Statics

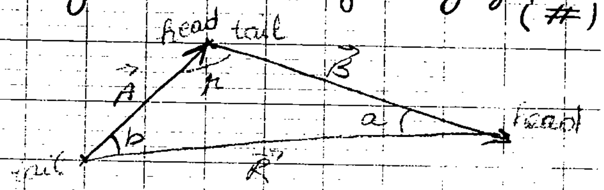
Force vectors

A vector is define by the Followings:

- Line of action
- Direction
- Point of application
- Magnitude

Vector Addition

$\vec{A} + \vec{B} = \vec{R}$: classical way → the triangle of forces (#)



Law of sin :

$$\frac{R}{\sin \hat{r}} = \frac{A}{\sin \hat{a}} = \frac{B}{\sin \hat{b}}$$

Law of cos :

$$R^2 = A^2 + B^2 - 2AB\cos \hat{r}$$
$$B^2 = A^2 + R^2 - 2AR\cos \hat{b}$$

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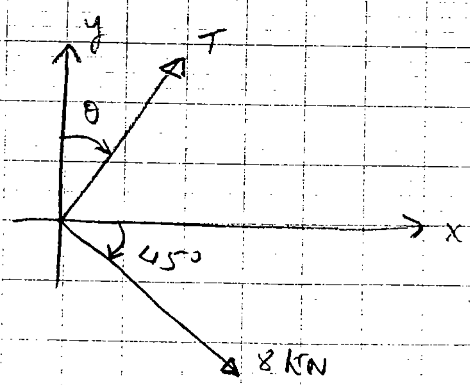
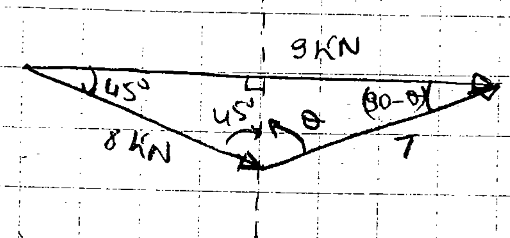
$R = 9\ KN$ (+x - AXIS)

$T = ??$ $\theta = ??$





sin

$$\frac{9}{\sin(45+\theta)} = \frac{T}{\sin 45} = \frac{8}{\sin(90-\theta)}$$

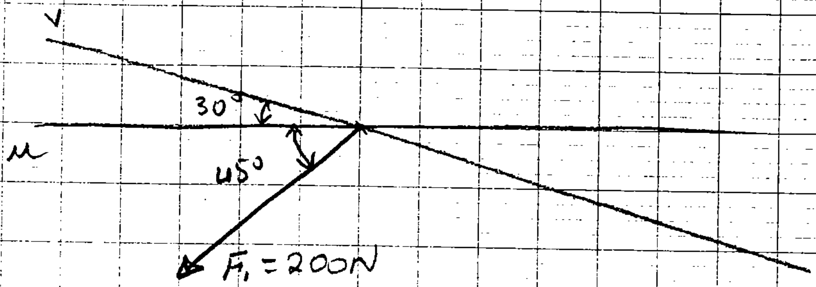
$$\Rightarrow T^2 = (9)^2 + (8^2) - 2(9)(8)\cos 45$$

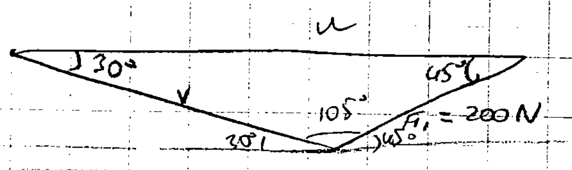
$$\Rightarrow T = \sqrt{9^2 + 8^2 - 2(9)(8)\cos 45} = 6.57 \text{ KN}.$$

$$\sin \Rightarrow \frac{9}{\sin(45+\theta)} = \frac{6,57}{\sin 45} = \frac{8}{\cos\theta} \Rightarrow \theta = \cos^{-1}\left[\frac{8\sin 45}{6.57}\right]$$

$$= 30.58^\circ$$

2-5)

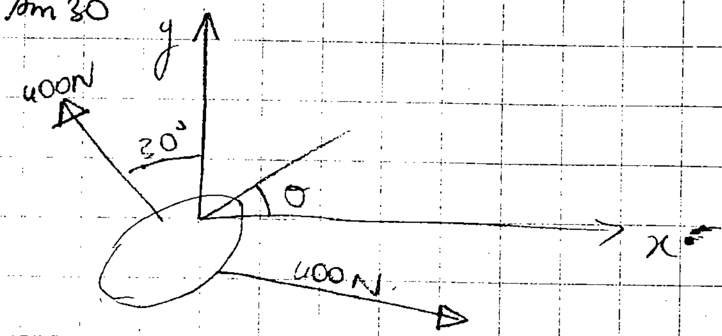


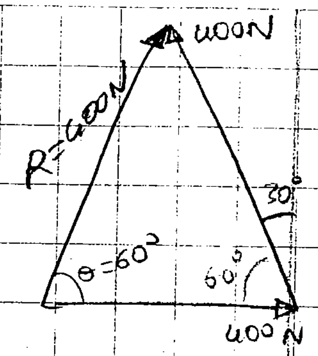


$$\frac{u}{\sin 105} = \frac{v}{\sin 45} = \frac{200}{\sin 30}.$$

$$\Rightarrow \begin{cases} u = \dfrac{200\sin 105}{\sin 30} = 386.37 \text{ N}. \\[2ex] v = \dfrac{200\sin 45}{\sin 30} = 282.84 \text{ N} \end{cases}$$

2-11)



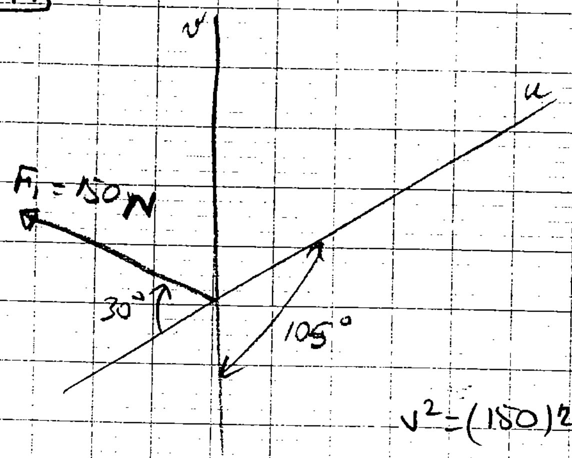


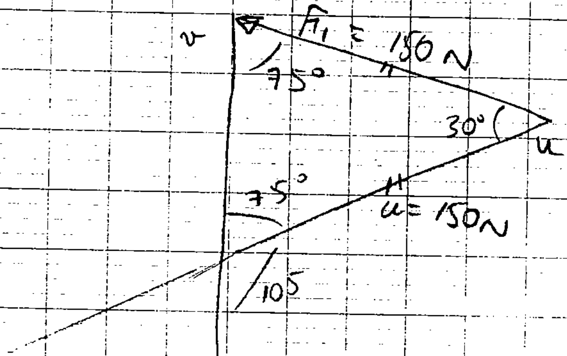
$$R^2 = (400)^2 + (400)^2 - 2(400)(400)\cos 60$$

$\Rightarrow R = 400\ N$

or directly

2-17)





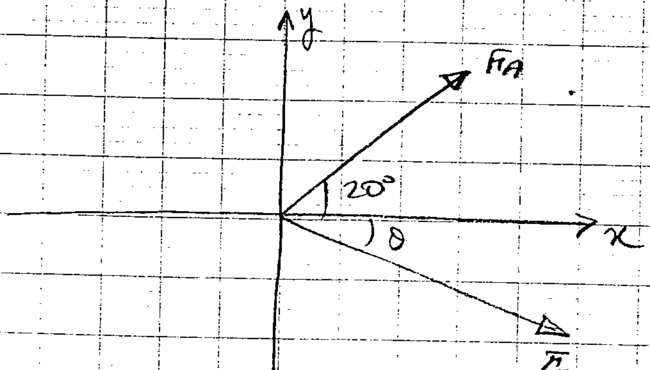
$$v^2 = (150)^2 + (150)^2 - 2(150)(150)\cos 30$$

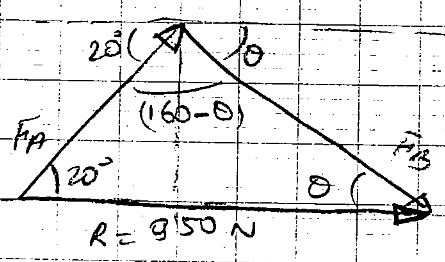
$\Rightarrow v = 77.65\ N$

2-19)

$F_A = ?$, $F_B = ?$, $\theta = ?$

$R = 950\ N$ (+ x-Axis)

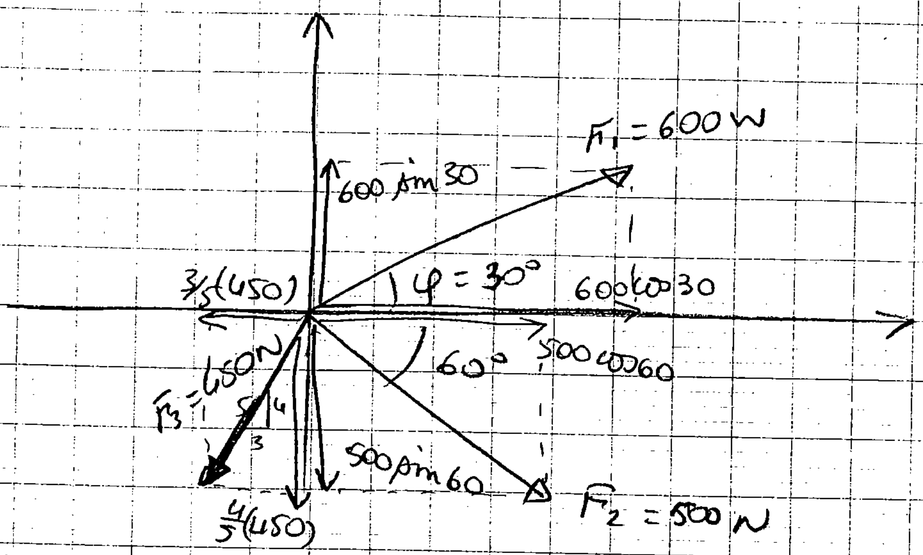


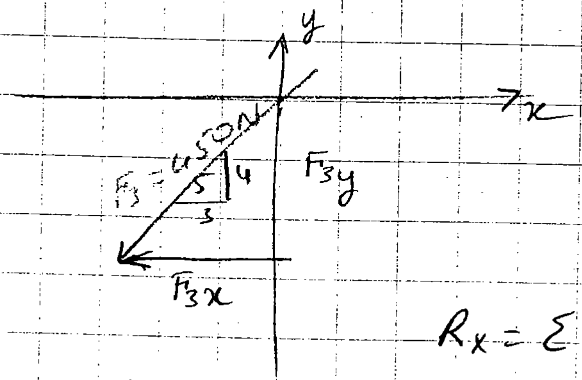


$$\frac{950}{\sin(160-\theta)} = \frac{F_A}{\sin\theta} = \frac{F_B}{\sin 20°}$$

$$F_B^2 = F_A^2 + 950^2 - 2(950)(F_A)\cos 20$$

2-33) $R = ?$ $\theta = \text{CW} + x\text{-Axis} = ?$



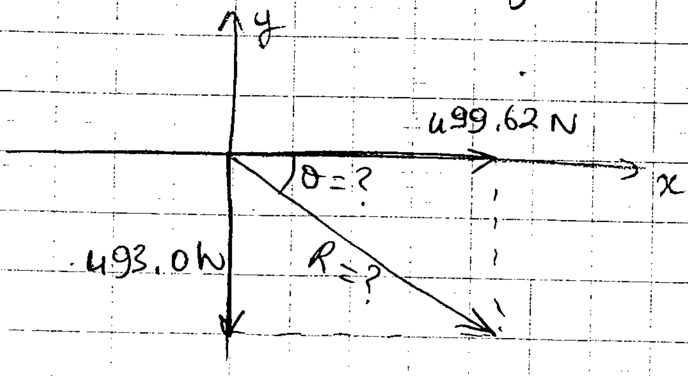


$$\frac{450}{5} = \frac{F_y}{4} = \frac{F_x}{3}$$

$F_y = 360$, $F_x = 270$

$$R_x = \Sigma F_x = 600\cos 30 + 500\cos 60 - \frac{3}{5}(450)$$
$$= 499.62\text{ N}$$

$$R_y = 600\sin 30 - 500\sin 60 - \frac{4}{5}(450) = -493.01\text{ N}.$$

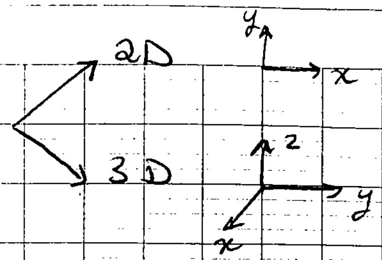


$$\theta = \tan^{-1}\left(\frac{493.0}{499.62}\right)$$
$$= 44.62°$$
$$R = \sqrt{R_x^2 + R_y^2}$$
$$= \sqrt{(499.62)^2 + (493.0)^2}$$
$$= 701.91\text{ N}$$

2-37) $R = 6\text{ kN}$ $\theta_{+\text{Axis}} = 30°$

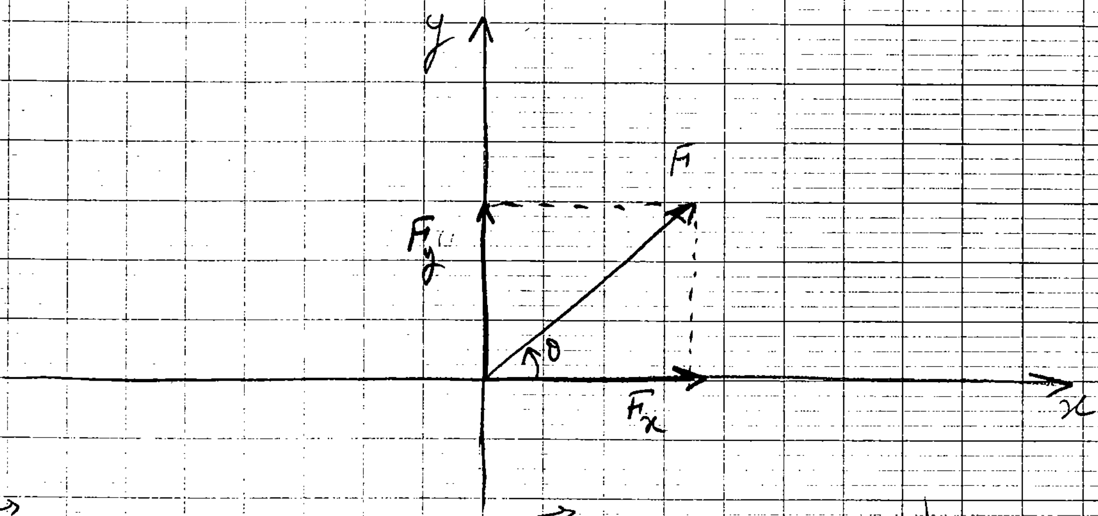
$F_2 = ?$ $\phi = ?$

Cartesian coordinate system



$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$

$\vec{i}, \vec{j}, \vec{k}$ are unit vectors on the x, y, z axis respectively.



