TEST 1
Fall 2016
(6 October, 2016)
CIE200 – STATICS
CLOSED BOOK, 75 MINUTES

Name: ______________________ **ID#:** ____________ **Section: 13**

NOTES

- 3 problems (10 pages).
- All your answers should be provided on the question sheets.
- **Two extra sheets is provided at the end.**
- **Ask for additional sheets if you need more space.**
- Some answers may require much less than the space provided.
- ***Do not*** use the back of the sheets for answers.
- ***Every FBD needed for the solution of a problem should be clearly shown.***
- ***Points will be deducted for any missing/ incomplete/incorrect FBD.***
- ***Points will be deducted for answers not supported by proper calculations.***

YOUR COMMENT(S)

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

YOUR GRADE

Problem I:	___/25
Problem II:	___/35
Problem III	___/40

....

TOTAL: /100

Problem I: *(25 points)*

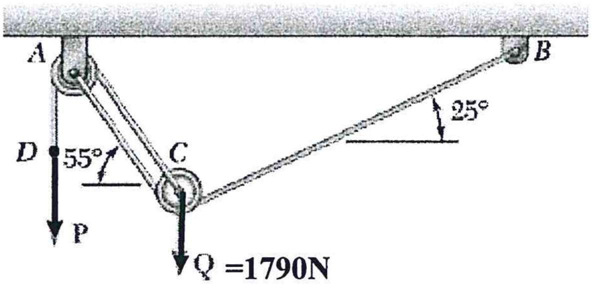


Figure I

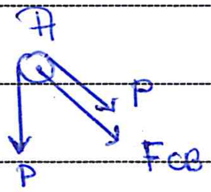
An 1790-N load Q is applied to the pulley C, which can roll on the cable ACB as shown in **Figure I**. The pulley is held in the position shown by a second cable CAD, which passes over the pulley A and supports a load P.

1. Determine the tension in cable ACB, and the magnitude of load P.

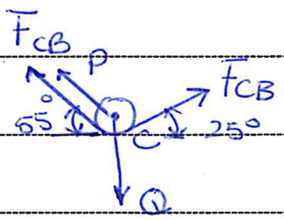
Note: FBD must be included

Calculations and/or Diagrams:

$F_{AC} = P$ (friction-less pulley)



F.B.D at A



F.B.D at C

$F_{CA} = F_{CB}$ (Frictionless pulley)

Equilibrium at C:

$\xrightarrow{+} \sum F_x = 0 \Rightarrow -F_{CB}\cos 55 - P\cos 55 + F_{CB}\cos 25 = 0$

$\Rightarrow P = F_{CB}(0.58)$ — Eq. 1

$+\uparrow \sum F_y = 0 \Rightarrow F_{CB}\sin 55 + P\sin 55 + F_{CB}\sin 25 = 1790$ — Eq. 2

Substitute Eq 1 in Eq 2 $\Rightarrow \boxed{F_{CB} = 1042.6 \text{ KN}}$

$\boxed{P = 604.7 \text{ KN}}$

Calculations and/or Diagrams:

Problem II: *(35 points)*

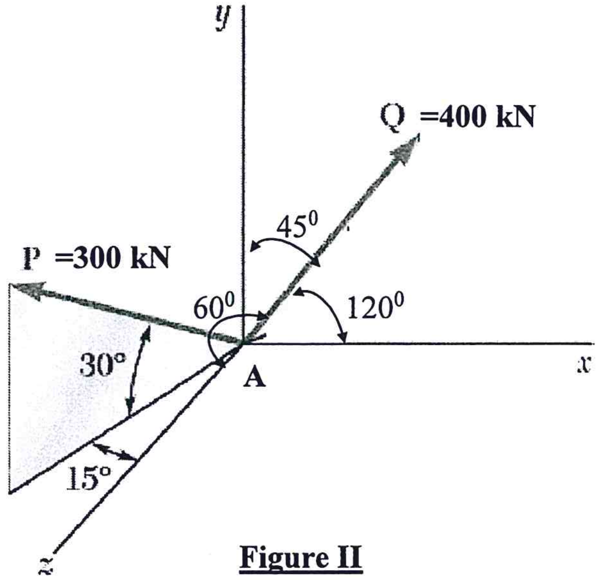


Figure II

For the two forces shown in **Figure II**:

1. Determine the magnitude and direction angles of the resultant force acting at A. Express your result as Cartesian vector. *(15 points)*
2. Determine the angles between ***P*** and ***Q***. *(5 points).*
3. Determine the projected components of the force ***P*** along and perpendicular to **Q**. Express the results as a Cartesian vector. *(15 points).*

Calculations and/or Diagrams:

Express P in Cartesian vector:

$P_x = -300\cos 30 \sin 15 = -67.24$ KN

$P_y = 300 \sin 30 = 150$ KN

$P_z = 300 \cos 30 \cos 15 = 250.95$ KN

$\vec{P} = \{-67.24\vec{i} + 150\vec{j} + 250.95$

Express Q in Cartesian vector.

$Q_x = 400\cos 120 = -200$ KN

$Q_y = 400 \cos 45 = 282.84$ KN

$Q_z = 400 \cos 60 = 200$ KN

$\vec{Q} = \{-200\vec{i} + 282.84\vec{j} + 200\vec{k}\}$

Calculations and/or Diagrams (cont'd):

$F_{Rx} = -67.24 - 200 = -267.24$ kN
$F_{Ry} = 150 + 282.84 = 432.84$ kN
$F_{Rz} = 250.95 + 200 = 450.95$ kN

$\boxed{\vec{F} = \{-267.24\vec{i} + 432.84\vec{j} + 450.9}$

$F_R = \sqrt{(-267.24)^2 + (432.84)^2 + (450.95)^2}$

$\Rightarrow \boxed{F_R = 679.8 \text{ kN}}$

Direction:

$\cos\alpha = -\frac{267.24}{679.8} \Rightarrow \boxed{\alpha = 113.15^\circ}$

$\cos\beta = \frac{432.84}{679.8} \Rightarrow \boxed{\beta = 50.45^\circ}$

$\cos\gamma = \frac{450.95}{679.8} \Rightarrow \boxed{\gamma = 48.44^\circ}$

2. $\vec{P} \cdot \vec{Q} = PQ\cos\theta$

$\{-67.24\vec{i} + 150\vec{j} + 250.95\vec{k}\} \cdot \{-200\vec{i} + 282.84\vec{j} + 200\vec{k}\} = (300)(400)\cos$

$\Rightarrow \boxed{\theta = 27.89^\circ}$

3.

$\vec{u}_Q = \frac{(-200\vec{i} + 282.84\vec{j} + 200\vec{k}\}}{400} = \{-\frac{1}{2}\vec{i} + 0.707\vec{j} + \frac{1}{2}\vec{k}\}$

$P_{//Q} = \vec{P} \cdot \vec{u}_Q = \{-67.24\vec{i} + 150\vec{j} + 250.95\vec{k}\} \cdot \{-\frac{1}{2}\vec{i} + 0.707\vec{j} + \frac{1}{2}\vec{k}\}$

$\Rightarrow \boxed{P_{//Q} = 265.15 \text{ KN}}$

or $P_{//Q} = P\cos\theta = 300\cos 27.89 = 265.15$ kN

Calculations and/or Diagrams (cont'd):

$\vec{P}_{//Q} = P_{//Q}\,\vec{u}_Q = 265.15\left\{-\frac{1}{2}\vec{i} + 0.707\vec{j} + \frac{1}{2}\vec{k}\right\}$

$\Rightarrow \boxed{\vec{P}_{//Q} = \left\{-132.58\,\vec{i} + 187.46\,\vec{j} + 132.85\,\vec{k}\right\}\text{ kN}}$

$\vec{P}_{\perp Q} = \vec{P} - \vec{P}_{//Q} = \left\{-67.24\,\vec{i} + 150\,\vec{j} + 250.95\,\vec{k}\right\} - \left\{-132.85\,\vec{i} + 187.46\,\vec{j} + 132.85\,\vec{k}\right\}$

$\boxed{\vec{P}_{\perp Q} = \left\{65.61\,\vec{i} - 37.46\,\vec{j} + 118.1\,\vec{k}\right\}}$

$\Rightarrow P_{\perp Q} = \sqrt{(65.61)^2 + (-37.46)^2 + (118.1)^2}$

$\Rightarrow \boxed{P_{\perp Q} = 140.2\text{ kN}}$

or $P_{\perp Q} = P\sin\theta = 300\sin 27.89$

$\Rightarrow P_{\perp Q} = 140.24\text{ kN}$

Problem III: *(40 points)*

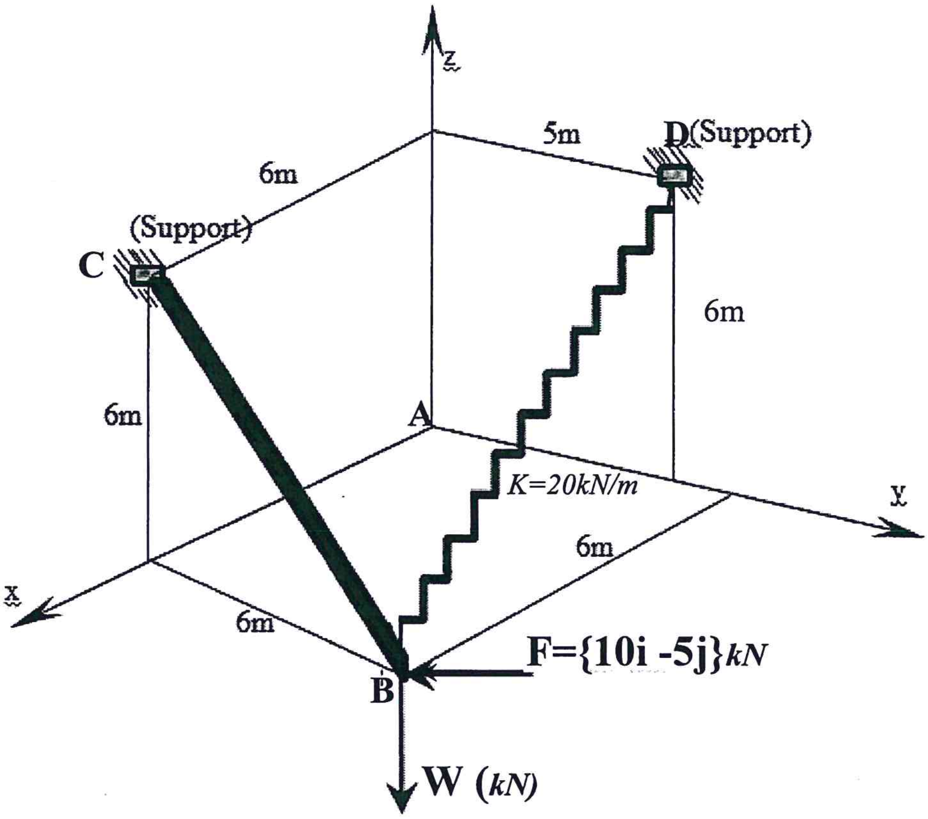


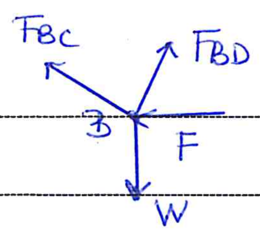
Figure III

The spring/chord system shown in Figure **III** is in equilibrium.

1. Determine the forces in spring ***BD***, chord ***BC*** and the weight ***W*** suspended at ***B***. *(30 points).*
2. What would be the un-stretched length of spring ***BD***? *(10 points).*

Calculations and/or Diagrams:

1)



FBD at B

coordinates

B(6, 6, 0) C (6, 0, 6) D (0, 5, 6)

Calculations and/or Diagrams (cont'd):

Express FBC in Cartesian Vector

$$\vec{F}_{BC} = F_{BC}\, u_{BC} = F_{BC}\left\{\frac{0\vec{i} - 6\vec{j} + 6\vec{k}}{\sqrt{(0)^2+(-6)^2+(6)^2}}\right\} = \{0\vec{i} - 0.707\vec{j} + 0.707\vec{k}\}$$

$$\boxed{\vec{F}_{BC} = \{0\vec{i} - 0.707 F_{BC}\vec{j} + 0.707 F_{BC}\vec{k}\}^{kN}}$$

Express FBD in Cartesian vector

$$\vec{F}_{BD} = F_{BD}\, u_{BD} = F_{BD}\left\{\frac{-6\vec{i} + (5-6)\vec{j} + 6\vec{k}}{\sqrt{(-6)^2+(-1)^2+(6)^2}}\right\} = \{-0.702\vec{i} - 0.117\vec{j} + 0.702\vec{k}\} F_{BD}$$

$$\therefore \boxed{\vec{F}_{BD} = \{-0.702 F_{BD}\vec{i} - 0.117 F_{BD}\vec{j} + 0.702 F_{BD}\vec{k}\}^{kN}}$$

$$\boxed{\vec{W} = \{0\vec{i} + 0\vec{j} - W\vec{k}\}}$$

$$\vec{F} = \{10\vec{i} - 5\vec{j} + 0\vec{k}\}$$

Equilibrium at B:

$$\overset{+}{\swarrow}\ \Sigma F_x = 0 \Rightarrow 0 - 0.702 F_{BD} + 0 + 10 = 0 \Rightarrow \boxed{F_{BD} = 14.245\ kN}$$

$$\overset{+}{\searrow}\ \Sigma F_y = 0 \Rightarrow -0.707(F_{BC}) - 0.117(14.245) + 0 - 5 = 0$$

$$\Rightarrow \boxed{F_{BC} = -9.43\ kN}$$

$$+\uparrow\ \Sigma F_z = 0 \Rightarrow 0.707(-9.43) + 0.702(14.245) - W + 0 = 0$$

$$\Rightarrow \boxed{W = 3.33\ kN}$$

2. $F_{BD} = \{-10\vec{i} - 1.67\vec{j} + 10\vec{k}\} \Rightarrow F_{BD} = k\Delta \Rightarrow 20 * \Delta = 14.245$

$$\Rightarrow \boxed{\Delta = 0.712\ m}$$

$$l_s = l_{BD} = \sqrt{(-6)^2+(1)^2+(6)^2} = 8.544\ m$$

$$\therefore \Delta = l_s - l_u \Rightarrow 0.712 = 8.544 - l_u \Rightarrow \boxed{l_u = 7.832\ m}$$