F_{AC})_z = -2.8461kN$$ **Ans.**



2–112. Determine the projected component of the force. **F_{AB}** = 560 N acting along cable *AC*. Express the result as a Cartesian vector



Force Vectors: The unit vectors $\mathbf{u}_{AB}$ and $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(-1.5-0)\mathbf{i}+(0-3)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-1.5-0)^2+(0-3)^2+(1-0)^2}} = -\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(1.5-0)\mathbf{i}+(0-3)\mathbf{j}+(3-0)\mathbf{k}}{\sqrt{(1.5-0)^2+(0-3)^2+(3-0)^2}} = \frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}$$

Thus, the force vector $\mathbf{F}_{AB}$ is given by

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 560\left(-\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}\right) = [-240\mathbf{i}-480\mathbf{j}+160\mathbf{k}]\,\text{N}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}_{AB}$ is

$$(F_{AB})_{AC} = \mathbf{F}_{AB}\cdot\mathbf{u}_{AC} = (-240\mathbf{i}-480\mathbf{j}+160\mathbf{k})\cdot\left(\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}\right)$$

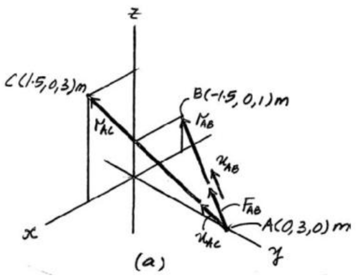
$$= (-240)\left(\frac{1}{3}\right)+(-480)\left(-\frac{2}{3}\right)+160\left(\frac{2}{3}\right)$$

$$= 346.67\,\text{N}$$

Thus, $(\mathbf{F}_{AB})_{AC}$ expressed in Cartesian vector form is

$$(\mathbf{F}_{AB})_{AC} = (F_{AB})_{AC}\,\mathbf{u}_{AC} = 346.67\left(\frac{1}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}\right)$$

$$= [116\mathbf{i}-231\mathbf{j}+231\mathbf{k}]\,\text{N}$$ **Ans.**



(a)