$\vec{F} = F_x \vec{i} + F_y \vec{j}$

$F_x = F\cos\theta$

$F_y = F\sin\theta$

$\vec{F} = (F\cos\theta)\vec{i} + (F\sin\theta)\vec{j}$

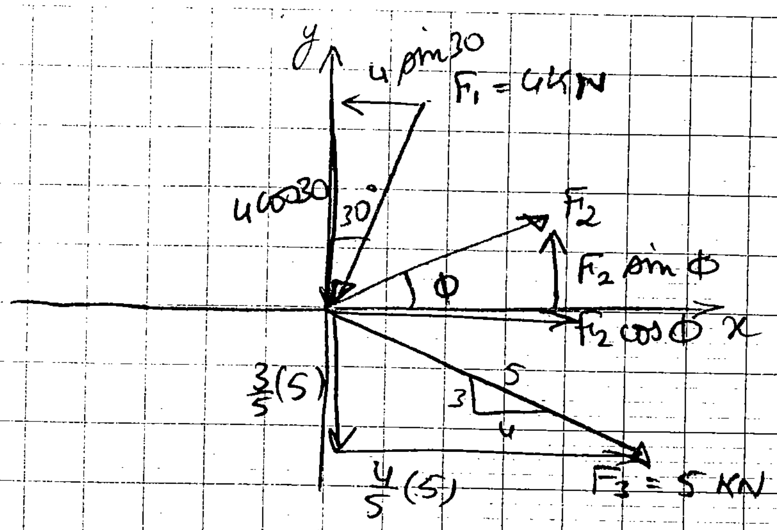
$$|\vec{F}| = F = \sqrt{(F_x)^2 + (F_y)^2}$$
$$= \sqrt{(F\cos\theta)^2 + (F\sin\theta)^2}$$
$$= \sqrt{F^2\cos^2\theta + F^2\sin^2\theta}$$
$$= \sqrt{F^2(\cos^2\theta + \sin^2\theta)}$$
$$= F$$

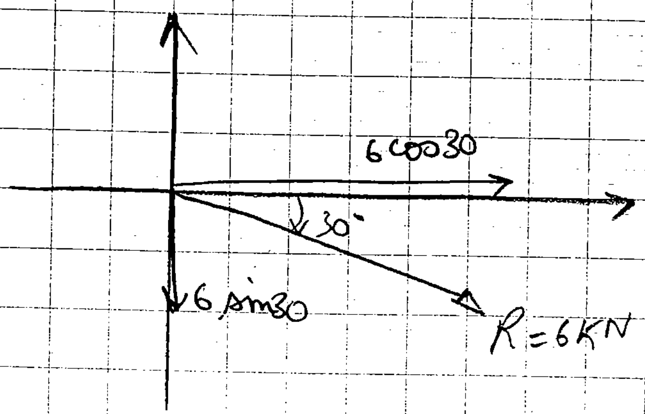
$$\vec{R} = \vec{F_1} + \vec{F_2}$$

$$\left.\begin{array}{l} \vec{R} = R_x\vec{i} + R_y\vec{j} \\ \vec{F_1} = F_{x_1}\vec{i} + F_{y_1}\vec{j} \\ \vec{F_2} = F_{x_2}\vec{i} + F_{y_2}\vec{j} \end{array}\right\} \Rightarrow R_x\vec{i} + R_y\vec{j} = F_{x_1}\vec{i} + F_{y_1}\vec{j} + F_{x_2}\vec{i} + F_{y_2}\vec{j}$$

$$\Rightarrow R_x\vec{i} + R_y\vec{j} = (F_{x_1} + F_{x_2})\vec{i} + (F_{y_1} + F_{y_2})\vec{j}$$

$$\Rightarrow \begin{cases} R_x = F_{x_1} + F_{x_2} \\ R_y = F_{y_1} + F_{y_2} \end{cases}$$





$\vec{R} = (6\cos 30\,\vec{i} - 6\sin 30\,\vec{j})\,kN$

$R_x = 6\cos 30 = \Sigma F_x$

$= -4\sin 30 + F_2\cos\phi + \frac{4}{5}(5$

$\Rightarrow F_2\cos\phi = 6\cos 30 + 4\sin 30$

$- \frac{4}{5}(5) = 3.20$

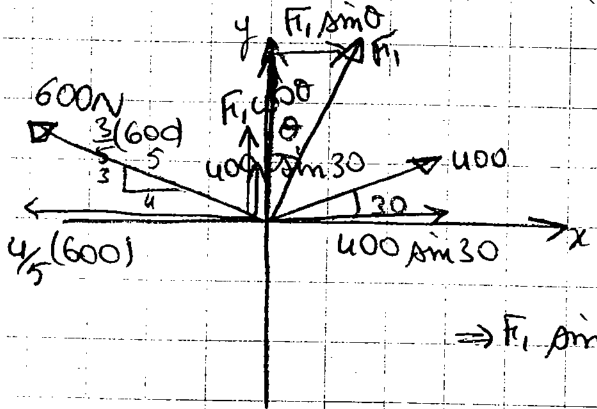
$R_y = -6\sin 30 = \Sigma F_y = -4\cos 30 + F_2\sin\phi - \frac{3}{5}(5)$

$\Rightarrow F_2\sin\phi = -6\sin 30 + 4\cos 30 + \frac{3}{5}(5) = 3.46$

$$\frac{F_2\sin\phi}{F_2\cos\phi} = \tan\phi = \frac{3.46}{3.20} \Rightarrow \phi = \tan^{-1}\left(\frac{3.46}{3.20}\right) = 47.24°$$

$$F_2 = \sqrt{(F_2\cos\phi)^2 + (F_2\sin\phi)^2} = \sqrt{(3.46)^2 + (3.20)^2} = 4.71\,KN$$

2-39) $R = 800N \uparrow$, $\vec{R} = (800\,\vec{j})\,N$



$R_x = 0 = \Sigma F_x$

$= F_1\sin\theta + 400\sin 30 - \frac{4}{5}(6$

$\Rightarrow F_1\sin\theta = -400\sin 30 + \frac{4}{5}(600)$

$= 133.59$

$R_y = 800 = \Sigma F_y = F_1 \cos\theta + 400 \sin 30 + \frac{3}{5}(600)$

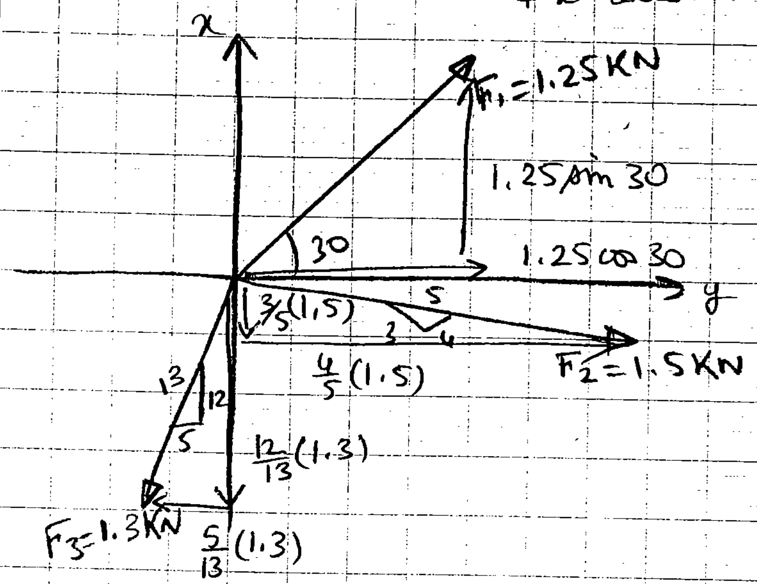
$\Rightarrow F_1 \cos\theta = -400 \sin 30 - \frac{3}{5}(600) + 800 = 240$

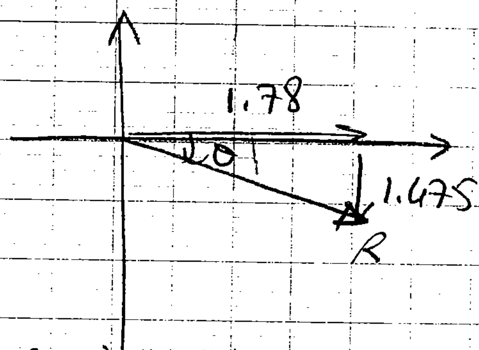
$\frac{F_1 \sin\theta}{F_1 \cos\theta} = \tan\theta = \frac{133.59}{240} \Rightarrow \theta = \tan^{-1}\left(\frac{133.59}{240}\right) = 29.10°$

$F_1 = \sqrt{(F_1 \cos\theta)^2 + (F_1 \sin\theta)^2} = \sqrt{(240)^2 + (133.59)^2} = 274.65\ N$

2-43) $F_1 = 1.25\ KN$, $\phi = 30°$

$R = ?$ $\qquad \theta \circlearrowleft_{+x\text{-axis}} = ?$



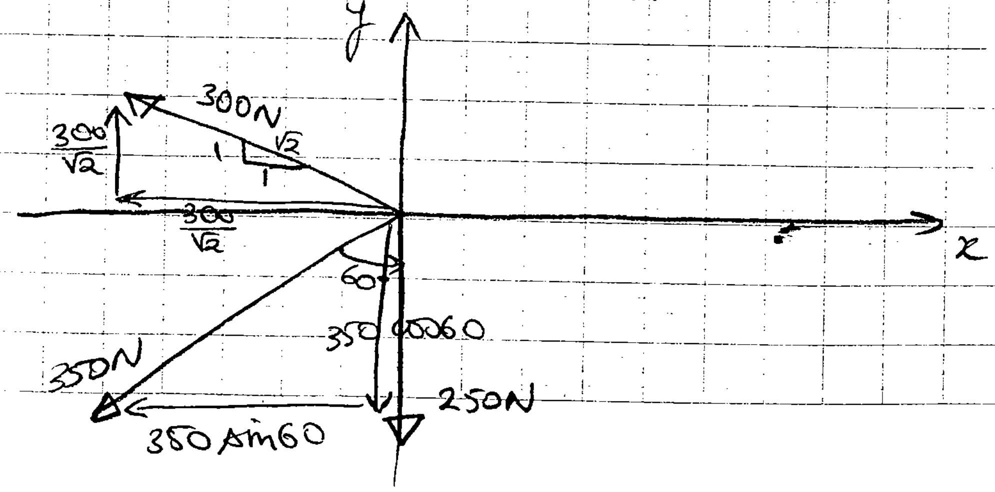


$R_x = \Sigma F_x = 1.25 \cos 30 + \frac{4}{5}(1.5) - \frac{5}{13}(1.3) = 1.78\ KN$

$R_y = \Sigma F_y = 1.25 \sin 30 - \frac{3}{5}(1.5) - \frac{12}{13}(1.3) = -1.475\ KN$

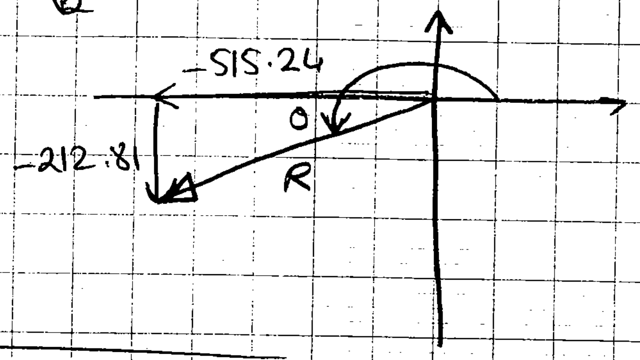
$\theta = \tan^{-1}\left(\frac{1.475}{1.78}\right) = 39.69°$, $R = \sqrt{(1.78)^2 + (1.475)^2} = 2.31\ K$

2-49) $R = ?$ $\qquad \theta \circlearrowleft +x\text{-axis} = ?$



$R_x = \sum F_x = -\frac{300}{\sqrt{2}} - 350 \sin 60 = -515.24\ N.$

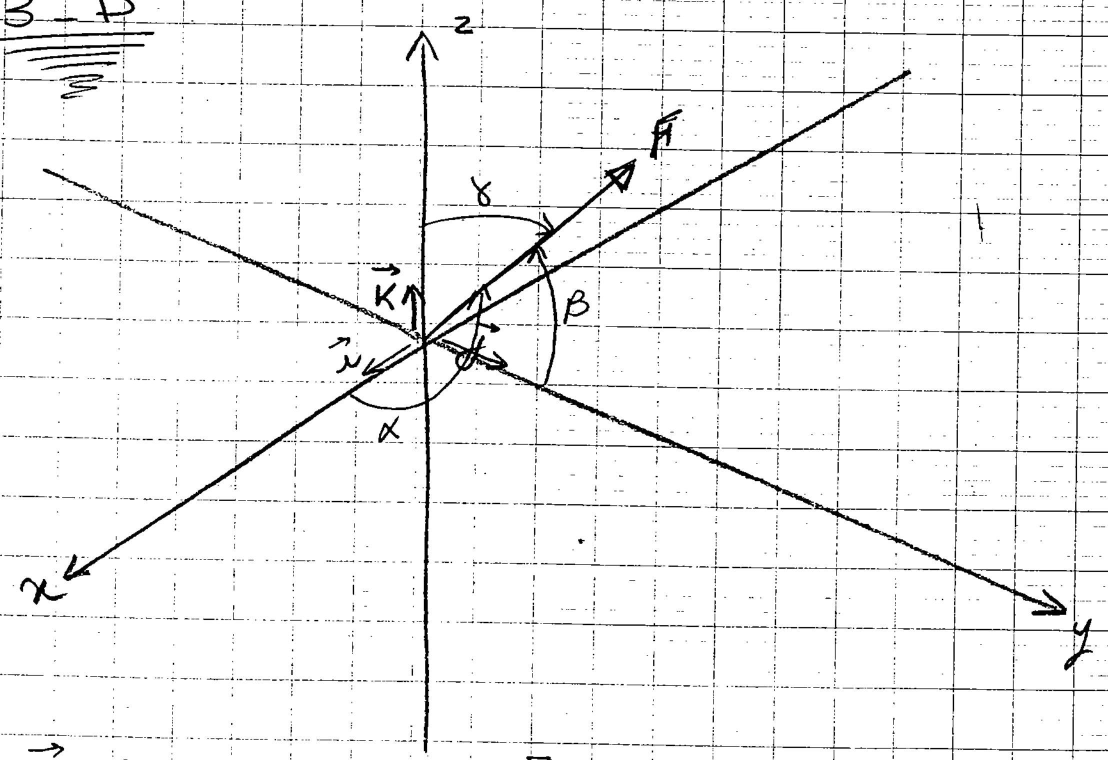
$R_y = \sum F_y = \frac{300}{\sqrt{2}} - 350 \cos 60 - 250 = -212.81 N$



$R = \sqrt{(-515.24)^2 + (-212.81)^2} = 557.48N$

$\theta = 180 + \tan^{-1}\left(\frac{212.81}{515.24}\right) = 202.45°$

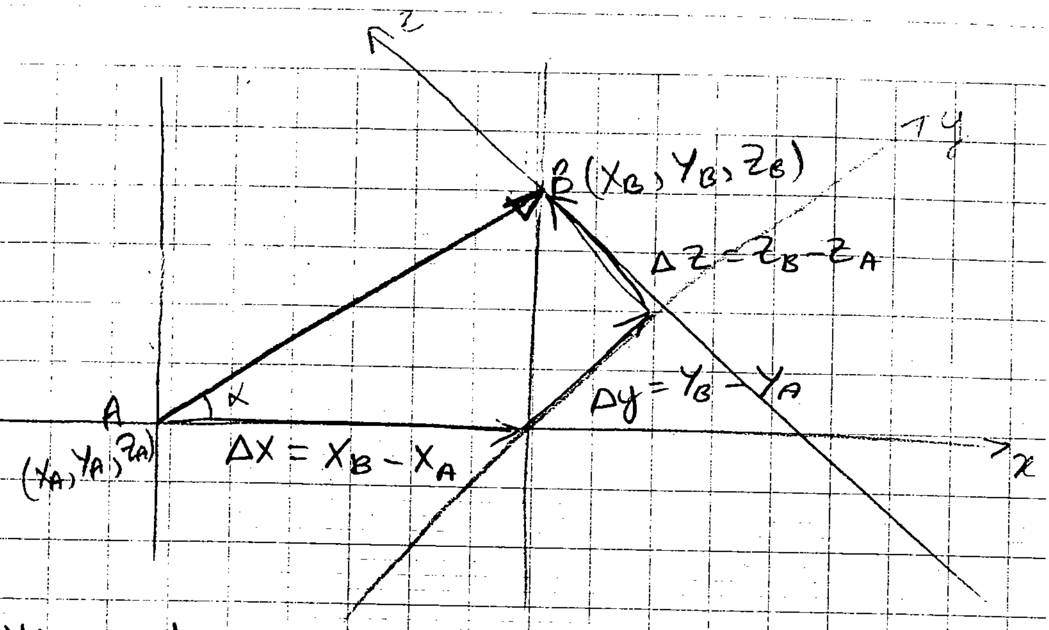
3 - D



$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$

$$\left.\begin{aligned} F &= \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2} \\ F_x &= F\cos\alpha \\ F_y &= F\cos\beta \\ F_z &= F\cos\gamma \end{aligned}\right\} \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

