TEST 2 – Solution

Fall 2015

(24 November, 2015)

CIE200 – STATICS

CLOSED BOOK, 75 MINUTES

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NOTES

- 3 problems (10 pages).
- All your answers should be provided on the question sheets.
- **One extra sheet is provided at the end.**
- **Ask for additional sheets if you need more space.**
- Some answers may require much less than the space provided.
- ***Do not*** use the back of the sheets for answers.
- ***Every FBD needed for the solution of a problem should be clearly shown.***
- ***Points will be deducted for any missing/ incomplete/incorrect FBD.***
- ***Points will be deducted for answers not supported by proper calculations.***

YOUR COMMENT(S)

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_ _

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

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YOUR GRADE

Problem I: 30 /30
Problem II: 35 /35
Problem III 35 /35

.....

TOTAL: 100 /100

Problem I: *(30 points)*

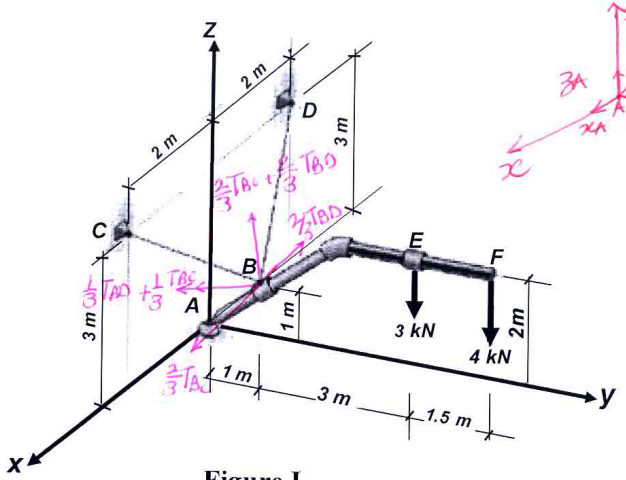
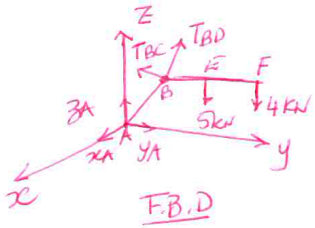


Figure I



The pipe assembly supports the forces shown in **Figure I**.

1. Determine the reactions at the ball and socket (***pin***) at **A**, and the forces in cables **BC** and **BD**. *(25 points)*

Note: FBD must be included

Calculations and/or Diagrams:

Coordinates: A(0,0,0) ; B(0, 1, 1) ; C(2, 0, 3)

D(-2, 0, 3) ; E(0, 4, 2) ; F(0, 5.5, 2)

Express each force in Cartesian vector:

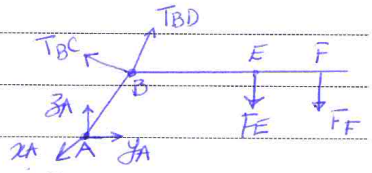
$\vec{T}_{BC} = T_{BC}\,\vec{u}_{BC} = T_{BC}\left\{\dfrac{2\vec{i} - 1\vec{j} + 2\vec{k}}{\sqrt{(2)^2+(-1)^2+(2)^2}}\right\} = \frac{2}{3}T_{BC} - \frac{1}{3}T_{BC} + \frac{2}{3}T_{BC}$

$\vec{T}_{BD} = T_{BD}\,\vec{u}_{BD} = T_{BD}\left\{\dfrac{-2\vec{i} - 1\vec{j} + 2\vec{k}}{\sqrt{(-2)^2+(-1)^2+(2)^2}}\right\} = -\frac{2}{3}T_{BD} - \frac{1}{3}T_{BD} + \frac{2}{3}T_{BD}$

$\vec{F}_E = \{0\vec{i} + 0\vec{j} - 3\vec{k}\}^{KN}$; $\vec{F}_F = \{0\vec{i} + 0\vec{j} - 4\vec{k}\}^{KN}$

Calculations and/or Diagrams:

at Pin $\vec{A} = \{x_A\vec{i} + y_A\vec{j} + z_A\vec{k}\}$



Equations of Equilibrium: F.B.D

$+\rightarrow \Sigma F_x = 0 \Rightarrow x_A - \frac{2}{3}T_{BC} - \frac{2}{3}T_{BD} = 0$ ... Eq. (1)

$+\uparrow \Sigma F_y = 0 \Rightarrow y_A - \frac{1}{3}T_{BC} - \frac{1}{3}T_{BD} = 0$ ... Eq. (2)

$+\uparrow \Sigma F_z = 0 \Rightarrow z_A + \frac{2}{3}T_{BC} + \frac{2}{3}T_{BD} - 3 - 4 = 0$ ... Eq. (3)

Scalar Approach:

$+\circlearrowleft \Sigma M_{x_A} = 0 \Rightarrow \frac{1}{3}T_{BD}(1) + \frac{1}{3}T_{BC}(1) + \frac{2}{3}T_{BC}(1) + \frac{2}{3}T_{BD}(1) - 3(4) - 4(5.5) =$

$\Rightarrow T_{BD} + T_{BC} - 34 = 0$ ... Eq. (4)

$+\circlearrowleft \Sigma M_{A_y} = 0 \Rightarrow \frac{2}{3}T_{BC}(1) - \frac{2}{3}T_{BD}(1) = 0 \Rightarrow T_{BC} = T_{BD}$ ... Eq (5)

$+\circlearrowleft \Sigma M_{A_z} = 0 \Rightarrow -\frac{2}{3}T_{BC}(1) + \frac{2}{3}T_{BD}(1) = 0 \Rightarrow T_{BC} = T_{BD}$ ... Eq (6)

Substitute Eq. (5) in (4) $\Rightarrow 2T_{BC} = 34 \therefore$ $\boxed{T_{BC} = 17\,KN}$

$\boxed{T_{BD} = 17\,KN}$

From Eq. (1) $\Rightarrow \boxed{x_A = 0}$; From Eq (3) $\Rightarrow z_A = -15.67\,KN$

From Eq. (2) $\Rightarrow \boxed{y_A = 11.33\,KN \rightarrow}$; $\boxed{z_A = +15.67\,KN \downarrow}$ $\vec{A} = \{0\vec{i} + 11.33\vec{j} - 15.67\vec{k}\}$

$\vec{T}_{BC} = \{11.33\vec{i} - 5.67\vec{j} + 11.33\vec{k}\}\,KN$

$\vec{T}_{BD} = \{-11.33\vec{i} - 5.67\vec{j} + 11.33\vec{k}\}\,KN$

Calculations and/or Diagrams (cont'd):

Or Using Vector Approach:

$\vec{r}_{AB} = \{0\vec{i} + 1\vec{j} + 1\vec{k}\}^m$ $\vec{r}_{AE} = \{0\vec{i} + 4\vec{j} + 2\vec{k}\}$

$\vec{r}_{AF} = \{0\vec{i} + 5.5\vec{j} + 2\vec{k}\}$

$\vec{M}_A = \vec{r}_{AB} \times (T_{BC} + T_{BD}) + \vec{r}_E \times \vec{F}_E + \vec{r}_{AF} \times \vec{F}_F$

$$\vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 1 \\ \frac{2}{3}T_{BC} - \frac{2}{3}T_{BD} & \frac{1}{3}T_{BC} - \frac{1}{3}T_{BD} & \frac{2}{3}(T_{BC}+T_{BD}) \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 4 & 2 \\ 0 & 0 & -3 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 5.5 & 2 \\ 0 & 0 & -4 \end{vmatrix}$$

$$= \left[\frac{2}{3}(T_{BC}+T_{BD}) + (T_{BC}+T_{BD})\frac{1}{3}\right]\vec{i} - \left(-\frac{2}{3}T_{BC} + \frac{2}{3}T_{BD}\right)\vec{j} - \frac{2}{3}(T_{BC}-T_{BD})\vec{k}$$

$$+ \{-12\vec{i} + 0\vec{j} + 0\vec{k}\} + \{-22\vec{i} + 0\vec{k} + 0\vec{k}\}$$

$\therefore +\circlearrowleft \Sigma M_{Ax} = 0 \Rightarrow T_{BC} + T_{BD} - 12 - 22 = 0$

$\therefore \boxed{T_{BC} + T_{BD} = 34\ kN}$ — Eq (4)

$+\circlearrowleft \Sigma M_{Ay} = 0 \Rightarrow \frac{2}{3}T_{BC} - \frac{2}{3}T_{BD} = 0 \Rightarrow \boxed{T_{BC} = T_{BD}}$ — Eq (5)

$+\circlearrowleft \Sigma M_{Az} = 0 \Rightarrow -\frac{2}{3}T_{BC} + \frac{2}{3}T_{BD} = 0 \Rightarrow \boxed{T_{BC} = T_{BD}}$ — Eq (6)

