TEST 1 – SOLUTION

Spring 2013-14
(27th March, 2014)
CIE200 – STATICS
CLOSED BOOK, 75 MINUTES

Name: _____ **ID#:** _____ **Section: 11**

NOTES

- 4 problems (13 pages).
- All your answers should be provided on the question sheets.
- Three extra sheets is provided at the end.
- Ask for additional sheets if you need more space.
- Some answers may require much less than the space provided.
- ***Do not*** use the back of the sheets for answers.
- ***Every FBD needed for the solution of a problem should be clearly shown.***
- ***Points will be deducted for any missing/ incomplete/incorrect FBD.***
- ***Points will be deducted for answers not supported by proper calculations.***

YOUR COMMENT(S)

_ _

_ _

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

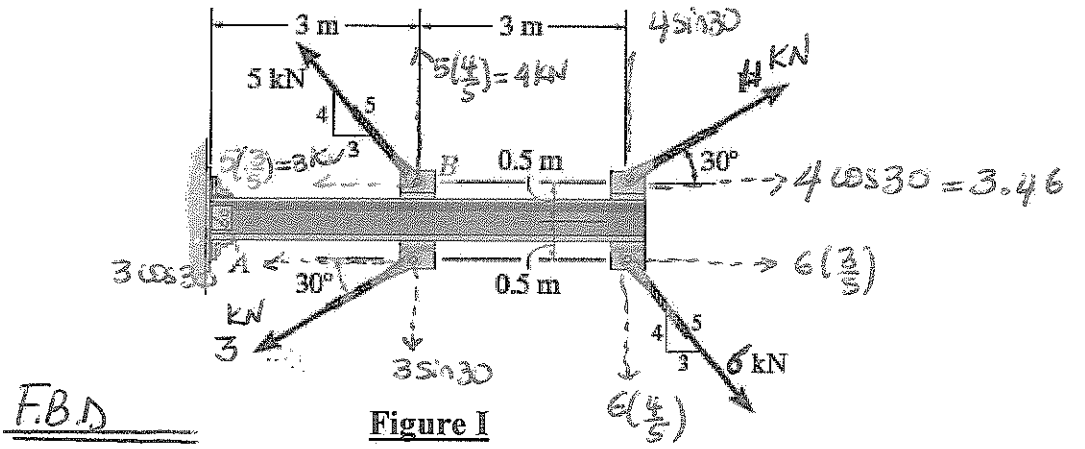
_ _

_ _

YOUR GRADE

Problem I:	___/20
Problem II:	___/35
Problem III	___/30
Problem IV	___/15
	.....
TOTAL:	/100

Problem I: *(20 points)*



FBD

Figure I

1. Determine the magnitude and direction of the resultant force for the system of forces shown in **Figure I**. *(20 points)*

Note: FBD must be included

Calculations and/or Diagrams:

$F_{Rx} = \Sigma F_x = -6\left(\frac{3}{5}\right) - 3\cos 30 + 4\cos 30 + 6\left(\frac{3}{5}\right) = 1.47\ kN$

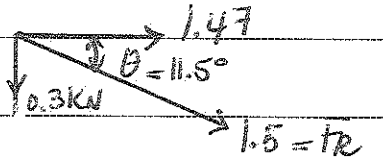
$F_{Ry} = \Sigma F_y = 5\left(\frac{4}{5}\right) - 3\sin 30 + 4\sin 30 - 6\left(\frac{4}{5}\right) = -0.3\ kN$

Magnitude:

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(1.47)^2 + (-0.3)^2} = 1.5\ kN \quad \therefore \boxed{F_R = 1.5\ kN}$

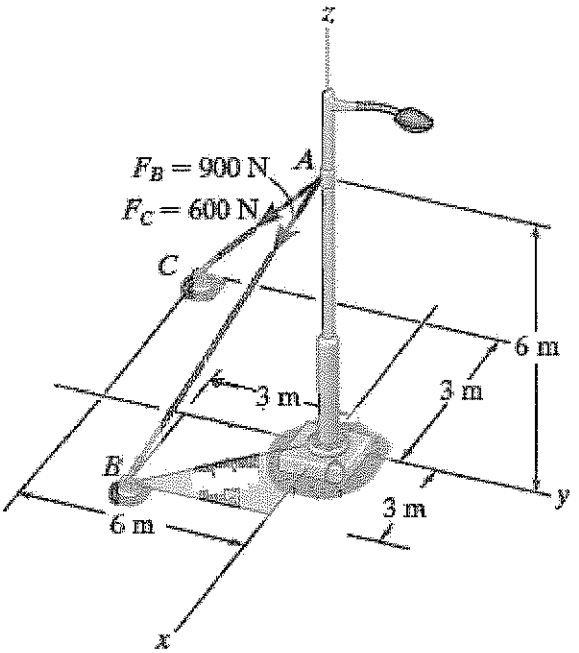
Direction:

$\tan\theta = \left|\frac{F_{Ry}}{F_{Rx}}\right| = \left|\frac{0.3}{1.47}\right| \rightarrow \boxed{\theta = 11.5^\circ}$



Problem II: *(35 points)*

Figure II



The light post is subjected to two forces as shown in **Figure II**.

1. Determine the magnitude and direction angles of the resultant force acting at A. *(20 points)*
2. Determine the projected component of the force in the cable AB along line AC. Express the result as a Cartesian vector. *(15 points)*

Note: FBD must be included for part 1.

Calculations and/or Diagrams:

(1) Coordinates: $A(0,0,6)$ $B(3,-3,0)$ & $C(-3,-6,0)$

Express each force in Cartesian vector:

$\vec{F}_{AB} = F_{AB}\,\vec{u}_{AB}$; $\vec{F}_{AC} = F_{AC}\,\vec{u}_{AC}$

$\vec{u}_{AB} = \dfrac{3\vec{i} - 3\vec{j} - 6\vec{k}}{\sqrt{(3)^2+(-3)^2+(-6)^2}} = 0.408\vec{i} - 0.408\vec{j} - 0.816\vec{k}$

$\therefore \vec{F}_{AB} = 900\{0.408\vec{i} - 0.408\vec{j} - 0.816\vec{k}\} = \{367.2\vec{i} - 367.2\vec{j} - 734.4\vec{k}\}$ N

$\vec{u}_{AC} = \dfrac{-3\vec{i} - 6\vec{j} - 6\vec{k}}{\sqrt{(-3)^2+(-6)^2+(-6)^2}} = -\frac{1}{3}\vec{i} - \frac{2}{3}\vec{j} - \frac{2}{3}\vec{k}$

Calculations and/or Diagrams (cont'd):

$\vec{F}_{AC} = F_{AC}\,\vec{u}_{AC} = 600\left\{-\frac{1}{3}\vec{i} - \frac{2}{3}\vec{j} - \frac{2}{3}\vec{k}\right\}$

$\therefore \vec{F}_{AC} = \{-200\,\vec{i} - 400\,\vec{j} - 400\,\vec{k}\}\ N$

$F_{Rx} = 367.2 - 200 = 167.2\ N$

$F_{Ry} = \Sigma F_y = -367.2 - 400 = -767.2\ N$

$F_{Rz} = \Sigma F_z = -734.4 - 400 = -1134.4\ N$

Magnitude:

$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2 + (F_{Rz})^2} = \sqrt{(167.2)^2 + (-767.2)^2 + (-1134.4)^2}$

$\therefore \boxed{F_R = 1379.64\ N}$

Direction:

$\cos\alpha = \frac{F_{Rx}}{F_R} = \frac{167.2}{1379.64} \Rightarrow \boxed{\alpha = 83.04^\circ}$

$\cos\beta = \frac{F_{Ry}}{F_R} = \frac{-767.2}{1379.64} \Rightarrow \boxed{\beta = 123.79^\circ}$

$\cos\gamma = \frac{F_{Rz}}{F_R} = \frac{-1134.4}{1379.64} \Rightarrow \boxed{\gamma = 145.31^\circ}$

(2) $\vec{F}_{AB/AC} = \vec{F}_{AB} \cdot \vec{u}_{AC} = \{367.2\,\vec{i} - 367.2\,\vec{j} - 734.4\,\vec{k}\} \cdot \left\{-\frac{1}{3}\vec{i} - \frac{2}{3}\vec{j} - \frac{2}{3}\vec{k}\right\}$

$\boxed{F_{AB/AC} = 612\ N}$

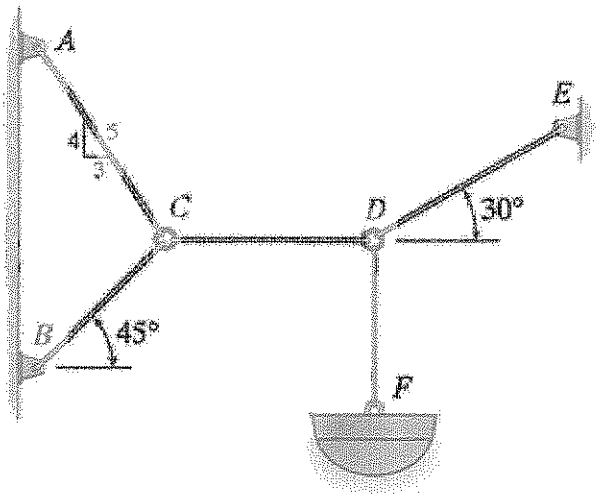
In Cartesian vector:

$\vec{F}_{AB/AC} = F_{AB/AC}\,\vec{u}_{AC} = 612\left\{-\frac{1}{3}\vec{i} - \frac{2}{3}\vec{j} - \frac{2}{3}\vec{k}\right\}$

$\therefore \boxed{\vec{F}_{AB/AC} = \{-204\,\vec{i} - 408\,\vec{j} - 408\,\vec{k}\}\ N}$

Problem III: *(30 points)*

Figure III

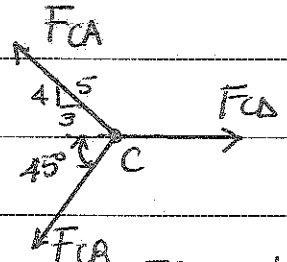


The system weights and cables shown in **Figure III** is in its equilibrium position.

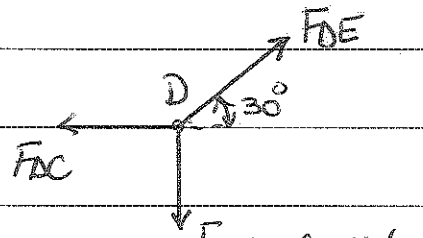
1. Calculate the force in each cable if the weight of the lamp at F is 20 kN.

Note: FBD mut be included

Calculations and/or Diagrams:



F.B.D at C



F.B.D at D

Equilibrium at D:

$+\uparrow \sum F_y = 0 \Rightarrow F_{DE}\sin 30 - 20 = 0 \Rightarrow \boxed{F_{DE} = 40\,kN}$

$+\rightarrow \sum F_x = 0 \Rightarrow -F_{DC} + F_{DE}\cos 30 = 0$

$\Rightarrow -F_{DC} + 40\cos 30 = 0 \Rightarrow \boxed{F_{DC} = 34.64\,kN}$

Calculations and/or Diagrams (cont'd):

Equilibrium at C.

$\pm \Sigma F_x = 0 \Rightarrow -F_{CA}\left(\frac{3}{5}\right) - F_{CB}\cos 45 + 34.64 = 0$ ... Eq①

$+\uparrow \Sigma F_y = 0 \Rightarrow F_{CA}\left(\frac{4}{5}\right) - F_{CB}\sin 45 = 0 \Rightarrow F_{CA} = 0.884 F_{CB}$ ... Eq②

Substitute Eq.② in ① $\Rightarrow -0.884 F_{CB}\left(\frac{3}{5}\right) - F_{CB}\cos 45 + 34.64 = 0$

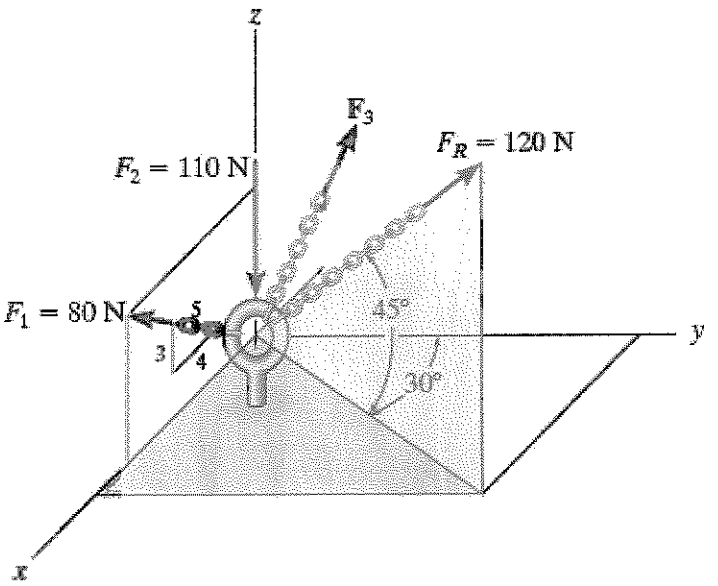
$\Rightarrow \boxed{F_{CB} \simeq 28\ KN}$

$\therefore F_{CA} = 0.884(28) = 24.752\ KN$

$\boxed{F_{CA} = 24.752\ KN}$

Problem IV: *(15 points)*

Figure IV



1. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ act on the ring. If the resultant force $\mathbf{F}_R$ has a magnitude of **120 N** and direction angles as shown, determine the magnitude and direction angles of force $\mathbf{F}_3$.

Calculations and/or Diagrams: Express each force in cartesian vector:

$\vec{F_1} = 80\left(\frac{4}{5}\right)\vec{i} + 0\vec{j} + 80\left(\frac{3}{5}\right)\vec{k}$

$\Rightarrow \vec{F_1} = 64\vec{i} + 0\vec{j} + 48\vec{k}$

$\Rightarrow \vec{F_2} = 0\vec{i} + 0\vec{j} - 110\vec{k}$

$\Rightarrow \vec{F_3} = F_3\cos\alpha\,\vec{i} + F_3\cos\beta\,\vec{j} + F_3\cos\gamma\,\vec{k}$

$\vec{F_R} = F_{Rx}\vec{i} + F_{Ry}\vec{j} + F_{Rz}\vec{k}$

where:

$F_{Rx} = 120\cos 45\sin 30 = 42.43$ KN

$F_{Ry} = 120\cos 45\cos 30 = 73.49$ KN

$F_{Rz} = 120\sin 45 = 84.85$ KN

$\therefore \vec{F_R} = 42.43\vec{i} + 73.49\vec{j} + 84.85\vec{k}$

EXTRA SHEET 1: Continued from page ____

Name: _____ **ID#:** _____

Calculations and/or Diagrams:

$F_{Rx} = \Sigma F_x$

$\Rightarrow 42.43 = 64 + 0 + F_3 \cos\alpha \Rightarrow F_3 \cos\alpha = -21.57\ KN = F_{3x}$

$F_{Ry} = \Sigma F_y$

$\Rightarrow 73.49 = 0 + 0 + F_3 \cos\beta \Rightarrow F_3 \cos\beta = F_{3y} = 73.49\ KN$

$F_{Rz} = \Sigma F_z$

$\Rightarrow 84.85 = 48 - 110 + F_3 \cos\gamma \Rightarrow F_3 \cos\gamma = 146.85\ KN$

$\therefore F_3 = \sqrt{F_{3x}^2 + F_{3y}^2 + F_{3z}^2} = \sqrt{(-21.57)^2 + (73.49)^2 + (146.85)^2}$

$\Rightarrow \boxed{F_3 = 165.62\ KN}$

$\therefore F_3 \cos\alpha = -21.57 \Rightarrow \cos\alpha = \frac{-21.57}{165.22} \Rightarrow \boxed{\alpha = 97.48^\circ}$

$F_3 \cos\beta = 73.49 \Rightarrow \cos\beta = \frac{73.49}{165.22} \Rightarrow \boxed{\beta = 63.6^\circ}$

$F_3 \cos\gamma = 146.85 \Rightarrow \cos\gamma = \frac{146.85}{165.22} \Rightarrow \boxed{\gamma = 27.28^\circ}$