α, β and γ are known as the directional angles



$\vec{r}_{AB}$ = Position vector from A to B

$= r_x \vec{i} + r_y \vec{j} + r_z \vec{k}$

where: $r_x = X_B - X_A$

$r_y = Y_B - Y_A$

$r_z = Z_B - Z_A$

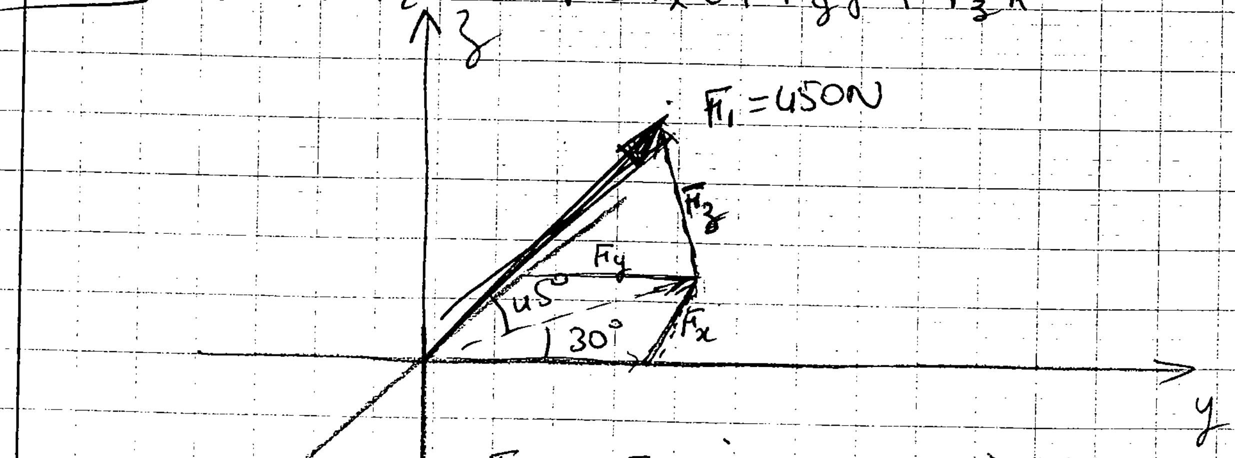
$\cos\alpha = \dfrac{\Delta x}{|\vec{AB}|}$

$\cos\beta = \dfrac{\Delta y}{|\vec{AB}|}$

$\cos\gamma = \dfrac{\Delta z}{|\vec{AB}|}$

$r = \sqrt{r_x^2 + r_y^2 + r_z^2} \Rightarrow \cos\alpha\,\vec{i} + \cos\beta\,\vec{j} + \cos\gamma\,\vec{k} = \dfrac{\vec{r}}{|\vec{r}|}$

2-59) γ For $\vec{F}_2$ $\qquad \vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$



$F_{1z} = F_1 \sin 45 = 450 \sin 45$

$F_{1xy} = F_1 \cos 45 = 450 \cos 45$

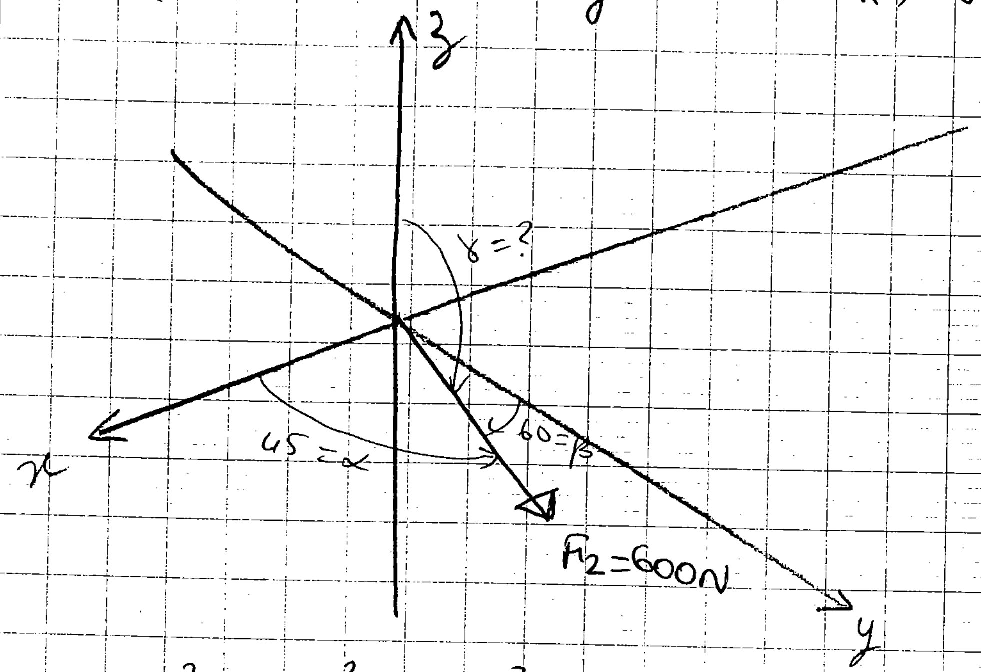
$F_{1y} = F_{1xy} \cos 30 = 450 \cos 45 \cos 30$

$F_{1x} = -F_{1xy} \sin 30 = -450 \cos 45 \sin 30$

HW 2-72 / 2-52 / 2-46 / 2-41

$\vec{F_1} = [(-450 \cos 45 \sin 30)\vec{i} + (450 \cos 45 \cos 30)\vec{j} + 450 \sin 45 \vec{k}]$ N

$= (-159.10\vec{i} + 275.57\vec{j} + 318.20\vec{k})$ N



$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos\gamma = \sqrt{1 - \cos^2\alpha - \cos^2\beta}$

$\gamma = \cos^{-1}(\sqrt{1 - \cos^2 45 - \cos^2 60}) = 60°$

$\vec{F_2} = F_2(\cos\alpha\,\vec{i} + \cos\beta\,\vec{j} + \cos\gamma\,\vec{k})$

$= 600(\cos 45\,\vec{i} + \cos 60\,\vec{j} + \cos 60\,\vec{k})$

$\vec{F_2} = 424.26\,\vec{i} + 300\vec{j} + 300\,\vec{k}$

2-60) $\vec{R} = \vec{F_1} + \vec{F_2} \Rightarrow R_x = \Sigma F_x = -159.10 + 424.26 = 265.$

$R_y = \Sigma F_y = 275.57 + 300 = 575.57$ N

$R_z = \Sigma F_z = 318.20 + 300 = 618.20$ N

$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(265.60)^2 + (575.57)^2 + (618.20)^2} = 885.30$

$R_x = R\cos\alpha \Rightarrow \alpha = \cos^{-1}\left(\frac{265.60}{885.30}\right) = 72.57°$

$R_y = R\cos\beta \Rightarrow \beta = \cos^{-1}\left(\frac{575.57}{885.30}\right) = 49.45°$

$R_z = R\cos\gamma \Rightarrow \gamma = \cos^{-1}\left(\frac{618.20}{885.30}\right) = 45.71°$

2-61 $\bullet\ \vec{F}_1 = \left[\frac{4}{5}(3)\vec{i} + \frac{3}{5}(3)\vec{k}\right]$ KN

$\vec{F}_1 = (2.4\vec{i} + 1.8\vec{k})$ KN

$\bullet\ \vec{F}_2 = F_2(\cos\alpha\,\vec{i} + \cos\beta\,\vec{j} + \cos\gamma\,\vec{k})$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\beta = \cos^{-1}(\sqrt{1-\cos^2\alpha - \cos^2\gamma})$

$= \cos^{-1}(\sqrt{1-\cos^2 60 - \cos^2 120})$

$= 45°$

$\vec{F}_2 = 2(\cos 60\,\vec{i} + \cos 45\,\vec{j} + \cos 120\,\vec{k})$

$\vec{F}_2 = (\vec{i} + 1.41\vec{j} - \vec{k})$ KN

2-62

$\vec{R} = \vec{F}_1 + \vec{F}_2 \Rightarrow$

$$\begin{cases} R_x = \Sigma F_x = 2.4 + 1 = 3.4 \\ R_y = \Sigma F_y = 0 + 1.41 = 1.41 \\ R_z = \Sigma F_z = 1.8 - 1 = 0.8 \end{cases}$$

$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{3.4^2 + 1.41^2 + 0.8^2} = 3.77$ KN

$R_x = R\cos\alpha \Rightarrow \alpha = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{3.4}{3.77}\right) = 25.60°$

$R_y = R\cos\beta \Rightarrow \beta = \cos^{-1}\left(\frac{R_y}{R}\right) = \cos^{-1}\left(\frac{1.41}{3.77}\right) = 67.97°$

$R_z = R\cos\gamma \Rightarrow \gamma = \cos^{-1}\left(\frac{R_z}{R}\right) = \cos^{-1}\left(\frac{0.8}{3.77}\right) = 77.75°$

2-65

$\vec{R} = \vec{F}_1 + \vec{F}_2 = (-100\vec{k})$ N

$F_2 = ?\quad \alpha, \beta, \gamma = ?$

$\vec{F}_1 = (-60\cos 50\cos 30\,\vec{i} + 60\cos 50\sin 30\,\vec{j} - 60\sin 50\,\vec{k})$ N

$\Rightarrow \vec{F}_1 = (-33.40\vec{i} + 19.28\vec{j} - 45.96\vec{k})$ N

$\vec{F}_2 = F_2(\cos\alpha\,\vec{i} + \cos\beta\,\vec{j} + \cos\gamma\,\vec{k})$ N

$\vec{R} = \vec{F}_1 + \vec{F}_2 = (-100\vec{K})\,N \Rightarrow$

$R_x = \Sigma F_x = 0 = -33.40 + F_2 \cos\alpha \Rightarrow F_2 \cos\alpha = 33.40 = F_{2x}$

$R_y = \Sigma F_y = 0 = 19.28 + F_2 \cos\beta \Rightarrow F_2 \cos\beta = -19.28 = F_{2y}$

$R_z = \Sigma F_z = -100 = -45.96 + F_2 \cos\gamma \Rightarrow F_2 \cos\gamma = -54.04 = F_{2z}$

$F_2 = \sqrt{33.40^2 + (-19.28)^2 + (-54.04)^2} = 68.39\,N.$

$\alpha = \cos^{-1}\left(\frac{33.40}{68.39}\right) = 59.80°$

$\beta = \cos^{-1}\left(\frac{-19.28}{68.39}\right) = 106.37°$

$\gamma = \cos^{-1}\left(\frac{-54.04}{68.39}\right) = 144.49°$

2-75) $F_1 = ?$, $\alpha_1 = ?$, $\beta_1 = ?$, $\gamma_1 = ?$

$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

$\vec{F}_1 = F_1 (\cos\alpha_1 \vec{i} + \cos\beta_1 \vec{j} + \cos\gamma_1 \vec{K})$

$\vec{F}_2 = -200\vec{K}N$

$\vec{F}_3 = -300\vec{j}\,N$

$F_1 \neq 0$

$R_x = \Sigma F_x = 0 = F_1 \cos\alpha_1 \Rightarrow \alpha_1 = 90°$

$R_y = \Sigma F_y = 0 = F_1 \cos\beta_1 - 300 \Rightarrow F_1 \cos\beta_1 = 300$

$R_z = \Sigma F_z = 0 = F_1 \cos\gamma_1 - 200 \Rightarrow F_1 \cos\gamma_1 = 200$

$F_1 = \sqrt{0^2 + 300^2 + 200^2} = 360.56\,N$

$\beta_1 = \cos^{-1}\left(\frac{360.56}{300}\right) = 33.69°$

$\gamma_1 = \cos^{-1}\left(\frac{360.56}{200}\right) = 56.31°$

2-79) $F_3 = ?$, $\alpha_3 = ?$, $\beta_3 = ?$, $\gamma_3 = ?$

$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (9\vec{j})\,KN$

$\vec{F}_1 = (12\cos 30\,\vec{j} - 12 \sin 30 \vec{K})\,KN$

$= (10.39\vec{j} - 6\vec{K})\,KN$

$\vec{F}_2 = \left(-\frac{12}{13}(10)\vec{i} + \frac{5}{13}(10)\vec{K}\right) KN$

$= (-9.23\vec{i} + 3.85\vec{K})\,KN$

$\vec{F}_3 = F_3 (\cos\alpha_3 \vec{i} + \cos\beta_3 \vec{j} + \cos\gamma_3 \vec{K})$

$$R_x = \sum F_x = 0 = -9.23 + F_3 \cos\alpha_3 \Rightarrow F_3\cos\alpha_3 = 9.23$$
$$R_y = \sum F_y = 9 = 10.39 + F_3 \cos\beta_3 \Rightarrow F_3 \cos\beta_3 = -1.39$$
$$R_z = \sum F_z = 0 = -6 + 3.85 + F_3\cos\gamma_3 \Rightarrow F_3\cos\gamma_3 = 2.15$$

$$F_3 = \sqrt{9.23^2 + (-1.39)^2 + 2.15^2} = 9.58 \text{ KN}$$

$$\alpha_3 = \cos^{-1}\left(\frac{9.23}{9.58}\right) = 15.50°$$

$$\beta_3 = \cos^{-1}\left(\frac{-1.39}{9.58}\right) = 98.34°$$

$$\gamma_3 = \cos^{-1}\left(\frac{2.15}{9.58}\right) = 77.03°$$

2-83) $F_3 = ?$, $\alpha, \beta, \gamma = ?$

$$\vec{F}_1 = \left(\tfrac{4}{5}(80)\vec{i} + \tfrac{3}{5}(80)\vec{K}\right) N$$
$$= (64\vec{i} + 48\vec{K}) N$$
$$\vec{F}_2 = (-110\vec{K}) N$$
$$\vec{F}_3 = F_3(\cos\alpha\,\vec{i} + \cos\beta\,\vec{j} + \cos\gamma\,\vec{K}) N$$

$$\begin{cases} R_z = 120 \sin 45 \\ R_{xy} = 120\cos 45 \\ R_x = R_{xy}\sin 30 = 120\cos 45 \sin 30 \\ R_y = R_{xy}\cos 30 = 120\cos 45\cos 30 \end{cases}$$

$$\vec{R} = (120\cos 45\sin 30)\vec{i} + (120\cos 45\cos 30)\vec{j} + (120\sin 45)\vec{K}$$
$$= (42.43\vec{i} + 73.48\vec{j} + 84.85\vec{K}) N$$

$$R_x = \sum F_x = 42.43 = 64 + F_3\cos\alpha \Rightarrow F_3\cos\alpha = -21.57$$
$$R_y = \sum F_y = 73.48 = 0 + 0 + F_3\cos\beta \Rightarrow F_3\cos\beta = 73.48$$
$$R_z = \sum F_z = 84.85 = 48 - 110 + F_3\cos\gamma \Rightarrow F_3\cos\gamma = 146.85$$

$$F_3 = \sqrt{(-21.57)^2 + 73.48^2 + 146.85^2} = 165.62 N$$

$$\alpha = \cos^{-1}\left(\frac{-21.57}{165.62}\right) = 97.48° ;\ \beta = \cos^{-1}\left(\frac{73.48}{165.62}\right) = 63.66° ;\ \gamma = \cos^{-1}\left(\frac{146.8}{165.6}\right)$$
$$= 27.54°$$

Position vectors

$$\vec{F}_{AB} = F_{AB}\left(\frac{\vec{r}_{AB}}{r_{AB}}\right)$$

$$\vec{r}_{AB} = r_x \vec{i} + r_y \vec{j} + r_z \vec{K}$$

$$r_x = X_B - X_A$$

$$r_y = Y_B - Y_A$$

$$r_z = Z_B - Z_A$$

$$r = \sqrt{r_x^2 + r_y^2 + r_z^2}$$

2.89) A(0, 0, 1.2) m

B(0.9, −0.9, 0.75) m

C(0.6, 1.2, 0) m

$$\vec{R} = \vec{F}_{AB} + \vec{F}_{AC}$$

$R = ?$, $\alpha, \beta, \gamma = ?$

$$\vec{F}_{AB} = F_{AB}\left[\frac{(X_B - X_A)\vec{i} + (Y_B - Y_A)\vec{j} + (Z_B - Z_A)\vec{K}}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (Z_B - Z_A)^2}}\right]$$

$$= 3\,KN\left(\frac{0.9\vec{i} - 0.9\vec{j} - 0.45\vec{K}}{\underbrace{\sqrt{0.9^2 + (-0.9)^2 + (-0.45)^2}}_{1.35}}\right)$$

$$\vec{F}_{AB} = \cancel{2.22\vec{i} - 2.22\vec{j} - 1}\ (2.0\vec{i} - 2.0\vec{j} - 1.0\vec{K})\ KN$$

$$\vec{F}_{AC} = F_{AC}\left[\frac{(X_C - X_A)\vec{i} + (y_C - y_A)\vec{j} + (z_C - z_A)\vec{K}}{\sqrt{(x_C - x_A)^2 + (y_C - y_A)^2 + (z_C - z_A)^2}}\right]$$

$$= 3.75\left[\frac{0.6\vec{i} + 1.2\vec{j} - 1.2\vec{K}}{\underbrace{\sqrt{0.6^2 + 1.2^2 + (-1.2)^2}}_{1.8}}\right]$$

$$\vec{F}_{AC} = (1.85\vec{i} + 2.50\vec{j} - 2.50\vec{K})\ KN$$

$$\vec{R} = \vec{F}_{AB} + \vec{F}_{AC} \Rightarrow$$

$R_x = \Sigma F_x = 2 + 1.25 = 3.25$
$R_y = \Sigma F_y = -2 + 2.5 = 0.50$
$R_z = \Sigma F_z = -1 - 2.5 = -3.50$

$$R = \sqrt{3.25^2 + 0.50^2 + (-3.50)^2} = 4.80 \, KN$$

$$\alpha = \cos^{-1}\left(\frac{3.25}{4.80}\right) = 47.38°$$

$$\beta = \cos^{-1}\left(\frac{0.50}{4.80}\right) = 84.02°$$

$$\gamma = \cos^{-1}\left(\frac{-3.50}{4.80}\right) = 136.82°$$

2-91) $A(0, 0, 6)$
$B(4.5 \sin 45°, -4.5 \cos 45°, 0) = B(3.18, -3.18, 0)$
$C(-3, -6, 0)$

$$\vec{F}_{AB} = F_{AB}\left(\frac{\vec{r}_{AB}}{r_{AB}}\right) = F_{AB}\left[\frac{(x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right]$$

$$= 900 \times \frac{3.18\vec{i} - 3.18\vec{j} - 6\vec{k}}{\sqrt{(3.18)^2 + (-3.18)^2 + (-6)^2}} = \{381.60\vec{i} - 381.60\vec{j} - 720\vec{k}\} \, N$$

$$\vec{F}_{AC} = F_{AC}\left(\frac{\vec{r}_{AC}}{r_{AC}}\right) = F_{AC}\left[\frac{(x_C - x_A)\vec{i} + (y_C - y_A)\vec{j} + (z_C - z_A)\vec{k}}{\sqrt{(x_C - x_A)^2 + (y_C - y_A)^2 + (z_C - z_A)^2}}\right]$$

$$= 600 \times \frac{-3\vec{i} - 6\vec{j} - 6\vec{k}}{\sqrt{(-3)^2 + (-6)^2 + (-6)^2}} = [-200\vec{i} - 400\vec{j} - 400\vec{k}] \, N$$