Problem II: *(35 points)*

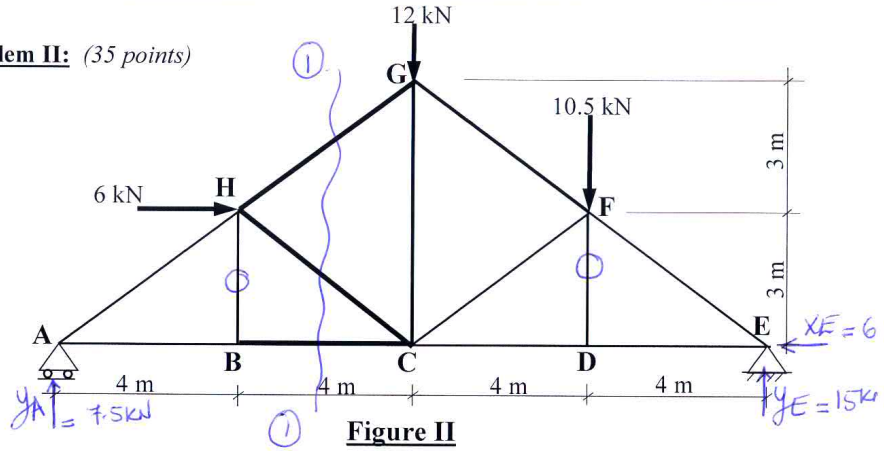


Figure II

For the truss shown in **Figure II**:

1) Determine the external reactions at the roller support **A** and the pin at **E**. *(5 points)*
2) Determine the force in members **BC**, **HC**, **HG** ***using the section method***. Also solve for the force in members **GF**, and **GC** using the appropriate method of analysis. *(25 points)*
3) Indicate Zero-force members. *(5 points)*

Note: FBD must be included

Calculations and/or Diagrams:

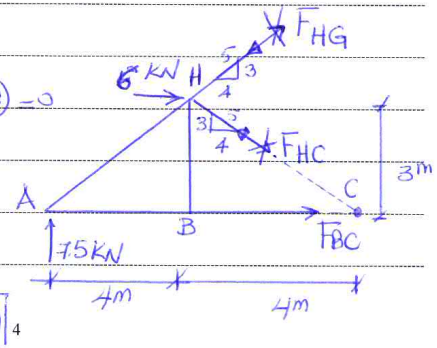
1. $\pm\sum F_x = 0 \Rightarrow 6 - X_E = 0 \Rightarrow \boxed{X_E = 6\,kN \leftarrow}$

$+\circlearrowleft \sum M_A = 0 \Rightarrow -6(3) - 12(8) - 10.5(12) + Y_E(16) = 0$

$\therefore \boxed{Y_E = 15\,kN \uparrow}$

$+\uparrow \sum F_y = 0 \Rightarrow Y_A - 12 - 10.5 + 15 = 0 \quad \therefore \boxed{Y_A = 7.5\,kN \uparrow}$

2. Sec (1) (1)



$+\circlearrowleft \sum M_H = 0 \Rightarrow F_{BC}(3) - 7.5(4) = 0$

$\boxed{F_{BC} = 10\,kN\,(T)}$

$+\circlearrowleft \sum M_C = 0 \Rightarrow -F_{HG}\left(\frac{4}{5}\right)(3) - F_{HG}\left(\frac{3}{5}\right)(4) - 6(3) - 7.5(8) = 0$

$\therefore \boxed{F_{HG} = -16.25\,kN = 16.25\,kN\,(C)}$ 4

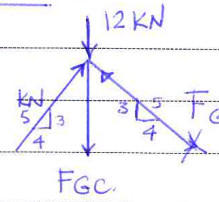
Calculations and/or Diagrams (cont'd):

$+\uparrow \sum F_y = 0 \Rightarrow 7.5 - 16.25 \times \frac{3}{5} - F_{HC}\left(\frac{3}{5}\right) = 0 \Rightarrow F_{HC} = 3.75\ KN$

$\therefore \boxed{F_{HC} = 3.75\ KN\ (C)}$

Check! $\pm \sum F_x = 0 \Rightarrow -3.75 \times \frac{4}{5} - 16.25 \times \frac{4}{5} + 10 = 0$ O.K

Equilibrium at Joint G



$\pm \rightarrow \sum F_x = 0 \Rightarrow 16.25\left(\frac{4}{5}\right) + F_{GF}\left(\frac{4}{5}\right) = 0 \Rightarrow F_{GF} = 16.25\ KN$

$\therefore \boxed{F_{GF} = 16.25\ KN\ (C)}$

$+\uparrow \sum F_y = 0 \Rightarrow -12 + 16.25\left(\frac{3}{5}\right) + 16.25\left(\frac{3}{5}\right) - F_{GC} = 0$

$\therefore \boxed{F_{GC} = 7.5\ KN\ (T)}$

3. Zero force members are: HB & DF.

Calculations and/or Diagrams (cont'd):

