TEST 1 – SOLUTION
Spring 2015
(5 March, 2015)
CIE200 – STATICS
CLOSED BOOK, 75 MINUTES

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NOTES

- 3 problems (11 pages).
- All your answers should be provided on the question sheets.
- Three extra sheets is provided at the end.
- Ask for additional sheets if you need more space.
- Some answers may require much less than the space provided.
- ***Do not*** use the back of the sheets for answers.
- ***Every FBD needed for the solution of a problem should be clearly shown.***
- ***Points will be deducted for any missing/ incomplete/incorrect FBD.***
- ***Points will be deducted for answers not supported by proper calculations.***

YOUR COMMENT(S)

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

YOUR GRADE

Problem I:	30 /30
Problem II:	40 /40
Problem III	30 /30

TOTAL: 100 /100

Problem I: *(30 points)*

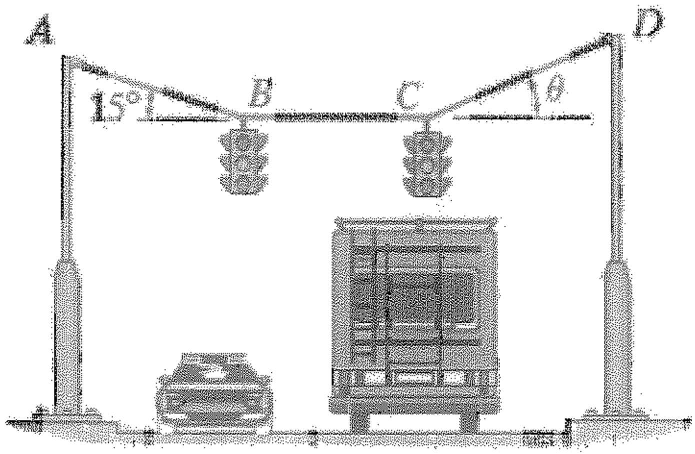


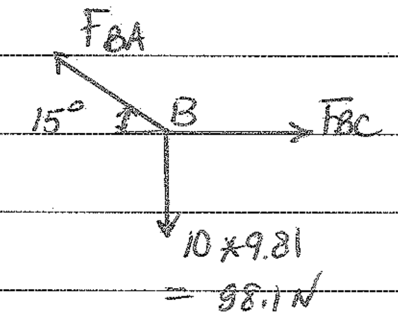
Figure I

The system shown in Figure I is in equilibrium.

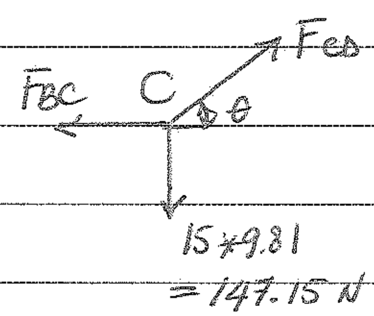
Determine the force in cables **AB**, **BC**, and **CD** necessary to support the 10-kg and 15-kg traffic lights at B and C, respectively. Also find the angle **θ**.

Note: FBD must be included

Calculations and/or Diagrams:



FBD at B



FBD at C

Equilibrium at B:

$\xrightarrow{+} \sum F_x = 0 \Rightarrow -F_{BA}\cos 15 + F_{BC} = 0$ — (1)

$+\uparrow \sum F_y = 0 \Rightarrow F_{BA}\sin 15 = 98.1 \Rightarrow \boxed{F_{BA} = 379.03\ N}$

From Eq. (1) $\Rightarrow 379.03 \cos 15 = F_{BC}$

$\Rightarrow \boxed{F_{BC} = 366.11\ N}$

Calculations and/or Diagrams:

Equilibrium at C:

$\xrightarrow{+} \sum F_x = 0 \Rightarrow -366.11 + F_{CD}\cos\theta \Rightarrow F_{CD}\cos\theta = 366.11$ ... (2)

$+\uparrow \sum F_y = 0 \Rightarrow F_{CD}\sin\theta = 147.15$ ... (3)

$$\frac{(3)}{(2)} \Rightarrow \frac{F_{CD}\sin\theta}{F_{CD}\cos\theta} = \frac{147.15}{366.11} \Rightarrow \tan\theta = 0.402$$

$$\Rightarrow \boxed{\theta = 21.9^\circ}$$

From Eq. (1) $\Rightarrow F_{CD}\cos 21.9 = 366.11$

$$\Rightarrow \boxed{F_{CD} = 394.6\,N}$$

Problem II: *(40 points)*

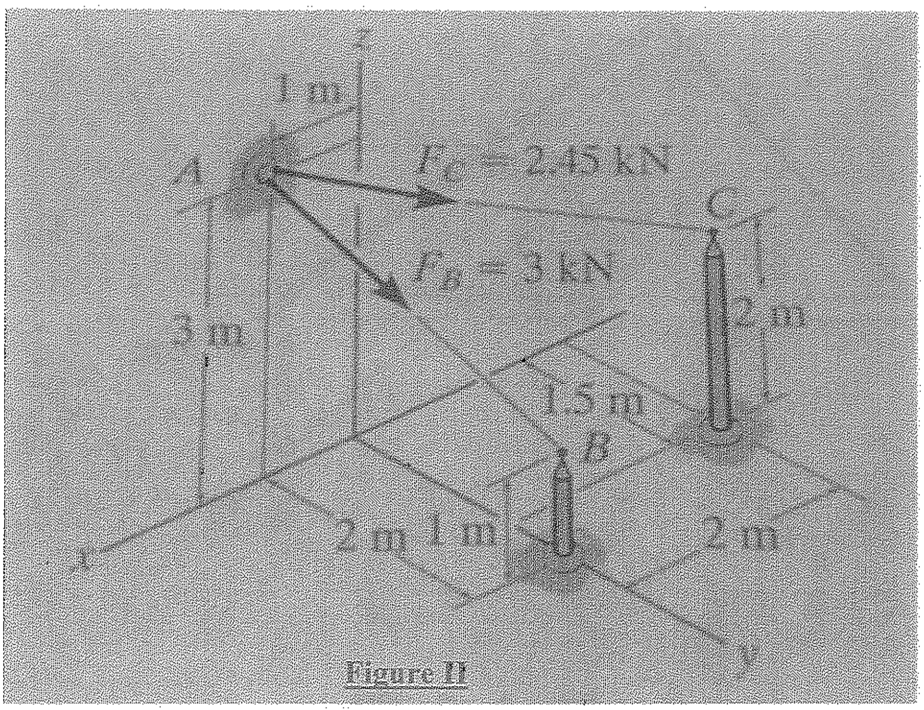


Figure II

The system shown in **Figure II** is subjected to two forces:

1. Determine the magnitude and direction angles of the resultant force acting at A. Express your result as Cartesian vector. *(20 points)*
2. Determine the projected components of the force F_C along and perpendicular to line AB. Express the result as a Cartesian vector for the parallel component only. *(15 points)*.
3. Determine the angles between F_B and F_C. *(5 points)*.

Calculations and/or Diagrams:

Coordinates: $A(1,0,3)$; $B(0,2,1)$; $C(-2,1.5,2)$

Express F_B & F_C in Cartesian vector: $\vec{F} = F\vec{u}$

$$\vec{u}_{AC} = \frac{-3\vec{i} + 1.5\vec{j} - 1\vec{k}}{\sqrt{(-3)^2+(1.5)^2+(-1)^2}} = -0.857\vec{i} + 0.429\vec{j} - 0.286\vec{k}$$

$$\vec{u}_{AB} = \frac{-1\vec{i} + 2\vec{j} - 2\vec{k}}{\sqrt{(-1)^2+(2)^2+(2)^2}} = -\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{2}{3}\vec{k}$$

$$\vec{F_B} = F_B\,\vec{u}_{AB} = 3\left\{-\frac{1}{3}\vec{i} + 2/3\,\vec{j} - 2/3\,\vec{k}\right\}$$

$$\rightarrow \boxed{\vec{F_B} = \{-1\vec{i} + 2\vec{j} - 2\vec{k}\}\ kN}$$

Calculations and/or Diagrams (cont'd):

$\vec{F_C} = F_C\, \vec{u_{AC}} = 2.45\{-0.857\vec{i} + 0.429\vec{j} - 0.286\vec{k}\}$

$\boxed{\vec{F_C} = \{-2.1\vec{i} + 1.05\vec{j} - 0.7\vec{k}\}\ kN}$

Resultant Force:

$F_{Rx} = -3.1\ kN$

$F_{Ry} = 2 + 1.05 = 3.05\ kN$

$F_{Rz} = -2 - 0.7 = -2.7\ kN$

$F_R = \sqrt{(-3.1)^2 + (3.05)^2 + (-2.7)^2} = 5.11\ kN$

$\Rightarrow \boxed{F_R = 5.11\ kN}$ $\boxed{\vec{F_R} = \{-3.1\vec{i} + 3.05\vec{j} - 2.7\vec{k}\}\ kN}$

Direction angles:

$\cos\alpha = \dfrac{-3.1}{5.11} \Rightarrow \boxed{\alpha = 127.35°}$

$\cos\beta = \dfrac{3.05}{5.11} \Rightarrow \boxed{\beta = 53.35°}$

$\cos\gamma = \dfrac{-2.7}{5.11} \Rightarrow \boxed{\gamma = 121.9°}$

2. $F_{C/AB} = \vec{F_C} \cdot \vec{u_{AB}} = \{-2.1\vec{i} + 1.05\vec{j} - 0.7\vec{k}\} \cdot \{\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{2}{3}\vec{k}\}$

$\Rightarrow F_{C/AB} = 1.85\ kN$

Expressed in cartesian: $\vec{F_C} = F_C\, \vec{u_{AB}} = 1.85\{\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{2}{3}\vec{k}\}$

$\Rightarrow \vec{F_C} = \{0.62\vec{i} + 1.233\vec{j} - 1.233\vec{k}\}\ kN$

$F_{\perp} = \sqrt{F_C^2 - F_{C/AB}^2} = \sqrt{(2.45)^2 - (1.85)^2} \Rightarrow \boxed{F_{\perp/AB} = 1.606\ kN}$

Calculations and/or Diagrams (cont'd):

3. $\vec{F_C} \cdot \vec{F_B} = F_C \cdot F_B \cos\theta$

$\{-2.1\vec{i} + 1.05\vec{j} - 0.7\vec{k}\} \cdot \{-1\vec{i} + 2.0\vec{j} - 2.0\vec{k}\} = (2.45)(3)\cos\theta$

$\Rightarrow \boxed{\theta = 40.37°}$

Problem III: *(30 points)*

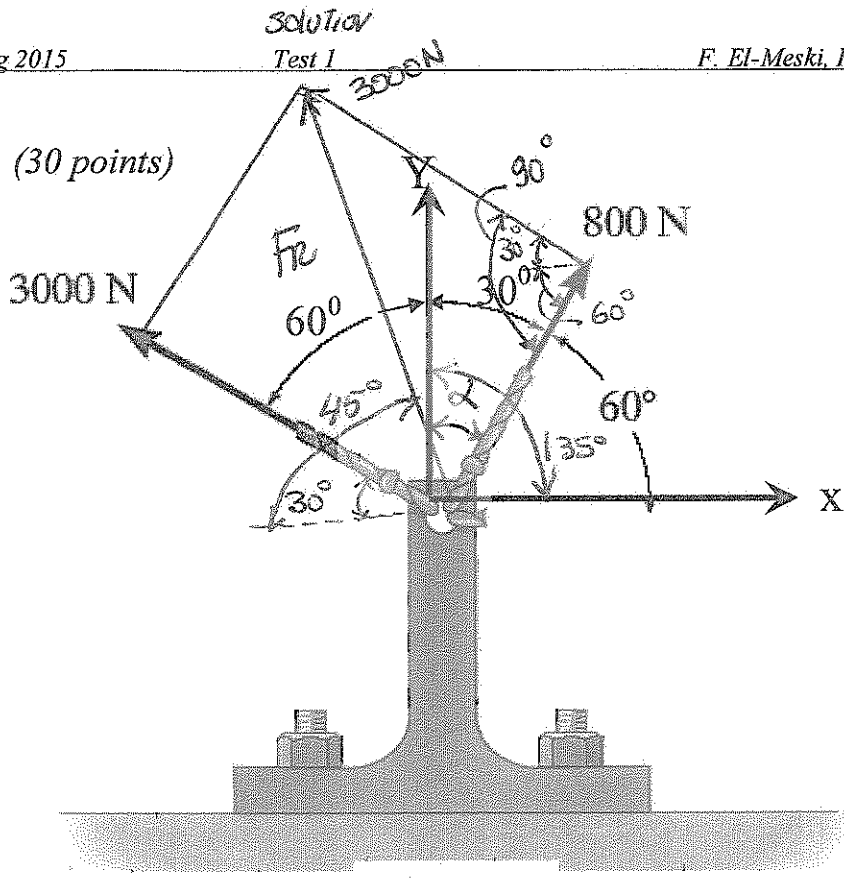


Figure III

The three forces are applied to the bracket shown in Figure III.

1 – Using Parallelogram, determine the magnitude and direction of the resultant force. *Express your result as Cartesian vector*. *(20 points)*

2- Using Projection, determine the magnitude and direction of the resultant force. Compare results with Part 1 *(10 points)*

Calculations and/or Diagrams:

Cosine Law:

$$F_R = \sqrt{(800)^2 + (3000)^2 - 2(800)(3000)\cos 90^\circ}$$

$$\Rightarrow F_R = 3104.83\ N$$

Sine Law:

$$\frac{3104.83}{\sin 90} = \frac{3000}{\sin\alpha} \Rightarrow \alpha = 75^\circ$$

$\Rightarrow$ direction of the resultant is 135° measured counterclockwise from x-axis

or 45° clockwise from the -x-axis

Calculations and/or Diagrams (cont'd):

2. $F_{1x} = 800\cos 60 = 400\ N$

$F_{1y} = 800\sin 60 = 692.82\ N$

$F_{2y} = -3000\cos 30 = -2598.08\ N$

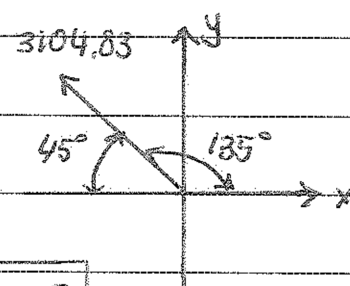
$F_{2y} = 3000\sin 30 = 1500\ N$

$F_{Rx} = 400 - 2598.08 = -2198.08\ N$

$F_{Ry} = 692.82 + 1500 = 2192.82\ N$

$F_R = \sqrt{(-2198.08)^2 + (2192.08)^2} = 3104.83\ N$ (same as Q1)

$\tan\alpha = \dfrac{2192.82}{2198.82} \Rightarrow \alpha \cong 45°$



$$\boxed{\vec{F}_R = \{-2198.08\,\vec{i} + 2192.82\,\vec{j}\}}$$