$\vec{R} = \vec{F}_{AB} + \vec{F}_{AC} \Rightarrow$

$$\begin{cases} R_x = \Sigma F_x = 381.60 - 200 = 181.60 \\ R_y = \Sigma F_y = -381.60 - 400 = -781.60 \\ R_z = \Sigma F_z = -720 - 400 = -1120 \end{cases}$$

$$R = \sqrt{181.60^2 + (-781.60)^2 + (-1120)^2} = 1377.78 \, N$$

$$\alpha = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{181.60}{1377.78}\right) = 82.43^\circ$$

$$\beta = \cos^{-1}\left(\frac{R_y}{R}\right) = \cos^{-1}\left(\frac{-781.60}{1377.78}\right) = 124.56^\circ$$

$$\gamma = \cos^{-1}\left(\frac{R_z}{R}\right) = \cos^{-1}\left(\frac{-1120}{1377.78}\right) = 144.38^\circ$$

Q.95 ~~A (3 sin~~ $B(1.5, -2.1, 0)$

$z_A = 3\sin 70$

$xy_A = 3\cos 70$

$x_A = xy_A \sin 30^\circ = 3\cos 70^\circ \sin 30^\circ$

$y_A = xy_A \cos 30^\circ = 3\cos 70^\circ \cos 30^\circ$

$A(-3\cos 70 \sin 30, 3\cos 70 \cos 30, 3\sin 70)$

$\Rightarrow A(-0.51, 0.89, 2.82)$

$$\vec{F} = \vec{F}_{AB} = F_{AB}\left[\frac{(x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{K}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right]$$

$$= 675\left[\frac{2.01\vec{i} - 2.99\vec{j} - 2.82\vec{K}}{\sqrt{(2.01)^2 + (-2.99)^2 + (-2.82)^2}}\right]$$

$$\vec{F} = (296.88\vec{i} - 441.63\vec{j} - 416.52\vec{K})\ N$$

$$\alpha = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{296.88}{675}\right) = 63.91^\circ$$

$$\beta = \cos^{-1}\left(\frac{F_y}{F}\right) = \cos^{-1}\left(\frac{-441.63}{675}\right) = 130.86^\circ$$

$$\gamma = \cos^{-1}\left(\frac{F_z}{F}\right) = \cos^{-1}\left(\frac{-416.52}{675}\right) = 128.10^\circ$$

2-97 $F_{AB} = F_A = 300\,N$, $F_{CD} = F_C = 250\,N$

$B(0, 0, 0)$

$D(-0.5, 0, 0)$

$A(0, -(1+1.5\cos30), 1.5\sin30) = A(0, -2.30, 0.75)\,m$

$C(-2.5, -2.30, 0.75)\,m$

$$\vec{F}_{AB} = \text{---}$$

$$= 300\left[\frac{0\vec{i} + 2.30\vec{j} - 0.75\vec{k}}{\sqrt{(2.3)^2 + (-0.75)^2}}\right]$$

$$= (285.12\,\vec{j} - 92.98\,\vec{k})\ N$$

$$\vec{F}_{CD} = \text{---}$$

$$= 250\left[\frac{2\vec{i} + 2.30\vec{j} - 0.75\vec{k}}{\sqrt{(2)^2 + (2.30)^2 + (-0.75)^2}}\right]$$

$$= (159.24\,\vec{i} + 183.12\,\vec{j} - 59.71\,\vec{k})\ N$$

2-99 $B(-2, 0, 3)$, $C(3, 0, 2)$, $A(0, 6, 0)$

$\vec{R} = (-R_y\vec{j})\ N$

$$\vec{F}_B = \vec{F}_{AB} = F_{AB}\left(\frac{\vec{r}_{AB}}{r_{AB}}\right) = F_B\left[\frac{-2\vec{i} - 6\vec{j} + 3\vec{k}}{\sqrt{(-2)^2 + (-6)^2 + (3)^2}}\right]$$

$$\vec{F}_C = \vec{F}_{AC} = F_{AC}\left(\frac{\vec{r}_{AC}}{r_{AC}}\right) = F_C\left[\frac{3\vec{i} - 6\vec{j} + 2\vec{k}}{\sqrt{3^2 + (-6)^2 + 2^2}}\right]$$

$\vec{W} = (-1500\,\vec{k})\ N$

$$\vec{R} = \vec{F}_B + \vec{F}_C + \vec{W} \Rightarrow \begin{cases} R_x = \Sigma F_x = 0 = -\frac{2}{7}F_B + \frac{3}{7}F_C & (1) \\ R_y = \Sigma F_y = -R_y = -\frac{6}{7}F_B - \frac{6}{7}F_C & (2) \\ R_z = \Sigma F_z = 0 = \frac{3}{7}F_B + \frac{2}{7}F_C - 1500 & (3) \end{cases}$$

$$① \Rightarrow \begin{cases} -\frac{2}{7} F_B + \frac{3}{7} F_C = 0 & (\times 2) \\ \frac{3}{7} F_B + \frac{2}{7} F_C = 1500 & (\times 3) \end{cases}$$
$③ \Rightarrow$

$$\frac{13}{7} F_C = 3000$$
$$F_C = 1615.38 \text{ N}$$

$① \Rightarrow F_B = \frac{7}{2}\left(\frac{3}{7}\right) F_C = 2423.08 \text{ N}$

$② \Rightarrow R_y = -\frac{6}{7}(2423.08) - \frac{6}{7}(1615.38) = -3461.53 \text{ N}$

2-101) $\vec{F} = (-120\vec{i} - 90\vec{j} - 80\vec{k})$ N $= \vec{F}_{AO}$

$l_{AO} = 1.02$ m

$A(x, y, z)$, $O(0, 0, 0)$

$\vec{F} = \vec{F}_{AO} = -120\vec{i} - 90\vec{j} - 80\vec{k} = F_{AO}\left(\frac{\vec{r}_{AO}}{r_{AO}}\right)$

$F_{AO} = \sqrt{(-120)^2 + (-90)^2 + (-80)^2} = 170$

$l_{AO} = r_{AO} = 1.02$

$\vec{F}_{AO} = \frac{170}{1.02}(-x\vec{i} - y\vec{j} - z\vec{k})$

$x = \frac{1.02}{170}(120) = 0.72$ m

$y = \frac{1.02}{170}(90) = 0.54$ m

$z = \frac{1.02}{170}(80) = 0.48$ m.

HW: 2-104 / 2-98 / 2-96 / 2-92 / 2-114

Dot Product

$\vec{A} \cdot \vec{B} = AB \cos \widehat{A,B}$

$\vec{P} \cdot \vec{Q} = PQ \cos \widehat{P,Q}$

$= (P_x \vec{i} + P_y \vec{j} + P_z \vec{k}) \cdot (Q_x \vec{i} + Q_y \vec{j} + Q_z \vec{k})$

$= (P_x \vec{i}) \cdot (Q_x \vec{i}) + \ldots$

$\vec{P} \cdot \vec{Q} = P_x Q_x + P_y Q_y + P_z Q_z$

$\vec{i} \cdot \vec{i} = 1$, $\vec{j} \cdot \vec{i} = 0$, $\vec{k} \cdot \vec{i} = 0$

$\vec{i} \cdot \vec{j} = 0$, $\vec{j} \cdot \vec{j} = 1$, $\vec{k} \cdot \vec{j} = 0$

$\vec{i} \cdot \vec{k} = 0$, $\vec{j} \cdot \vec{k} = 0$, $\vec{k} \cdot \vec{k} = 1$

2-115) $F = 600N$, $F \perp DE$

$F_{EB} = F = 600N$

$B(0,2,0)$, $E(4,5,-2)$

$D(4,2,-2)$

$\vec{r}_{EB} = -4\vec{i} - 3\vec{j} + 2\vec{k}$

$\Rightarrow r_{EB} = \sqrt{(-4)^2 + (-3)^2 + (2)^2} = 5.39$

$\vec{r}_{ED} = -3\vec{j}$

$\Rightarrow r_{ED} = 3$

$\hat{\theta} = \widehat{EB,ED} = \cos^{-1}\left[\dfrac{\vec{r}_{EB} \cdot \vec{r}_{ED}}{(r_{EB})(r_{ED})}\right] = \cos^{-1}\left[\dfrac{(-4\vec{i} - 3\vec{j} + 2\vec{k})(-3\vec{j})}{(5.39)(3)}\right]$

$= 56.18°$

Proj on DE $= 600 \cos 56.18 = 333.95 N$

Proj $\perp$ DE $= 600 \sin 56.18 = 448.47 N$

2-119) $\vec{F} = (-500\vec{k})\, N \Rightarrow \vec{r}_F = -r_z \vec{k}$

$F_1 = ?$ = Proj on OA , $F_2 = ?$ = Proj $\perp$ OA

$\vec{r}_{AO} = ?$

$O(0,0,0)$, $A(20,40,40)$

$\vec{r}_{AO} = -20\vec{i} - 40\vec{j} - 40\vec{k} \Rightarrow r_{AO} = \sqrt{(-20)^2 + (-40)^2 + (-40)^2} = 60$

$\theta = \cos^{-1}\left[\dfrac{\vec{r}_{AO} \cdot \vec{r}_F}{(r_{AO})(r_F)}\right] = \cos^{-1}\left[\dfrac{(-20\vec{i} - 40\vec{j} - 40\vec{k})(-500\vec{k})}{(60)(500)}\right]$

r_3

$\theta = \cos^{-1}\left(\frac{40 r_z}{60 r_z}\right) = \cos^{-1}\left(\frac{2}{3}\right) = 48.19°$

$F_1 = F\cos\theta = 500\cos 48.19° = 333.33N$
$F_2 = F\sin\theta = 500\sin 48.19° = 372.67N$

2-121 $F_{AC} = 3KN$ $\qquad F_z = ?$

$$\vec{F}_{AC} = F_{AC}\left(\frac{\vec{r}_{AC}}{r_{AC}}\right) = F_{AC}\left(\frac{r_x\vec{i} + r_y\vec{j} + r_z\vec{k}}{r_{AC}}\right)$$
$$= F_x\vec{i} + F_y\vec{j} + F_z\vec{k} = F(\cos\alpha\vec{i} + \cos\beta\vec{j} + \cos\gamma\vec{k})$$

gr

$A(0, 0, 9)$
$C(3\sin 30, 3\cos 30, 0)$
$\Rightarrow C(1.5, 2.60, 0)$
$\vec{r}_{AC} = (1.5\vec{i} + 2.6\vec{j} - 9\vec{k}) \Rightarrow r_{AC} = \sqrt{(1.5)^2 + (2.6)^2 + (-9)^2}$
$= 9.48\ m$

$\vec{r} = r_z\vec{k}$
$$\cos\theta = \cos(\widehat{\vec{F}_{AC}, z}) = \left[\frac{\vec{r}_{AC}\cdot\vec{r}}{(r_{AC})(r)}\right] = \left[\frac{(1.5\vec{i} + 2.6\vec{j} - 9\vec{k})(r_z\vec{k})}{(9.48)(r_z)}\right] = \frac{-9r_z}{9.48 r_z} = \frac{-9}{9.48}$$
$$F_z = F\cos\theta = 3KN\left(\frac{-9}{9.48}\right) = -2.85\ KN$$

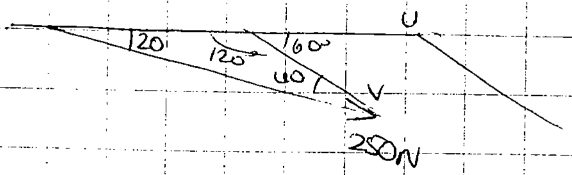
2-137 $\hat{\theta} = \widehat{BA, BC} = ?$

$A(-0.9, 0, 0)$ $\qquad C(1.8, 1.2, -0.6)$
$B(0, 0, 0)$

$\vec{r}_{BA} = -0.9\vec{i} \Rightarrow r_{BA} = 0.9$
$\vec{r}_{BC} = 1.8\vec{i} + 1.2\vec{j} - 0.6\vec{k} \Rightarrow r_{BC} = \sqrt{(1.8)^2 + (1.2)^2 + (-0.6)^2}$
$= 2.24$

$$\theta = \cos^{-1}\left[\frac{\vec{r}_{BA}\cdot\vec{r}_{BC}}{(r_{BA})(r_{BC})}\right] = \cos^{-1}\left[\frac{(-0.9\vec{i})(1.8\vec{i} + 1.2\vec{j} - 0.6\vec{k})}{(0.9)(2.24)}\right]$$
$$= \cos^{-1}\left[\frac{(-0.9)(1.8)}{(0.9)(2.24)}\right] = 143.47°$$

2-141)



$$\frac{250}{\sin 120} = \frac{U}{\sin 40} = \frac{V}{\sin 20}$$

$$\Rightarrow \begin{cases} U = \dfrac{250 \sin 40}{\sin 120} = 185.56\ N \\[2ex] V = \dfrac{250 \sin 20}{\sin 120} = 98.73 N \end{cases}$$

2-143) $F_{AB} = F_B = 400N$

$F_{AC} = F_C = 400N$

$F_{DE} = F_E = 350N$

$A(5, 0, 0)$ $C(0, -2, 3)$ $E(0, 0, 3)$

$B(0, 2, 3)$ $D(2, 0, 0)$

$$\vec{F}_{AB} = F_{AB}\left(\frac{\vec{r}_{AB}}{r_{AB}}\right) = 400\left[\frac{-5\vec{i}+2\vec{j}+3\vec{k}}{\sqrt{(-5)^2+(2)^2+(3)^2}}\right] =$$

$$\vec{F}_{AC} = F_{AC}\left(\frac{\vec{r}_{AC}}{r_{AC}}\right) = 400\left[\frac{-5\vec{i}-2\vec{j}+3\vec{k}}{\sqrt{(-5)^2+(-2)^2+(3)^2}}\right] =$$

$$\vec{F}_{DE} = F_{DE}\left(\frac{\vec{r}_{DE}}{r_{DE}}\right) = 350\left[\frac{-2\vec{i}+3\vec{k}}{\sqrt{(-2)^2+(3)^2}}\right] =$$

$\vec{F}_{AB} = (-324.68\vec{i} + 129.87\vec{j} + 194.81\vec{k})\ N$

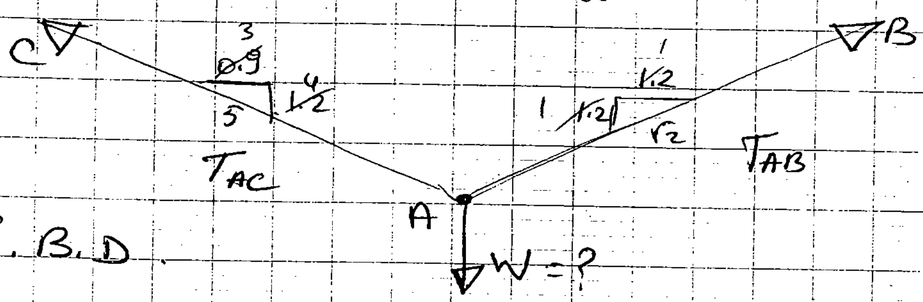
$\vec{F}_{AC} = (-324.68\vec{i} - 129.87\vec{j} + 194.81\vec{k})\ N$

$\vec{F}_{DE} = (-193.91\vec{i} + 291.22\vec{k})\ N$

3 - Equilibrium of a Particle

$$\sum_{i=1}^{n=\infty} \vec{F_i} = \vec{0} = \vec{R} \iff \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

3-9) $T_{AC_{max}} = 1500\,N$, $T_{AB_{max}} = 1250\,N$



F.B.D.

$\sum F_x = 0 = \frac{1}{\sqrt{2}} T_{AB} - \frac{3}{5} T_{AC}$ —— ①

$\sum F_y = 0 = \frac{1}{\sqrt{2}} T_{AB} + \frac{4}{5} T_{AC} - W$

Let $T_{AB} = T_{AB_{max}} = 1250\,N$ in ① $\Rightarrow T_{AC} = \frac{5}{3\sqrt{2}} T_{AB} = \frac{5}{3\sqrt{2}}(1250)$

$= 1473.14\,N < T_{AC_{max}} = 1500\,N$

OK

Let $T_{AC} = T_{AC_{max}} = 1500\,N$ in ① $\Rightarrow T_{AB} = \frac{3\sqrt{2}}{5} T_{AC} = \frac{3\sqrt{2}}{5}(1500)$

$= 1272.79\,N > T_{AB_{max}} = 1250$ N.G

if we use $T_{AB} = 1250\,N$ and $T_{AC} = 1473.14\,N$ in ② $\Rightarrow$

$W = \frac{1}{\sqrt{2}}(1250) + \frac{4}{5}(1473.14) = 2062.40\,N$

3-11) $\vec{F} = -\frac{4}{5}(8)\vec{i} - \frac{3}{5}(8)\vec{j}$

$\vec{T} = T\cos\alpha\,\vec{i} - T\sin\alpha\,\vec{j}$

$\sum F_x = 0 = -\frac{4}{5}(8) + T\cos\alpha$ —— ①

$\sum F_y = 0 = -\frac{3}{5}(8) + 9 - T\sin\alpha$ —— ②

① $\Rightarrow T\cos\alpha = \frac{4}{5}(8) = 6.4$

② $\Rightarrow T\sin\alpha = 9 - \frac{3}{5}(8) = 4.2$

$T = \sqrt{(T\cos\alpha)^2 + (T\sin\alpha)^2} = \sqrt{(6.4)^2 + (4.2)^2} = 7.7\,kN$

in ① $\Rightarrow \alpha = \cos^{-1}\left(\frac{6.4}{7.7}\right) = 33.78°$

$\theta = \alpha + \tan^{-1}\left(\frac{3}{4}\right)$

$= 33.78 + \tan^{-1}\left(\frac{3}{4}\right)$

$= 70.65°$

3-15) $M_D = ? \Rightarrow \frac{W_D}{g} = M_D$

$l_{AB} = 3m$ = unstretched

$l'_{AB} = \sqrt{(3)^2 + (4)^2} = 5\,m$

$\Delta_{AB} = l'_{AB} - l_{AB} = 5 - 3 = 2\,m$

$F_{AB} = K_{AB}\,\Delta_{AB} = \left(30\,\frac{N}{m}\right)(2\,m) = 60\,N$

$\Sigma F_x = 0 = \frac{4}{5}(60) - \frac{1}{\sqrt{2}}F_{AC} \Rightarrow F_{AC} = 48\sqrt{2}\,N$

$\Sigma F_y = 0 = \frac{3}{5}(60) + \frac{1}{\sqrt{2}}F_{AC} - W_D \Rightarrow W_D = \frac{3}{5}(60) + \frac{1}{\sqrt{2}}(48\sqrt{2})$

$= 84\,N$

$M_D = \frac{W_D}{g} = \frac{84}{9.81} = 8.56\,Kg$

3-17) $T_{CA} = T$ $\theta = ?$ $T_{CA} = ?$

$T_{CB} = 2T$ $M = 10\,kg$ $T_{CB} = ?$

$W = 10g = 98.1\,N$

$\Sigma F_x = 0 = 2T\cos\theta - T\cos 30$ --- (1)

$\Rightarrow 2\cos\theta = \cos 30$

$\cos\theta = \frac{\cos 30}{2}$

$\theta = \cos^{-1}\left(\frac{\cos 30}{2}\right) = 64.34°$

$\Sigma F_y = 0 = 2T\sin\theta + T\sin 30 - 98.1$

$\Rightarrow T = \left(\frac{98.10}{2\sin\theta + \sin 30}\right) = \frac{98.10}{2\sin 64.34 + \sin 30} = 42.60\,N$

3-19) $M_D = 20\,Kg \Rightarrow W_D = M_D \cdot g = (20)(9.81) = 196.2\,N$

$F = 100\,N \qquad d = ?$

$Hyp = \sqrt{(2)^2 + (d+1.5)^2}$
$= \sqrt{4 + d^2 + 2.25 + 3d}$
$= \sqrt{6.25 + 3d + d^2}$

EQUILIBRIUM in A $\Rightarrow$

$\Sigma F_x = 0 = 100 - \left(\frac{2}{HYP}\right) F_{AB} \Leftrightarrow \left(\frac{2}{HYP}\right) F_{AB} = 100$ ——— ①

$\Sigma F_y = 0 = \left(\frac{d+1.5}{HYP}\right) F_{AB} - 196.20 \Leftrightarrow \left(\frac{1.5+d}{HYP}\right) F_{AB} = 196.20$ —

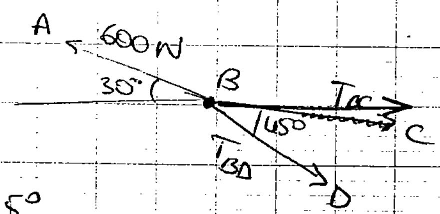
$\frac{②}{①} \Leftrightarrow \frac{1.5+d}{2} = \frac{196.20}{100}$

$150 + 100d = 2(196.20)$

$d = \frac{2(196.20) - 150}{100} = 2.42\,m$

3-21) $T_{max} = 600N$

Let $T_{AB} = 600N$



Equilibrium at B $\Rightarrow$

$\Sigma F_y = 0 = 600 \sin 30° - T_{BD} \sin 45°$

$\Rightarrow T_{BD} = \frac{600 \sin 30}{\sin 45} = 424.26\,N < 600\,N$

$\Sigma F_x = 0 = -600 \cos 30 + T_{BC} + T_{BD} \cos 45°$

$\Rightarrow T_{BC} = 600 \cos 30 - (424.26) \cos 45°$

$T_{BC} = 219.61\,N < 600\,N$

at D $\Rightarrow$

$\Sigma F_x = 0 = T_{DC} \sin 60 - 424.26 \sin 45°$

$\Rightarrow T_{DC} = \frac{424.26 \sin 45}{\sin 60} = 346.40 < 600\,N$

$\Sigma F_y = 0 = 424.26 \cos 45 + T_{DC} \cos 60 - W_D$

$\Rightarrow W_D = 424.26 \cos 45 + (346.40) \cos 60 = 473.20\,N$

$\Rightarrow M_D = \frac{W_D}{g} = \frac{473.20}{9.81} = 48.23 kg.$

3-45) $W = Mg = (100\ kg)(9.81) = 981\ N \Rightarrow \vec{W} = (-981\vec{k})N$

$A(0,0,0)$ $C(0,-2,0)$

$B(2.5,0,0)$ $D(-2,2,1)$

$$\vec{T_{AD}} = T_{AD}\left(\frac{\vec{r_{AD}}}{r_{AD}}\right) = T_{AD}\left(\frac{-2\vec{i}+2\vec{j}+\vec{k}}{\underbrace{\sqrt{(-2)^2+(2)^2+(1)^2}}_{3}}\right)$$

$$\vec{T_{AB}} = T_{AB}\left(\frac{\vec{r_{AB}}}{r_{AB}}\right) = T_{AB}\left(\frac{2.5\vec{i}}{\sqrt{(2.5)^2}}\right) = T_{AB}\vec{i}$$

$$\vec{T_{AC}} = T_{AC}\left(\frac{\vec{r_{AC}}}{r_{AC}}\right) = T_{AC}\left(\frac{-2\vec{j}}{\sqrt{(-2)^2}}\right) = T_{AC}\vec{j}$$

equilibrium at A $\Rightarrow$

$\Sigma F_x = 0 = -\frac{2}{3}T_{AD} + T_{AB}$ —— ①

$\Sigma F_y = 0 = \frac{2}{3}T_{AD} - T_{AC}$ —— ②

$\Sigma F_z = 0 = \frac{T_{AD}}{3} - 981$ —— ③

③ $\Rightarrow T_{AD} = (3)(981) = 2943\ N$

in ① $\Rightarrow T_{AB} = \frac{2}{3}T_{AD} = \frac{2}{3}(2943) = 1962\ N$

in ② $\Rightarrow T_{AC} = \frac{2}{3}T_{AD} = 1962\ N.$

3-49) $T_{max} = 2250\ N$

$\vec{W_A} = (-W\vec{k})\ N$

$$\vec{T_{AB}} = T_{AB}\left(\frac{\vec{r_{AB}}}{r_{AB}}\right) = T_{AB}\left(\frac{-0.6\vec{i}+0.3\vec{j}+0.6\vec{k}}{\sqrt{(-0.6)^2+(0.3)^2+(0.6)^2}}\right) = T_{AB}\left(\frac{-2\vec{i}+\vec{j}+2\vec{k}}{3}\right)$$

$$\vec{T_{AC}} = T_{AC}\left(\frac{\vec{r_{AC}}}{r_{AC}}\right) = T_{AC}\left(\frac{-0.6\vec{i}-0.6\vec{j}+0.3\vec{k}}{\sqrt{(-0.6)^2+(-0.6)^2+(0.3)^2}}\right) = T_{AC}\left(\frac{-2\vec{i}-2\vec{j}+\vec{k}}{3}\right)$$

$$\vec{T_{AD}} = T_{AD}\left(\frac{\vec{r_{AD}}}{r_{AD}}\right) = T_{AD}\left(\frac{0.9\vec{i}}{\sqrt{(0.9)^2}}\right) = T_{AD}\vec{i}$$

$A(0,0,0)\ m$ $D(0.9,0,0)$

$B(-0.6,0.3,0.6)\ m$

$C(-0.6,-0.6,0.3)\ m$

equilibrium at A ⇒

$\sum F_x = 0 = -\frac{2}{3}T_{AB} - \frac{2}{3}T_{AC} + T_{AD}$ ——— ①

$\sum F_y = 0 = \frac{T_{AB}}{3} - \frac{2}{3}T_{AC}$ ——— ②

$\sum F_z = 0 = \frac{2}{3}T_{AB} + \frac{T_{AC}}{3} - W$ ——— ③

① ⇒ $T_{AD} = \frac{2}{3}(T_{AB} + T_{AC})$ let $T_{AD} = 2250\,N$

⇒ $T_{AB} + T_{AC} = \frac{3}{2}(2250) = 3375\,N$

(−) ② ⇒ $T_{AB} - 2T_{AC} = 0$

$3T_{AC} = 3375$

$T_{AC} = 1125\,N < T_{max}$

② ⇒ $T_{AB} = 2T_{AC} = 2(1125) = 2250\,N = T_{max}$

③ ⇒ $W = \frac{2}{3}T_{AB} + \frac{1}{3}T_{AC}$

$= \frac{2}{3}(2250) + \frac{1}{3}(1125)$

$W = 1875\,N$

3.55) $M = 50\,kg \Rightarrow W = Mg = 50(9.81) = 490.5\,N \Rightarrow \vec{W} = (-490.5$

$x = 2\,m$, $z = 1.5\,m$

$A(0, 6, 0)$, $B(0, 6+y, 0)$

$C(2, 0, 3)$, $D(-2, 0, 1.5)$

$\vec{T}_{AB} = T_{AB}\left(\frac{\vec{r}_{AB}}{r_{AB}}\right) = T_{AB}\,\vec{j}$

$\vec{T}_{AC} = T_{AC}\left(\frac{\vec{r}_{AC}}{r_{AC}}\right) = T_{AC}\left(\frac{2\vec{i} - 6\vec{j} + 3\vec{k}}{\sqrt{(2)^2 + (-6)^2 + (3)^2}}\right)$

$\vec{T}_{AD} = T_{AD}\left(\frac{\vec{r}_{AD}}{r_{AD}}\right) = T_{AD}\left(\frac{-2\vec{i} - 6\vec{j} + 1.5\vec{k}}{\sqrt{(-2)^2 + (-6)^2 + (1.5)^2}}\right)$

6.5

Equilibrium at A :

$\sum F_x = 0 = \frac{2}{7}T_{AC} - \frac{2}{6.5}T_{AD}$ ——— ①

$\sum F_y = 0 = T_{AB} - \frac{6}{7}T_{AC} - \frac{6}{6.5}T_{AD}$ ——— ②

$\sum F_z = 0 = \frac{3}{7}T_{AC} + \frac{1.5}{6.5}T_{AD} - 490.5$ ——— ③

$3\times ① + 4 \times ③ \Rightarrow \frac{18}{7} T_{AC} = 4(490.5)$

$T_{AC} = \frac{(4)(490.5)(7)}{18} = 763\ N$

$① \Rightarrow T_{AD} = \frac{6.5}{7} T_{AC} = \frac{6.5}{7}(763) = 708.5\ N$

$② \Rightarrow T_{AB} = \frac{6}{7}(763) + \frac{6}{6.5}(708.5) = 1308\ N$

3_57) $T_{max} = 15\ KN$

$W = Mg \Rightarrow \vec{W} = (-W\vec{K})\ KN$

$A(0,0,12)$

$B(4,-6,0)$

$C(-6,-4,0)$

$D(-4,6,0)$

$$\vec{T}_{AB} = T_{AB}\left(\frac{4\vec{i}-6\vec{j}-12\vec{K}}{\sqrt{(4)^2+(-6)^2+(-12)^2}}\right) = T_{AB}\left(\frac{2\vec{i}-3\vec{j}-6\vec{K}}{7}\right)$$

&

$$\vec{T}_{AC} = T_{AC}\left(\frac{-6\vec{i}-4\vec{j}-12\vec{K}}{\sqrt{(-6)^2+(-4)^2+(-12)^2}}\right) = T_{AC}\left(\frac{-3\vec{i}-2\vec{j}-6\vec{K}}{7}\right)$$

$$\vec{T}_{AD} = T_{AD}\left(\frac{-4\vec{i}+6\vec{j}-12\vec{K}}{\sqrt{(-4)^2+(6)^2+(-12)^2}}\right) = T_{AD}\left(\frac{-2\vec{i}+3\vec{j}-6\vec{K}}{7}\right)$$

$\vec{F}_A = (W\vec{K})\ KN$

Equilibrium at A

$\Sigma F_x = 0 = \frac{2}{7}T_{AB} - \frac{3}{7}T_{AC} - \frac{2}{7}T_{AD}$ — ①

$\Sigma F_y = 0 = -\frac{3}{7}T_{AB} - \frac{2}{7}T_{AC} + \frac{3}{7}T_{AD}$ — ②

$\Sigma F_z = 0 = -\frac{6}{7}T_{AB} - \frac{6}{7}T_{AC} - \frac{6}{7}T_{AC} + W$ — ③

$$\Leftrightarrow \begin{cases} 2T_{AB} - 3T_{AC} - 2T_{AD} = 0 & ① \\ -3T_{AB} - 2T_{AC} + 3T_{AD} = 0 & ② \\ T_{AB} + T_{AC} + T_{AC} = \frac{7}{6}W & ③ \end{cases}$$

Let $T_{AB} = 15\ KN \Rightarrow (3T_{AC} + 2T_{AD} = 30) \times 2$

$(-2T_{AC} + 3T_{AD} = 45) \times 3$

$13\,T_{AD} = 2(30) + 3(45)$

$\Rightarrow T_{AD} = \cancel{27.86\ N > 15}$ 15 KN OK

$\Rightarrow T_{AC} = \frac{45-45}{2} = 0 < 15$ OK

③ $\Rightarrow W = \frac{6}{7}(15) + \frac{6}{7}(15) = \frac{12}{7}(15) = 25.71\ KN.$

$$M = \frac{25.71 \times 1000}{9.81} = 2620.79\ kg.$$

Hw: 3-60 / 3-50 / 3-30 / 3-24 / 3-6

3-63 $T_{AB_{max}} = T_{AC_{max}} = 2500\ N.$

$z_{max} = ?$ $\quad M = 100\ kg \Rightarrow W = Mg = (100)(9.81) = 981\ N$

$F = ?$ $\quad y = 2.4\ m \quad \vec{W}_A = (-981\vec{k})\ N$

$$\vec{T}_{AB} = T_{AB}\left(\frac{-1.5\vec{i} - 2.4\vec{j} + (1.2-z)\vec{k}}{\sqrt{(-1.5)^2 + (-2.4)^2 + (1.2-z)^2}}\right)$$

$$\vec{T}_{AC} = T_{AC}\left(\frac{1.5\vec{i} - 2.4\vec{j} + (1.2-z)\vec{k}}{\sqrt{(1.5)^2 + (-2.4)^2 + (1.2-z)^2}}\right)$$

$\vec{F}_A = (F\vec{j})\ N$

$A(0, 2.4, z)$

$B(-1.5, 0, 1.2)$

$C(1.5, 0, 1.2)$

$$H = \sqrt{(1.5)^2 + (2.4)^2 + (1.2-z)^2}$$
$$= \sqrt{2.25 + 5.76 + (1.2)^2 - 2.4z + z^2}$$
$$= \sqrt{9.45 - 2.4z + z^2}$$

Equilibrium at A:

$\Sigma F_x = 0 = -\frac{1.5}{H} T_{AB} + \frac{1.5}{H} T_{AC} \Rightarrow T_{AB} = T_{AC} = T$ ①

$\Sigma F_y = 0 = -\frac{2.4}{H} T - \frac{2.4}{H} T + F \Rightarrow F = \frac{4.8T}{H}$ ②

$\Sigma F_z = 0 = \left(\frac{1.2-z}{H}\right) T + \left(\frac{1.2-z}{H}\right) T - 981 \Rightarrow$ ③

$$\left(\frac{2.4-2z}{H}\right) T = 9.81$$

Let $T = 2500N \Rightarrow$ ③: $\frac{2.4-2z}{H} \times 2500 = 9.81$

$$(2.4-2z)(2500) = 981\left(\sqrt{9.45-2.4z+z^2}\right)$$

$$\Leftrightarrow \left[(2.4)^2 - 9.6z + 4z^2\right]\left(\frac{2500}{981}\right)^2 = 9.45 - 2.4z + z^2$$

$$\Leftrightarrow -27.94 + 59.95z - 24.98z^2 = 0$$

$$\Leftrightarrow 24.98z^2 - 59.95z + 27.94 = 0$$

$$\Rightarrow z = 1.77\,m, \quad z = 0.63\,m.$$

use $z_{(max)} = 1.77\,m$

in ② $\Rightarrow F = \dfrac{(4.8)(2500)}{\sqrt{9.45 - 2.4(1.77) + (1.77)^2}} = 4156.53\,N$

3_75) $P = ?$ $\quad \alpha, \beta, \gamma = ?$ For F_3

$F_3 = 200\,kN$

$$\vec{F_1} = 360\left(\frac{-\vec{i} - 7\vec{j} + 4\vec{k}}{\sqrt{(-1)^2+(-7)^2+(4)^2}}\right) kN$$

$$= (-44.31)\vec{i} - (310.19)\vec{j} + (177.34)\vec{k} \quad kN$$

$\vec{F_2} = (-120\vec{j})\,kN$

$\vec{F_4} = (-300\vec{k})\,kN$

$\vec{P} = P(\cos 20^\circ \vec{j} + \sin 20^\circ \vec{k})$

Let $\vec{F_3} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$

$= F(\cos\alpha\vec{i} + \cos\beta\vec{j} + \cos\gamma\vec{k})$

Equilibrium :

$\Sigma F_x = 0 = -44.31 + F_x \Rightarrow F_x = 44.31\,kN$ ①

$\Sigma F_y = 0 = -310.19 - 120 + P\cos 20 + F_y$

$\Rightarrow F_y = 430.19 - P\cos 20$ ②

$\Sigma F_z = 0 = 177.34 - 300 + P\sin 20^\circ + F_z$

$\Rightarrow F_z = +122.66 - P\sin 20$ ③

BUT $(F_3)^2 = (200)^2 = F_x^2 + F_y^2 + F_z^2$

$\Rightarrow F_y^2 + F_z^2 = \cancel{(44.31)^2}\ 200^2 - (44.31)^2 = 38\,036.62$ ④

② and ③ in ④ $\Rightarrow (430.19 - P\cos 20)^2 + (122.66 - P\sin 20)^2 = 38036.6$

$\Leftrightarrow (430.19)^2 - 2(430.19)P\cos 20 + P^2\cos^2 20 + (122.66)^2 - 2(122.66)P\sin 2$

$+ P^2\sin^2 20 = 38036.62$

$\Leftrightarrow P^2 - 892.40\,P + 162072.29 = 0$

$\Rightarrow P = 638.61,\ 253.79$ KN.

	P(KN) 638.61	253.79
$F_y = 430.19 - P\cos 20$	-169.91	191.71
$F_z = 122.66 - P\sin 20$	-95.76	35.86
	✓	X

$P = 638.61$ KN

$\alpha = \cos^{-1}\left(\frac{F_x}{F_3}\right) = \cos^{-1}\left(\frac{44.31}{200}\right) = 77.20°$

$\beta = \cos^{-1}\left(\frac{F_y}{F_3}\right) = \cos^{-1}\left(\frac{-169.91}{200}\right) = 148.16°$

$\gamma = \cos^{-1}\left(\frac{F_z}{F_3}\right) = \cos^{-1}\left(\frac{-95.76}{200}\right) = \cancel{103.85°}\ 118.44°$.

3-77) $F_1,\ F_2,\ F_3 = ?$

$\vec{F_1} = F_1(\cos 60°\,\vec{i} + \cos 135\,\vec{j} + \cos 60\,\vec{k})$ N

$\vec{F_2} = (F_2\,\vec{i})$ N

$\vec{F_3} = (-F_3\,\vec{j})$ N

$\vec{Q} = (-200\vec{k})$ N

$\vec{R} = 800\left(-\frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}\right) = (-480\vec{i} + 640\vec{j})$ N

Equilibrium at P:

$\Sigma F_x = 0 = F_1\cos 60° + F_2 - 480$ —— ①

$\Sigma F_y = 0 = F_1\cos 135 - F_3 + 640$ —— ②

$\Sigma F_z = 0 = F_1\cos 60 - 200$ —— ③

③ $\Rightarrow F_1 = \frac{200}{\cos 60} = 400$ N

The above in ① $\Rightarrow F_2 = 480 - 400\cos 60 = 280$ N

Also in ② $\Rightarrow F_3 = 400\cos 135 + 640 = 357.16\,N$

3_79)

$\vec{F}_{OA} = \vec{F}_1 = F_1(-\sin 45\,\vec{i} + \cos 45\,\vec{j})$ N

$\vec{F}_{BO} = \vec{F}_2 = F_2(-\cos 40°\,\vec{j} + \sin 40°\,\vec{k})$ N

$\vec{F}_3 = (F_3\,\vec{j})$ N

$\vec{F}_4 = (-200\,\vec{k})$ N.

Equilibrium.

$\sum F_x = 0 = -F_1 \sin 45 \Rightarrow F_1 = 0$

$\sum F_y = 0 = \cancel{F_1 \cos 45} - F_2 \cos 40 + F_3$ ②

$\sum F_z = 0 = F_2 \sin 40 - 200 \Rightarrow F_2 = \frac{200}{\sin 40} = 311.14\,N$

② $\Rightarrow F_3 = 311.14\cos 40 = 238.34\,N$.

4- Force System Resultants.

A Moment = (Force) x (Distance).

Distance has to be ⊥ to the line of action of force.

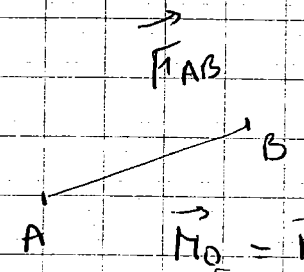
3-D : C.C.W = Positive.

Cross product :

$$\vec{P} \times \vec{Q} = (P)(Q)\sin(\widehat{P,Q})$$

$\vec{i} \times \vec{i} = 0$	$\vec{j} \times \vec{i} = -\vec{k}$	$\vec{k} \times \vec{i} = \vec{j}$
$\vec{i} \times \vec{j} = \vec{k}$	$\vec{j} \times \vec{j} = 0$	$\vec{k} \times \vec{j} = -\vec{i}$
$\vec{i} \times \vec{k} = -\vec{j}$	$\vec{j} \times \vec{k} = \vec{i}$	$\vec{k} \times \vec{k} = 0$

In general :



$$\vec{M}_O = \vec{r}_{OA} \times \vec{F}_{AB}$$

O^{*}

$$\vec{M}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (r_y F_z - r_z F_y)\vec{i} + (-r_x F_z + r_z F_x)\vec{j} + (r_x F_y - r_y F_x)\vec{k}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$= (r_x \vec{i} + r_y \vec{j} + r_z \vec{k}) \times (F_x \vec{i} + F_y \vec{j} + F_z \vec{k})$$

$$= (r_x)(F_x)(\overset{0}{\cancel{\vec{i} \times \vec{i}}}) + (r_x)(F_y)(\overset{\vec{k}}{\cancel{\vec{i} \times \vec{j}}}) + (r_x)(F_z)(\overset{-\vec{j}}{\cancel{\vec{i} \times \vec{k}}})$$

$$+ (r_y)(F_x)(\overset{-\vec{k}}{\cancel{\vec{j} \times \vec{i}}}) + (r_y)(F_y)(\overset{0}{\cancel{\vec{j} \times \vec{j}}}) + (r_y)(F_z)(\overset{\vec{i}}{\cancel{\vec{j} \times \vec{k}}})$$

$$+ (r_z)(F_x)(\underset{\vec{j}}{\cancel{\vec{k} \times \vec{i}}}) + (r_z)(F_y)(\underset{-\vec{i}}{\cancel{\vec{k} \times \vec{j}}}) + (r_z)(F_z)(\underset{0}{\cancel{\vec{k} \times \vec{k}}})$$

$$\vec{M} = (r_y F_z - r_z F_y)\vec{i} + (r_z F_x - r_x F_z)\vec{j} + (r_x F_y - r_y F_x)\vec{k}.$$

4-5) $\sum M_O \curvearrowright = 0 = \left(\frac{4}{5} F\right)(3.6) - (150 \cos 45)(3.6+1.8)$

$\Rightarrow F = \dfrac{(150 \cos 45)(3.6+1.8)}{\left(\frac{4}{5}\right)(3.6)} = 198.87\,N.$

4-7) $F = 4\,KN$ $\quad M_A \curvearrowright = 10\,KN.m$

$\theta = ?$ $\quad 0 \le \theta \le 90°$

$\sum M_A \curvearrowright = 10\,kN.m = (4 \sin\theta)(3) - (4\cos\theta)(0.45)$

$\Rightarrow 12 \sin\theta - 1.8 \cos\theta = 10$

Let : $x = \cos\theta$ and $\sin\theta = \sqrt{1-x^2}$

$\Leftrightarrow \left(12\sqrt{1-x^2}\right)^2 = (10+1.8x)^2$

$\Leftrightarrow 144(1-x^2) = 100 + 36x + 3.24x^2$

$\Leftrightarrow 147.24x^2 + 36x \cancel{+100}^{-44} = 0$

$\Leftrightarrow x = 0.44, -0.68 = \cos\theta$ $\quad$ BUT $0 \le \theta \le 90°$

$\Rightarrow 0 \le \cos\theta \le 1$

use $x = 0.44 = \cos\theta$.

$\Rightarrow \theta = \cos^{-1}(0.44) = 63.90°$

4-11) $F = 6\,KN$

$\theta = 45°$ $\quad M_A = ?$ $\quad$ C.W

$\vec{F} = 6\cos 45\,\vec{i} - 6 \sin 45\,\vec{j}$.

$M_A \curvearrowright = (6\cos 45)(6) + (6\sin 45)(3) = 38.18\,KN.m$

4-15) $F_t = 650\,N$ $\quad \vec{F_t} = F_t(\cos 5\,\vec{i} + \sin 5\,\vec{j})$

$N_f = 400\,N$ $\quad M_A = ?$

$M_A \curvearrowright = (650 \cos 5°)(65\,mm) - (400)(100\,mm)$

$= 2.09\ N.mm$

4-37) $\vec{F_1} = \vec{F_A} = (-20\vec{i} + 10\vec{j} + 30\vec{K})\ N$

$\vec{M_o} = \vec{r_{OA}} \times \vec{F_A}$

O(0, 0, 0) m
A(0.6, 0.6, -0.4) m

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{K} \\ 0.6 & 0.6 & -0.4 \\ -20 & 10 & 30 \end{vmatrix}$$

$$= [(0.6)(30) - (-0.4)(10)]\vec{i} - [(0.6)(30) - (-0.4)(-20)]\vec{j} + [(0.6)(10) - (0.6)(-20)]\vec{K}$$

$$= (22\vec{i} - 10\vec{j} + 18\vec{K})\ N.m$$

4-41) $F_C = 420 N = F_{AC}$

$\vec{M_o} = \vec{r_{OA}} \times \vec{F_{AC}}$

O(0, 0, 0) m
A(0, 0, 6) m
C(2, -3, 0) m

$\vec{r_{OA}} = (6\vec{K})\ m$

$$\vec{F_{AC}} = F_{AC}\left(\frac{\vec{r_{AC}}}{r_{AC}}\right) = 420\left[\frac{2\vec{i} - 3\vec{j} - 6\vec{K}}{\underbrace{\sqrt{(2)^2 + (-3)^2 + (-6)^2}}_{7}}\right]$$

$$= (120\vec{i} - 180\vec{j} - 360\vec{K})\ N$$

k j i

$$\vec{M_o} = [(6\vec{K})\ m] \times [(120\vec{i} - 180\vec{j} - 360\vec{K})\ N]$$

$$= [(6)(120)\vec{j} - (-180)(6)\vec{i}]\ N.m$$

$$\vec{M_o} = [1080\vec{i} + 720\vec{j}]\ N.m$$

4-43) $\vec{M_o} = ?$

$\vec{F_A} = (-40\vec{i} - 100\vec{j} - 60\vec{K})\ N \Rightarrow F_A = 123.29\ N$

$\vec{F_B} = (-50\vec{i} - 120\vec{j} + 60\vec{K})\ N \Rightarrow F_B = 143.17\ N$

O(0, 0, 0) mm
A(150, 300, 0) mm
B(150, 600, -150) mm

$$\vec{M}_O = \vec{r}_{OA} \times \vec{F}_A$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 150 & 300 & 0 \\ -40 & -100 & -60 \end{vmatrix}$$

$$= 300(-60)\vec{i} - (150)(-60)\vec{j} + [150(-100) - 300(-40)]\vec{k}$$

$$= -18000\vec{i} + 9000\vec{j} - 3000\vec{k}$$

$$= (-18\vec{i} + 9\vec{j} - 3\vec{k})\ N.m$$

$$\vec{M}_O = \vec{r}_{OB} \times \vec{F}_B$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ & & \\ -50 & -120 & 60 \end{vmatrix}$$

$$= (18\vec{i} + 7.5\vec{j} + 30\vec{k})\ N.m$$

<u>Projection</u>: Moment about an axis

$\vec{M}_C$ about any point on the axis (AB)
$(C) \subset [A, B]$

$\vec{U}_{AB}$ = unit vector of $[AB] = \left(\dfrac{\vec{r}_{AB}}{r_{AB}}\right)$

$M_{AB} = \vec{U}_{AB} \cdot \vec{M}_C$

Moment about the X-Axis $\Rightarrow \vec{u} = \vec{i}$

$$\vec{M} \cdot \vec{i} = \vec{i} \cdot \vec{M} = (M_x\vec{i} + M_y\vec{j} + M_z\vec{k}) \cdot (\vec{i})$$

$$= M_x$$

4-51) $M_{AF} = ?$

$$M_{AF} = \vec{U}_{AF} \cdot \vec{M}_A = \vec{U}_{AF} \cdot (\vec{r}_{AB} \times \vec{F}_B)$$

$A(0, 0, 1.5)$ m
$B(0, 3, 1.5)$ m
$F(3, 3, 0)$ m

$$\vec{u}_{AF} = \frac{\vec{r}_{AF}}{r_{AF}} = \frac{3\vec{i} + 3\vec{j} - 1.5\vec{k}}{\sqrt{(3)^2 + (3)^2 + (-1.5)^2}}$$

$$\vec{u}_{AF} = \frac{2\vec{i} + 2\vec{j} - \vec{k}}{3}$$

$\vec{r}_{AB} = (3\vec{j})$ m

$$M_{AF} = \begin{vmatrix} 2/3 & 2/3 & -1/3 \\ 0 & 3 & 0 \\ -6 & 3 & 10 \end{vmatrix} = \left(\tfrac{2}{3}\right)(3)(10) - \left(-\tfrac{1}{3}\right)(3)(-6)$$

$$= 20 - 6 = 14\ N.m$$

Hw: 4-22 / 4-30 / 4-46 / 4-54 / 4-68.

4-53) $M = ? \Rightarrow \vec{U} = \vec{K}$

$\vec{M_O} = \vec{r_{OA}} \times \vec{F_A}$

$M_z = \vec{U_z} \cdot \vec{M_o} = \vec{U_z} \cdot (\vec{r_{OA}} \times \vec{F_A})$

$\vec{F_A} = (-60\vec{i} + 20\vec{j} + 15\vec{k})\, N$

$A(0.25 \sin 30, 0.25 \cos 30, 0.4)\, m \Leftrightarrow (0.125, 0.217, 0.4)\, m$

$$M_z = \begin{vmatrix} 0 & 0 & 1 \\ 0.125 & 0.217 & 0.4 \\ -60 & 20 & 15 \end{vmatrix} = (0.125)(1)(20) - (1)(0.217)(-60)$$
$$= 15.52\ N.m.$$

4-55) $M_{AC} = ?$ Due to $\vec{F} = \vec{F_B} = (4\vec{i} + 12\vec{j} - 3\vec{K})$ KN

$\vec{M_{AC}} = ?$

$\vec{M_{AC}} = \vec{U_{AC}} \cdot M_{AC}$

$= \vec{U_{AC}} \cdot [\vec{U_{AC}} \cdot \vec{M_A}]$

$= \vec{U_{AC}} \cdot [\vec{U_{AC}} \cdot (\vec{r_{AB}} \times \vec{F_B})]$

A(0, 0, 0)

B(4, 3, -2) m

C(4, 3, 0) m

$$\vec{U_{AC}} = \frac{\vec{r_{AC}}}{r_{AC}} = \frac{4\vec{i} + 3\vec{j}}{\underbrace{\sqrt{(4)^2 + (3)^2}}_{5}}$$

$$M_{AC} = \begin{vmatrix} 4/5 & 3/5 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

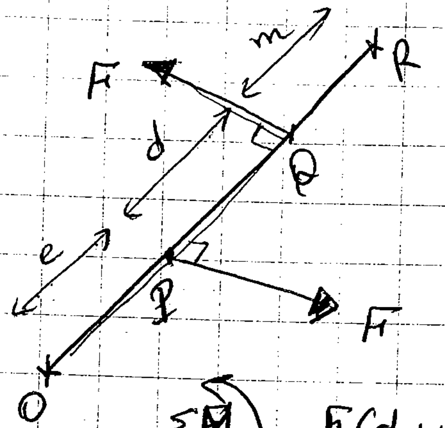
$$= \frac{4}{5}(-9 + 24) - \frac{3}{5}(-12 + 8)$$

$$= 14.4\ KN.m$$

$$\vec{M_{AC}} = M_{AC} \cdot \vec{U_{AC}} = (14.4)\left(\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}\right)$$
$$= (11.52\vec{i} + 8.64\vec{j})\ KN.m$$

Moment Due to a couple

Two opposite forces make up a couple i.e forces are equal in magnitude opposite in direction.



$\sum M_O = F(d+e) - F(e)$
$= Fd$

$\sum M_P = Fd$

$\sum M_Q = Fd$

$\sum M_R = F(d+m) - (F)(m) = F d$.

A moment due to a couple is constant regardless of where you are summing moment.

Usually: $M_c = F.d$

where d is distance ⊥ between the two forces.

4.75] $F = 2000N$

Old concepts

$M_C = \sum M_A = (1500 \cos 30 (0.6)) + (1500 \sin 30)(0.4) - (\frac{4}{5})(2000)(0.2)$
$-(\frac{3}{5})(2000)(0.6) + (\frac{3}{5})(2000)(0.4) - (1500 \cos 30)(0.2)$
$= 259.62$ N.m

New concept

$M_C = M_{c_1} + M_{c_2}$
$= (1500 \sin 30)(0.4) + (1500 \cos 30)(0.4)$
$- (\frac{4}{5})(2000)(0.2) - (\frac{3}{5})(2000)(0.2)$
$= 259.62$ N.m

4-81) $\overset{\curvearrowright}{M_C} = 15\ N.m$

$$\overset{\curvearrowright}{M_C} = M_{C_1} + M_{C_2} = (100 \sin 30)(300) + (100 \cos 30)(300) - (P \sin 15)(300) - (P \cos 15)(300)$$
$$= 15\ N.m \left(\frac{\times 10^3\ mm}{m}\right)$$

$$\Rightarrow P(\sin 15 + \cos 15) = 100 \sin 30 + 100 \cos 30 - \frac{15000}{300}$$
$$\Rightarrow P = \frac{100 \sin 30 + 100 \cos 30 - 50}{\sin 15 + \cos 15} = 70.71\ N$$

4-85) $M_C = ?$ — ΣM_O / ΣM_A

$$\overset{\curvearrowright}{M_C} = \Sigma \overset{\curvearrowright}{M_O} = (8 \cos 45 + 2 \cos 30)(1.5 + 1.8) - (8 \cos 45 + 2 \cos 30)(1.8) - (8 \sin 45 - 2 \sin 30)(0.3)$$
$$= 9.69\ KN.m$$

$$\overset{\curvearrowright}{M_C} = \Sigma \overset{\curvearrowright}{M_A} = (8 \cos 45 + 2 \cos 30)(1.5) - (8 \sin 45 - 2 \sin 30)(0.3)$$
$$= 9.69\ KN.m$$

* couple concept

$$\overset{\curvearrowright}{M_C} = (8 \cos 45 + 2 \cos 30)(1.5) - (8 \sin 45)(0.3) + (2 \sin 30)(0.3)$$
$$= 9.69\ KN.m.$$

4-93) $F = 80\ N$

O(0, 0, 0)

A(200, 300, 0)

B(300, 800, 0)

$$\vec{M_C} = \vec{M_O} = \vec{r}_{OA} \times \vec{F_A} + \vec{r}_{OB} \times \vec{F_B}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 200 & 300 & 0 \\ 0 & 0 & -80 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 300 & 800 & 0 \\ 0 & 0 & 80 \end{vmatrix}$$

$$= (40\vec{i} - 8\vec{j})\ N.mm$$

4-101 $M_1 = ?$, $M_2 = ?$, $M_3 = ?$

$\Sigma \vec{M}_c = \vec{0}$

$A(0.6, -0.6, 0.3)$ m

$\vec{M}_1 = M_1 \vec{j}$

$\vec{M}_2 = -M_2 \vec{i}$

$\vec{M}_3 = M_3 \left(\frac{\vec{r}_{OA}}{r_{OA}}\right)$

$M_4 = 225$ N.m

$$\begin{cases} M_z = -225 \sin 45 \\ M_{xz} = 225 \cos 45 \\ M_x = M_{xy} \sin 45 = 225 \cos 45 \sin 45 \\ M_y = -M_{xy} \cos 45 = -225 \cos 45 \cos 45 \end{cases}$$

$\Rightarrow \vec{M}_4 = \left((225 \cos 45 \sin 45)\vec{i} - 225 \cos 45 \cos 45 \vec{j} - 225 \sin 45 \vec{k}\right)$

$= (112.5\vec{i} - 112.5\vec{j} - 159.10\vec{k})$ N.m

$\vec{M}_3 = M_3 \left(\frac{0.6\vec{i} - 0.6\vec{j} + 0.3\vec{k}}{\sqrt{(0.6)^2 + (-0.6)^2 + (0.3)^2}}\right) = M_3\left(\frac{2}{3}\vec{i} - \frac{2}{3}\vec{j} + \frac{1}{3}\vec{k}\right)$

$\Sigma \vec{M}_c = \vec{0} \Rightarrow$

$\Sigma M_x = 0 = -M_2 + \frac{2}{3}M_3 + 112.5$ — (1)

$\Sigma M_y = 0 = M_1 - \frac{2}{3}M_3 - 112.5$ — (2)

$\Sigma M_z = 0 = \frac{1}{3}M_3 - 159.10$ — (3)

(3) $\Rightarrow M_3 = 3(159.10) = \cancel{377}\ 477.30$ N.m

in (1) $\Rightarrow M_2 = \frac{2}{3}(477.30) + 112.5 = 430.70$ N.m

in (2) $\Rightarrow M_1 = \frac{2}{3}(477.30) + 112.5 = 430.70$ N.m

4-103 $\Sigma \vec{M}_c = \vec{0} = \Sigma \vec{M}_O$

$= \vec{r}_{OA} \times \vec{F}_A + \vec{r}_{OB} \times \vec{F}_B + \vec{r}_{OC} \times \vec{F}_C + \vec{r}_{OD} \times \vec{F}_D$

$$\left(\begin{aligned} \vec{F}_A : F_x &= \tfrac{3}{5}(1250) = 750\text{ N} \\ F_y &= -\tfrac{4}{5}(1250) = -1000\text{ N} \end{aligned}\right)$$

$O(0,0,0)$ m

$A(0.9, 0, 0)$ m

$B(0.9, 1.2, 0)$ m

$C(0, 1.2, 0)$ m

$D(0, 1.2, 0.6)$ m

$\vec{F}_A = (750\vec{i} - 1000\vec{j})$ N

$\vec{F}_B = (F_1 \vec{j})$

$\vec{F}_C = (-F_2\vec{i} - F_1\vec{j})$

$\vec{F}_D = \left[(F_2 - 750)\vec{i} + 1000\vec{j}\right]$ N

$$\Sigma \vec{M}_C = \Sigma \vec{M}_O$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.9 & 0 & 0 \\ 750 & -1000 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.9 & 1.2 & 0 \\ 0 & F_1 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1.2 & 0 \\ -F_2 & -F_1 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1.2 & 0.6 \\ (F_2-750) & 1000 & 0 \end{vmatrix}$$

$$= -900\vec{k} + 0.9F_1\vec{k} + (-1.2F_2)\vec{k} + 600\vec{i} + 0.6(F_2-750)\vec{j}$$
$$-1.2(F_2-750)\vec{k}$$
$$= 600\vec{i}$$
$$\neq 0$$

ERROR.

4-107) $F = 100N$

$F_x = \frac{4}{5}(100) = 80$

$F_y = \frac{3}{5}(100) = 60$

$R_y = 100\cos 30 + 60 = \Sigma F_y = 146.60N$

$R_x = 80 - 100\sin 30 = \Sigma F_x = 30N$

$M_y = R_y(50 + 150\cos 40 + 35) = (146.60)(199.90)$
$= 29306.83 \text{ N.mm}$
$= 29.31 \text{ N.m}$

$M_x = R_x(150\sin 40) = 2892.54 \text{ N.mm}$
$= 2.89 \text{ N.m}$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(30)^2 + (146.60)^2} = 149.64 N$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{146.60}{30}\right) = 78.43°$

$M = 29.31 - 2.89 = 26.42 \text{ N.m}$

4-108)

$\left(\frac{3}{5}(250), \frac{4}{5}(250)\right) = (150, 200)$

$M_x = (200)(3)$ $M_y = (150)(0.5)$

$(500\cos 30,\ 500\sin 30)$

$\overset{\curvearrowleft}{M_x} = (500\cos 30)(2)$ $\qquad\qquad$ $\overset{\curvearrowright}{M_y} = (500\sin 30)(0.2)$

300:

$\overset{\curvearrowleft}{M_x} = (300)(1)$

$R_x = \Sigma F_x = 200 - 300 - 500\cos 30 = -533.01\,N$

$R_y = \Sigma F_y = -150 + 500\sin 30 = 100\,N$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-533.01)^2 + 100^2} = 542.31\,N$

$\theta = \tan^{-1}\left(\frac{100}{-533.01}\right) = 10.63°$

$\Sigma \overset{\curvearrowright}{M_C} = (500\sin 30)(0.2) + (150)(0.5) + (200)(3) - (300)(1)$
$\quad - (500\cos 30)(2)$
$\quad = -441.03\,N.m$

	Forces	Moments
4-111	200 N	$M = 0$
	750 N	$M_y = (750)(1.25)$
	200 N	$\overset{\curvearrowright}{M_x} = (200)(1)$
	$\frac{4}{5}(500) = 400$	$\overset{\curvearrowleft}{M_y} = (400)(2.5)$
	$\frac{3}{5}(500) = 300$	$\overset{\curvearrowright}{M_x} = (300)(1)$

$R_x = \Sigma F_x = 300 + 200 - 200 = 300\,N$

$R_y = \Sigma F_y = 400 - 750 = -350\,N$

$\Sigma \overset{\curvearrowright}{M} = (200)(1) - (400)(2.5) + (300)(1) + (750)(1.25)$
$\quad = 437.5\ N.m$

$R = \sqrt{(300)^2 + (350)^2} = 460.98\,N$

$\theta = \tan^{-1}\left(\frac{350}{300}\right) = 49.40°$

Hw: 4-114 / 4-112 / 4-88 / 4-86 / 4-74

4-113

$$\vec{M}_C = \vec{M}_O = \vec{r}_{OA} \times \vec{F}_{AB} + \vec{r}_{OC} \times \vec{F}_{CD} \quad , \quad F_D = 7\,KN,\; F_B = 5\,KN$$

$$\begin{cases} \vec{F}_{AB} = \vec{F}_B = F_B\left(\dfrac{\vec{r}_{AB}}{r_{AB}}\right) = (5\,KN)\left[\dfrac{6\vec{j} - 8\vec{k}}{\underbrace{\sqrt{(6)^2+(-8)^2}}_{10}}\right] = (3\vec{j} - 4\vec{k})\ KN \\ \vec{F}_{CD} = \vec{F}_D = F_{CD}\left(\dfrac{\vec{r}_{CD}}{r_{CD}}\right) = (7\,KN)\left[\dfrac{2\vec{i} - 3\vec{j} - 6\vec{k}}{\sqrt{(2)^2+(-3)^2+(-6)^2}}\right] = (2\vec{i} - 3\vec{j} - 6\vec{k})\ KN \end{cases}$$

O(0, 0, 0)
A(0, 0, 8) m
B(0, 6, 0) m
C(0, 0, 6) m
D(2, −3, 0) m

$$\Rightarrow \vec{R} = \Sigma\vec{F} = (2\vec{i} - 10\vec{k})\ KN$$

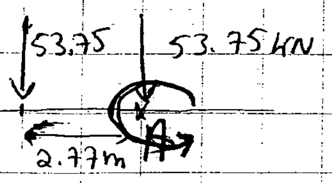
$$\vec{M}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 8 \\ 0 & 3 & -4 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 6 \\ 2 & -3 & -6 \end{vmatrix}$$

$$= -(3)(8)\vec{i} + (+3)(6)\vec{i} + (2)(6)\vec{j}$$
$$= (-6\vec{i} + 12\vec{j})\ KN$$

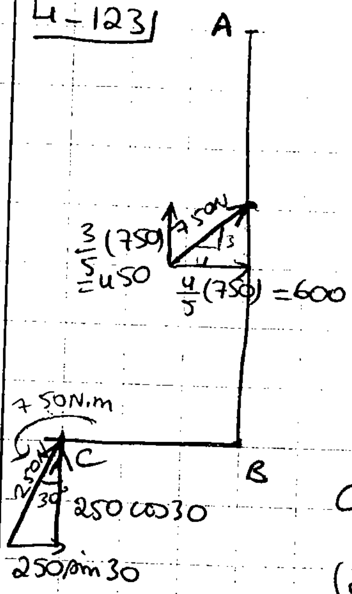
4-119

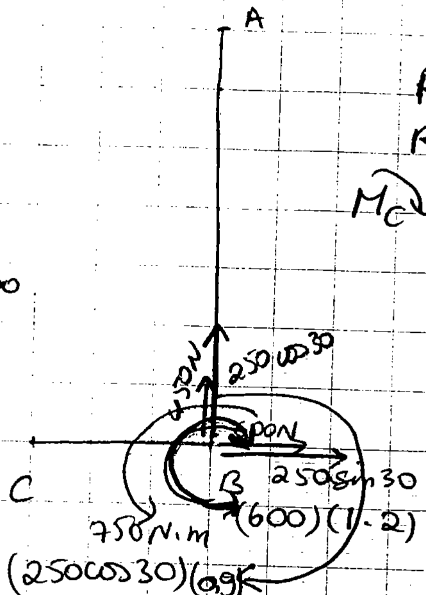
17.5 KN	$M\circlearrowright = (17.5)(4.2+1.8)$
27.5 KN	$M\circlearrowright = (27.5)(1.8)$
8.75 KN	$M\circlearrowleft = (8.75)(0.6)$
$R = 53.75$ KN	$M\circlearrowright = 149.25$ KN.m

$$d = \frac{M}{R} = \frac{149.25}{53.75} = 2.77\ m$$



4-123





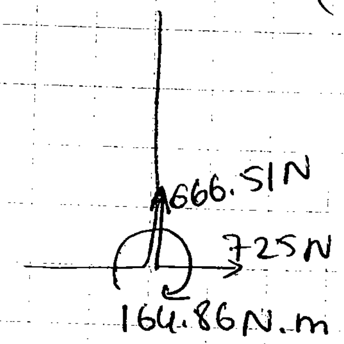
$R_x = 600 + 250 \sin 30 = 725N$

$R_y = 450 + 250 \cos 30 = 666.51N$

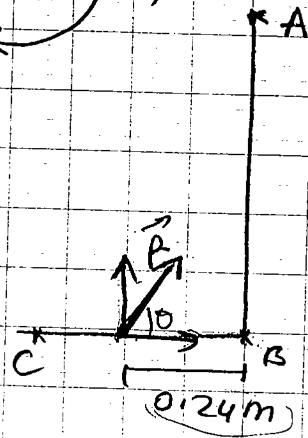
$M_C = (250\cos 30)(0.9) + (600)(1.2) - 750$