Problem III: *(35 points)*

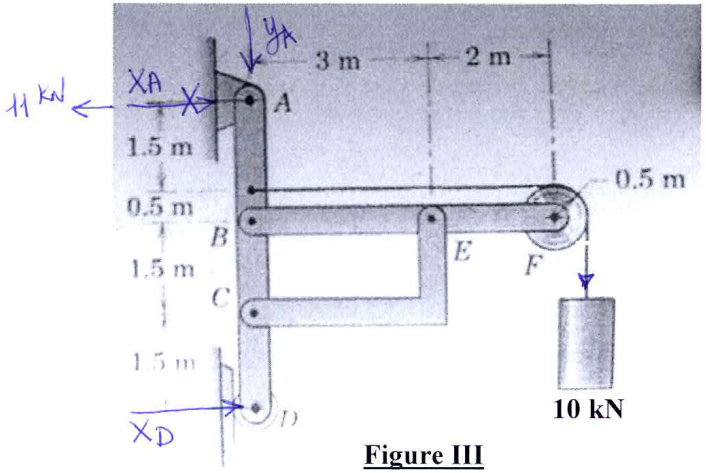


Figure III

The frame shown in **Figure III** is composed of three members **AD**, **CE**, and **BF**, connected with pins at **B**, **C**, and **E**. A frictionless pulley that has a radius of 0.5m is attached to member **BF** with a pin at **F**, and holds a 10kN.

1. Determine external reactions at pin support **A** and roller support **D**, and the internal forces at **B**, **C**, **E**, and **F**. *(35 points)*.

Note: FBD must be included

Calculations and/or Diagrams:

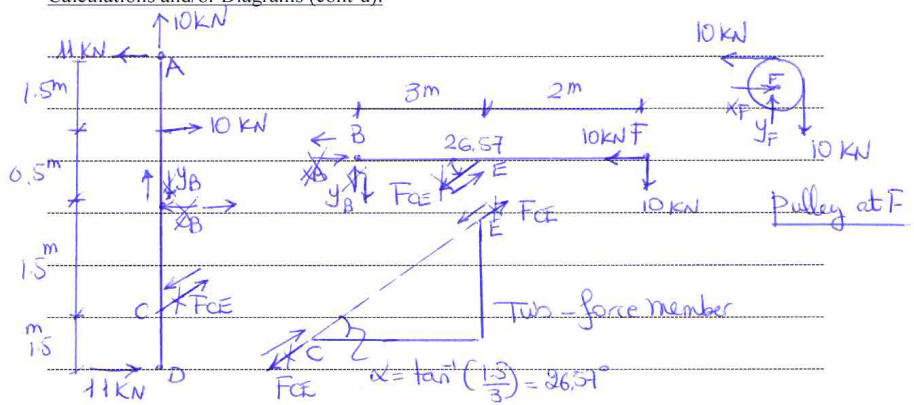
Use the whole frame:

$+\circlearrowleft \Sigma M_D = 0 \Rightarrow -x_A(5) - 10(5.5) \Rightarrow x_A = -11\,kN \therefore \boxed{x_A = 11\,kN \leftarrow}$

$+\rightarrow \Sigma F_x = 0 \Rightarrow -11 + x_D = 0 \Rightarrow \boxed{x_D = 11\,kN \rightarrow}$

$+\uparrow \Sigma F_y = 0 \Rightarrow -y_A - 10 = 0 \Rightarrow y_A = -10\,kN \therefore \boxed{y_A = 10\,kN \uparrow}$

Calculations and/or Diagrams (cont'd):



F.B.D

At Pulley F:

$\pm\rightarrow \sum F_x = 0 \Rightarrow -10 + x_F = 0 \Rightarrow \boxed{x_F = 10\ KN \rightarrow}$

$+\uparrow \sum F_y = 0 \Rightarrow -10 + y_F = 0 \Rightarrow \boxed{y_F = 10\ KN \uparrow}$

Part BEF.

$+\circlearrowleft \sum M_B = 0 \Rightarrow -F_{CE}\sin(26.57)(3) - 10(5) = 0$

$\Rightarrow F_{CE} = -37.26\ KN \therefore \boxed{F_{CE} = 37.26\ KN}$ (Compression member CE)

$+\circlearrowleft \sum M_E = 0 \Rightarrow -10(3) - y_B(3) = 0 \Rightarrow y_B = -6.67\ KN$

$\therefore \boxed{y_B = 6.67\ KN \downarrow}$

check! $+\uparrow \sum F_y = 0 \Rightarrow -6.67 + 37.26\sin 26.57 - 10 = 0$ ✓ o.k.

$\pm\rightarrow \sum F_x = 0 \Rightarrow x_B + 37.26\cos 26.57 - 10 = 0$

$\Rightarrow x_B = -23.32\ KN \therefore \boxed{x_B = 23.32\ KN \leftarrow}$

Use Part ABCD as check!

$\pm\rightarrow \sum F_x = 0 \Rightarrow -11 + 10 + 23.32 - 37.26\cos 26.57 + 11 = 0$ o.k ✓

$+\uparrow \sum F_y = 0 \Rightarrow 10 + 6.67 - 37.26\sin 26.57 = 0$ ✓ o.k.

EXTRA SHEET 1: Continued from page _____

Name: _____ **ID#:** _____

Calculations and/or Diagrams: