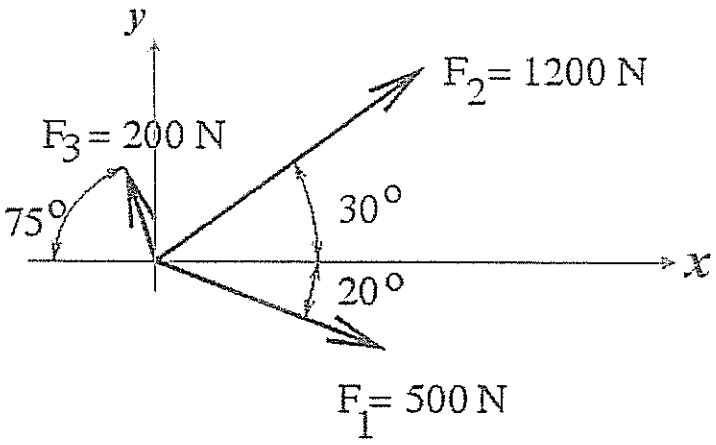
Problem I:



1) Determine the magnitude and direction of the resultant force for the system of forces shown using *the parallelogram law. (20 points)*
2) Determine the magnitude and direction of the resultant force for the system of forces shown using *the system of coplanar forces. (15points)*

Calculations:

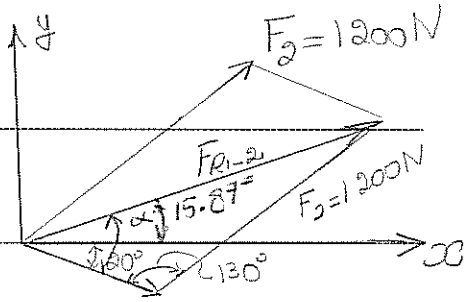
1- Magnitude and direction of $F_{R_{1-2}}$

Apply cosine Law:

$$F_{R_{1-2}} = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos 130}$$

$$\Rightarrow F_{R_{1-2}} = \sqrt{(500)^2 + (1200)^2 - 2(500)(1200)\cos 130}$$

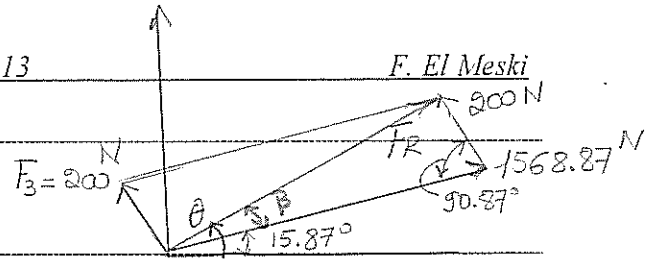
$$\Rightarrow F_{R_{1-2}} = 1568.87\ N$$



Apply sine Law: $\frac{1568.87}{\sin 130} = \frac{1200}{\sin \alpha} \Rightarrow \alpha = 35.87°$

∴ Direction of $F_{R_{1-2}}$ measured from x-axis = 35.87° - 20° = 15.87°

b) Magnitude and direction of F_R



Apply cosine Law: (Magnitude)

$$F_R = \sqrt{(200)^2 + (1568.87)^2 - 2(200)(1568.87)\cos 90.87^\circ} = 1584.5\ N$$

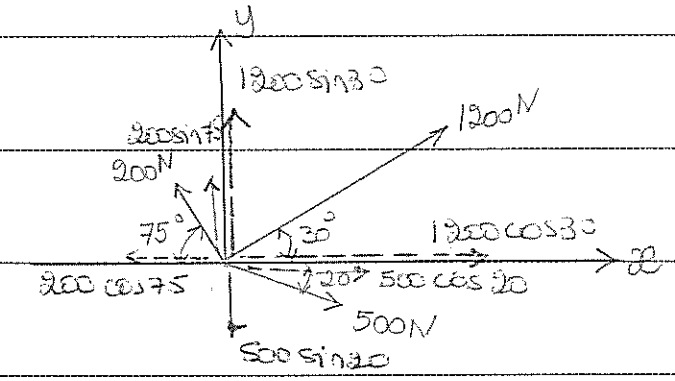
$$\Rightarrow \boxed{F_R = 1584.5\ N}$$

Direction: Apply sine law

$$\frac{1584.5}{\sin 90.87} = \frac{200}{\sin\beta} \Rightarrow \beta = 7.25^\circ$$

$$\Rightarrow \theta = 15.87 + 7.25 = 23.12^\circ \therefore \boxed{\theta = 23.12^\circ}$$

2



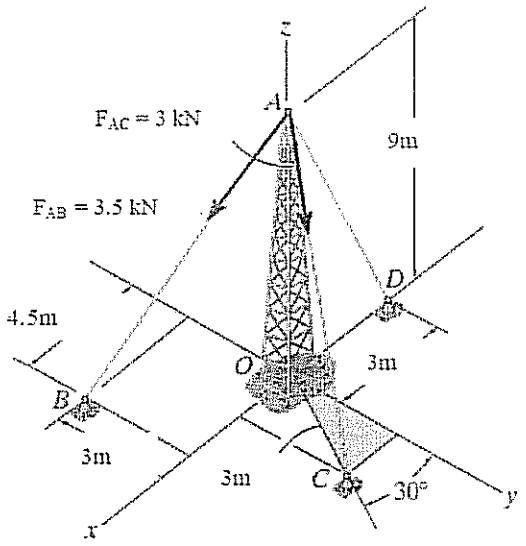
$$F_{Rx} = \Sigma F_x = 1200\cos 30 + 500\cos 20 - 200\cos 75 = 1457.31\ N$$

$$F_{Ry} = \Sigma F_y = 1200\sin 30 - 500\sin 20 + 200\sin 75 = 622.18\ N$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(1457.31)^2 + (622.18)^2} = 1584.5\ N \Rightarrow \boxed{F_R = 1584.5\ N}$$

$$\tan\theta = \left|\frac{F_{Ry}}{F_{Rx}}\right| = \frac{622.18}{1457.31} \Rightarrow \boxed{\theta = 23.12^\circ}$$

Problem II:



1) Determine the angle between the two forces F_{AC} and F_{AB}. *(20 points)*
2) Determine the magnitude of the projection component of force F_{AC} along line AD. *(15 points)*

Calculations:

1. $\vec{F_{AC}} \cdot \vec{F_{AB}} = F_{AC} F_{AB} \cos\theta$.

Coordinates: $A(0,0,9)$ $B(4.5,-3,0)$ $C(\underbrace{3\sin 30}_{1.5}, \underbrace{3\cos 30}_{2.6}, 0)$ $D(-3,0,0)$

Express F_{AB} as cartesian vector: $\vec{F_{AB}} = F_{AB}\, \vec{u_{AB}}$

$$\vec{u_{AB}} = \frac{(4.5-0)\vec{i} + (-3-0)\vec{j} + (0-9)\vec{k}}{\sqrt{(4.5)^2 + (-3)^2 + (-9)^2}} = \frac{4.5}{10.5}\vec{i} - \frac{3}{10.5}\vec{j} - \frac{9}{10.5}\vec{k}$$

$$\Rightarrow \vec{F_{AB}} = 3.5\left[\frac{4.5}{10.5}\vec{i} - \frac{3}{10.5}\vec{j} - \frac{9}{10.5}\vec{k}\right] = \{1.5\vec{i} - 1.0\vec{j} - 3.\vec{k}\}\ kN$$

Express F_{AC} as cartesian vector: $\vec{F}_{AC} = F_{AC}\,\vec{u}_{AC}$

$$\vec{u}_{AC} = \frac{(1.5-0)\vec{i} + (2.6-0)\vec{j} + (0-9)\vec{k}}{\sqrt{(1.5)^2 + (2.6)^2 + (-9)^2}} = 0.158\vec{i} + 0.274\vec{j} - 0.949\vec{k}$$

$$\vec{F}_{AC} = 3\left[0.158\vec{i} + 0.274\vec{j} - 0.949\vec{k}\right] = \left\{0.474\vec{i} + 0.822\vec{j} - 2.847\vec{k}\right\} \text{ KN}$$

$$\Rightarrow \vec{F}_{AB} \cdot \vec{F}_{AC} = F_{AB}\,F_{AC}\cos\theta$$

$$\left\{1.5\vec{i} - 1\vec{j} - 3\vec{k}\right\} \cdot \left\{0.474\vec{i} + 0.822\vec{j} - 2.847\vec{k}\right\} = (3.5)(3)\cos\theta$$

$$(1.5)(0.474) + (-1)(0.822) + (-3)(-2.847) = (3.5)(3)\cos\theta$$

$$\Rightarrow \boxed{\theta = 36.6^\circ}$$

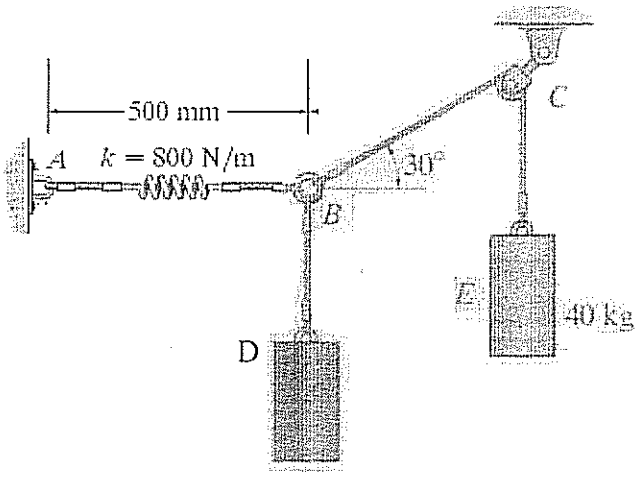
2- $F_{AC/AD} = \vec{F}_{AC} \cdot \vec{u}_{AD}$

$$\vec{u}_{AD} = \frac{(-3-0)\vec{i} + (0-0)\vec{j} + (0-9)\vec{k}}{\sqrt{(-3)^2 + 0 + (-9)^2}} = -0.316\vec{i} + 0\vec{j} - 0.95\vec{k}$$

$$F_{AC/AD} = \left\{0.474\vec{i} + 0.822\vec{j} - 2.847\vec{k}\right\} \cdot \left\{-0.316\vec{i} + 0\vec{j} - 0.95\vec{k}\right\}$$

$$\boxed{F_{AC/AC} = 2.55 \text{ KN}}$$

Problem III:

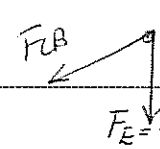


If the mass of cylinder *E* is *40 kg*, determine the mass of cylinder *D* and the unstretched length of spring AB, in order to hold the assembly in the position shown. *(30 points)*

Note: FBD must be included

Calculations:

Equilibrium at C: $F_E = F_{CB} \Rightarrow$

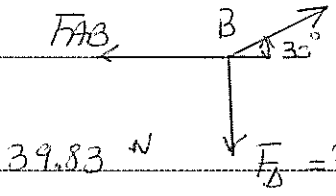


$\therefore \boxed{F_{CB} = 392.4 \text{ N}}$

F.B.D

Equilibrium at B:

$+\rightarrow \sum F_x = 0 \Rightarrow F_{BC}\cos 30^\circ - F_{AB} = 0$



$\Rightarrow F_{AB} = F_{BC}\cos 30 = 392.4 \cos 30 = 339.83$ N

$\Rightarrow \boxed{F_{AB} = 339.83 \text{ N}}$

$+\uparrow \sum F_y = 0 \Rightarrow F_{BC} \sin 30 - F_D = 0$

$\Rightarrow F_D = F_{BC} \sin 30 = 392.4 \sin 30$

$\therefore F_D = 196.2\ N \rightarrow W_D = \frac{196.2}{9.81} = 20\ kg$

$\Rightarrow \boxed{W_D = 20\ kg}$

Unstretched length in AB:

$F_{AB} = k\Delta \Rightarrow 339.83 = 800 * \Delta \Rightarrow \Delta = 0.425\ m$

$l_u = l_f - \Delta = 0.5 - 0.425 = 0.075\ m = 75\ mm$

$\therefore \boxed{l_u = 75\ mm}$

Good Luck!