$= 164.86 \text{ N.m}$

$d = \frac{164.86}{666.51} = 0.24m$



$\Longleftrightarrow$



$R = \sqrt{(666.51)^2 + (164.86)^2} = 986.81N$

$\theta = \tan^{-1}\left(\frac{666.51}{164.86}\right) = 42.59°$

4-125

175 cos 30	$M = (175 \cos 30)(0.6)$
175 sin 30	$M = 0$
100	$M = (100)(1.2+0.6)$
125	$M = (125)(0.9)$

$\vec{R_x} = \Sigma \vec{F_x} = 125 + 175 \sin 30 = 212.5N$

$R_y \downarrow = \Sigma F_y \downarrow = 100 + 175 \cos 30 = 251.55N$

$M_C = (175\cos 30)(0.6) + (100)(1.2+0.6) - (125)(0.9)$

$= 158.43 \text{ N.m}$

$d = \frac{158.43}{251.55} = 0.63 \text{ m}$

$R = \sqrt{(251.55)^2 + (212.5)^2} = 329.29\ N$

$\theta = \tan^{-1}\left(\dfrac{251.55}{212.5}\right) = 49.81^\circ$

Q-129) $F_1 = 30\,KN$ $\qquad C(x, y) = ?$

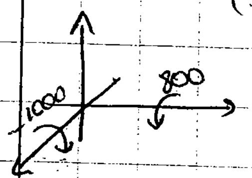
$F_2 = 40\,KN$

$\vec{M}_0 = \vec{r}_{OA} \times \vec{F}_A + \vec{r}_{OB} \times \vec{F}_B + \vec{r}_{OC} \times \vec{F}_C + \vec{r}_{OD} \times \vec{F}_D$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10 & 0 & 0 \\ 0 & 0 & -20 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 3 & 0 \\ 0 & 0 & -50 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 11 & 0 \\ 0 & 0 & -30 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10 & 13 & 0 \\ 0 & 0 & -40 \end{vmatrix}$$

$$= +(10)(20)\vec{j} + (3)(-50)\vec{i} - (4)(-50)\vec{j} + (11)(-30)\vec{i} + (13)(-40)\vec{i} - (10)(-40)\vec{j}$$

$$= (-1000\,\vec{i} \cancel{+900\,\vec{j}} + 800\,\vec{j}\,)\ KN.m$$



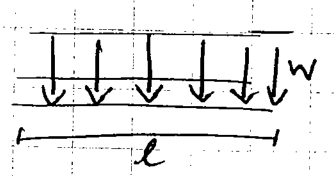
$R_z \downarrow = 50 + 20 + 30 + 40 = 140\,KN$

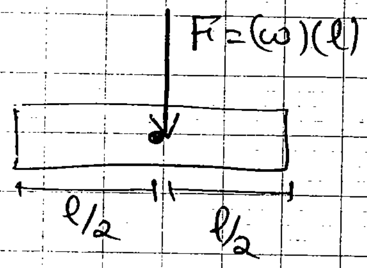
$X = \left|\dfrac{M_y}{R_z}\right| = \dfrac{800}{140} = 5.71\,m$

$y = \left|\dfrac{M_x}{R_z}\right| = \dfrac{1000}{140} = 7.14\,m$

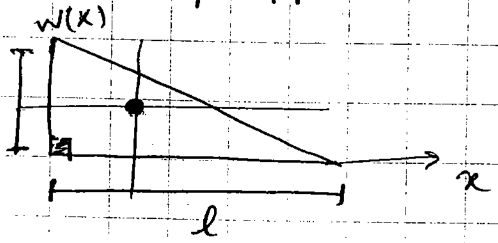
$C(5.71, 7.14)\,m$

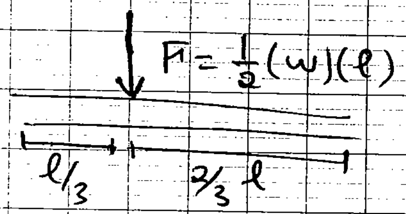
Force reduction :





Area under load = Force
Point of app. center of load shape





4-163) $M_c = 0$, $d = ?$

$750 : \frac{4}{5}(750) = \cancel{1000}600$ $\quad M = (1.2)(\cancel{1}600)$

$\frac{3}{5}(750) = 450$ $\quad \cancel{= 1200\ N.m}$

$500 : 500 \cos 30$ $\quad M = (500 \cos 30)(d)$

$500 \sin 30$

$$\Sigma M_c = (1.2)(\overset{600}{\cancel{1000}}) - (500 \cos 30)(d) = 0$$

$$\Rightarrow d = \frac{(1.2)(\cancel{1}600)}{500 \cos 30} = \_\_$$

4-171) $A(8, 8, 0)$ m $\quad P(6, -6, 10)$

$C(0, 0, 4)$ m

$F_{AC} = 120$ KN

$$\vec{F}_{AC} = 120 \left[\frac{-8\vec{i} - 8\vec{j} + 4\vec{k}}{\underbrace{\sqrt{(-8)^2 + (-8)^2 + (4)^2}}_{12}} \right] = (-80\vec{i} - 80\vec{j} + 40\vec{k}) \text{ KN}$$

$$\vec{M}_P = \vec{r}_{PA} \times \vec{F}_{AC}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 14 & -10 \\ -80 & -80 & 40 \end{vmatrix} = [(14)(40) - (-10)(-80)]\vec{i} - $$

$[(2)(40)-(-10)(-80)]\vec{j} + [(2)(-80)-(14)(-80)]\vec{K}$

$= (-240\vec{i} + 720\vec{j} + 960\vec{K})$ kN.m

Hw (4-124 / 4-134 / 4-164 / 4-168 / 4-172)

Chap. 5

Rigid Bodies (action = reaction)

3-D:

$$\sum \vec{F} = \vec{0} \Rightarrow \begin{cases} \sum F_x = 0 & ① \\ \sum F_y = 0 & ② \\ \sum F_z = 0 & ③ \end{cases}$$

$$\sum \vec{M} = \vec{0} \Rightarrow \begin{cases} \sum M_x = 0 & ④ \\ \sum M_y = 0 & ⑤ \\ \sum M_z = 0 & ⑥ \end{cases}$$

2-D:

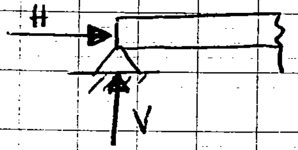
① $\sum \overrightarrow{F} = \sum F_x = 0$

② $\sum F\uparrow = \sum F_y = 0$

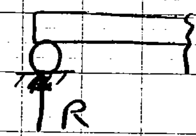
⑥ $\sum \overset{\frown}{M} = 0$

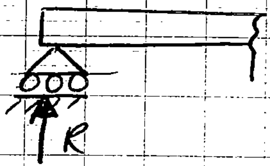
Support condition (2-D)

PIN:
(Hinge)

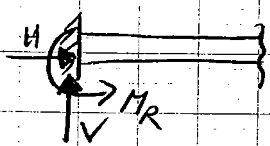


Roller:

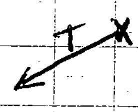




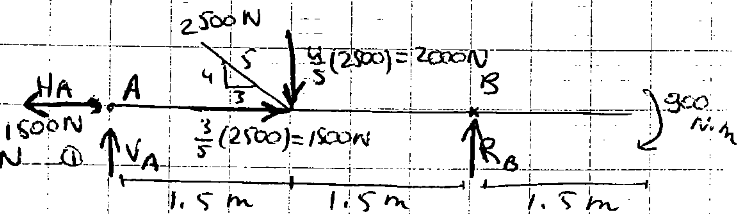
Fixed:



Cable:



5-111



$\sum \vec{F} = 0 = H_A + 1500 \Rightarrow H_A = -1500$ N ①

$\sum F\uparrow = 0 = V_A + R_B - 2000$ ②

$\sum \overset{\frown}{M_A} = 0 = (2000)(1.5) - (3)(R_B) + 900$ ③

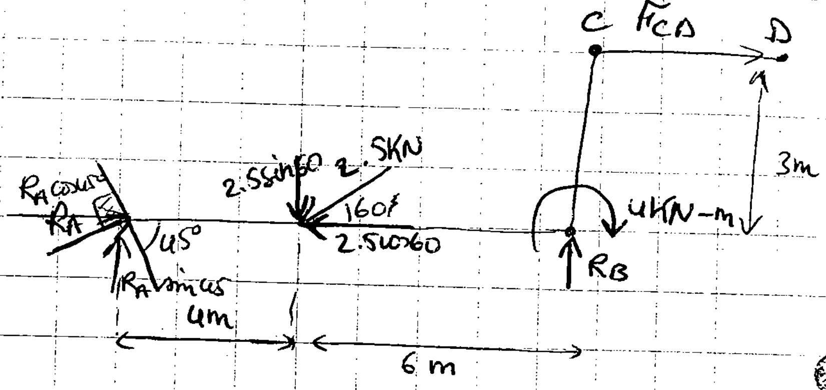
$③ \Rightarrow R_B = \dfrac{(2000)(1.5) + 900}{3} = 1300$ N

$\sum \overset{\frown}{M_B} = 0 = (3)(V_A) - (2000)(1.5) + 900$

$\Rightarrow V_A = \dfrac{(2000)(1.5) - 900}{3} = 700$ N

Check : $\Sigma F\uparrow = 700 + 1300 = 2000 = 0$ ✓

5_16



$\Sigma \vec{F} = 0 = R_A \cos 45 - 2.5 \cos 60 + F_{CD}$ —— ①

$\Sigma F\uparrow = 0 = R_A \sin 45 - 2.5 \sin 60 + R_B$ —— ②

$\Sigma M_C = 0 = (R_A \sin 45)(10) - (R_A \cos 45)(3) - (2.5 \sin 60)(6)$
$+ (2.5 \cos 60)(3) + 4$ —— ③

③ $\Rightarrow R_A = \dfrac{-(2.5 \sin 60)(6) + (2.5 \cos 60)(3) + 4}{3 \cos 45 - 10 \sin 45}$

$= 1.06$ KN

in ① $\Rightarrow F_{CD} = 2.5 \cos 60 - 1.06 \cos 45 = 0.50$ KN.

in ② $\Rightarrow R_B = 2.5 \sin 60 - 1.06 \sin 45 = 1.42$ KN

Check :

$\Sigma M_A = (2.5 \sin 60)(4) - (1.42)(10) + 4 + (0.50)(3)$
$= -0.04$

5_21 $\Sigma \vec{F} = 0 = -\frac{3}{5}(T_{BC}) + H_A$ —— ①

$\Sigma F\uparrow = 0 = -\frac{4}{5} T_{BC} + V_A - 60$ —— ②

$\Sigma M_A = 0 = (60)(1) + 30 - (\frac{4}{5} T_{BC})(1) - (\frac{3}{5} T_{BC})(3)$ —— ③

③ $\Rightarrow T_{BC} = \dfrac{(60)(1) + 30}{(\frac{4}{5})(1) + (\frac{3}{5})(3)} = 34.62$ KN.

in ① $\Rightarrow H_A = \frac{3}{5}(34.62) = 20.78$ KN

in ② $\Rightarrow V_A = \frac{4}{5}(34.62) + 60 = 87.70$ KN

Check : $\Sigma M_B = (60)(2) + 30 - (87.70)(1) - (20.78)(3) = -0.04$

5-23) $F = 200N$

$H_A, V_A = ?$, $R_C = ?$

$$\sum \overrightarrow{F} = 0 = H_A + R_C\cos 15 - 200\cos 60 \quad \text{--- } ①$$

$$\sum F\uparrow = 0 = V_A - R_C\sin 15 + 200\sin 60 \quad \text{--- } ②$$

$$\sum M_A \circlearrowright = 0 = (R_C\sin 15)(600\sin 15 + 200) + (R_C\cos 15)(600\cos 15) - (200\sin 60)(800) \quad \text{--- } ③$$

$$③ \Rightarrow R_C = \frac{(200\sin 60)(800)}{(600\sin 15 + 200)(\sin 15) + (600\cos 15)(\cos 15)}$$

$$= \frac{(200\sin 60)(800)}{600\sin^2 15 + 200\sin 15 + 600\cos^2 15}$$

$$= \frac{(200\sin 60)(800)}{600 + 200\sin 15}$$

$$R_C = 212.60\ N$$

in ① $\Rightarrow H_A = 200\cos 60 - 212.60\cos 15 = -105.36\ N$

in ② $\Rightarrow V_A = -200\sin 60 + 212.60\sin 15 = -118.18\ N$

5-27) $R_C = ?$

$F_{AB} = ?$ $40°$

$$\sum \overrightarrow{F} = 0 = 2 + F_{AB}\cos 40 + H_C \quad \text{--- } ①$$

$$\sum F\uparrow = 0 = 6 + F_{AB}\sin 40 + V_C \quad \text{--- } ②$$

$$\sum M_C \circlearrowright = 0 = (6)(1000\tan 20) - (2)(1000) + (F_{AB}\sin 40)(400\tan 20) - (F_{AB}\cos 40)(400) \quad \text{--- } ③$$

$$③ \Rightarrow F_{AB} = \frac{(6)(1000\tan 20) - (2)(1000)}{400\cos 40 - 400\sin 40\tan 20} = 0.86\ KN$$

in ① $\Rightarrow H_C = -2 - (0.86)(\cos 40) = -2.66\ KN$

in ② $\Rightarrow V_C = -6 - (0.86)(\sin 40) = -6.43\ KN$

5-37] $M = 50 kg \Rightarrow W = (50)(9.81) = 490.5 N$

$F_A = \frac{1}{2}(w_A)(0.45)$
$F_B = \frac{1}{2}(w_B)(0.3)$

$\Sigma M_A = 0 = (490.5)[3 + \frac{1}{3}(0.45)] - [\frac{1}{2}(w_B)(0.3)][\frac{1}{3}(0.45) + 3 + 6 + \frac{1}{3}(0.3)]$

$\Rightarrow w_B = \frac{(490.5)[3+\frac{1}{3}(0.45)]}{(\frac{1}{2})(0.3)[\frac{1}{3}(0.45)+3+6+\frac{1}{3}(0.3)]} = \cancel{111.36 N.m}$ 7546.15 N/m

$\Sigma M_B = 0 = [\frac{1}{2}(w_A)(0.45)][\frac{1}{3}(0.45)+3+6+\frac{1}{3}(0.3)] - [490.5][6+\frac{1}{3}(0.3)]$

check $\Sigma F_y = \dots = ?$

$\Rightarrow w_A = 1437.62 N/m$

5-41] $\Sigma M_A = 0 = (1500)(0.3) + (2250)(0.9) - R_B(1.2 \sin 30)$
$\Rightarrow R_B = 4125 N \Rightarrow H_A = 4125 N$

$\Sigma M_B = 0 = (V_A)(0.3+0.6+0.3+1.2\cos 30) - (4125)(1.2\sin 30) - (1500)(0.9+1.2\cos 30) - (2250)(0.3+1.2\cos 30)$

$\Rightarrow V_A = 3750 N$

Check: ......

Hw: 5-42 / 5-40 / 5-30 / 5-26 / 5-50.

5-55] $K = 5 KN/m$, $F_s = K\Delta \Rightarrow \Delta = (\frac{F_s}{K})$

$\theta = \tan^{-1}(\frac{\Delta_A - \Delta_B}{l_{AB}})$

$\Sigma M_A = 0 = (800)(1) - (F_B)(3)$

$\Rightarrow F_B = \frac{800}{3} N.$

$\Sigma M_B = 0 = (F_A)(3) - (800)(2)$

$\Rightarrow F_A = \frac{1600}{3} N$

$\Delta_A = \frac{F_A}{K} = \frac{1600/3}{5 \times 1000 \frac{N}{KN} \times \frac{1 m}{1000 mm}} = 106.67 mm$

$$\Delta_B = \frac{F_B}{K} = \frac{800/3}{5 \times 1000 \times \frac{1}{1000}} = 53.33\text{ mm}$$

$$\theta = \tan^{-1}\left(\frac{106.67 - 53.33}{3.10^3\text{ mm}}\right) = 1.02^\circ$$

5-59) $K = 15\text{ KN/m}$

$M = 40\text{ Kg} \Rightarrow W = (40)(9.81) = 392.4\text{ N}$

$\Sigma M_A = 0 = -(1)(F_B) + (392.4)(4)$

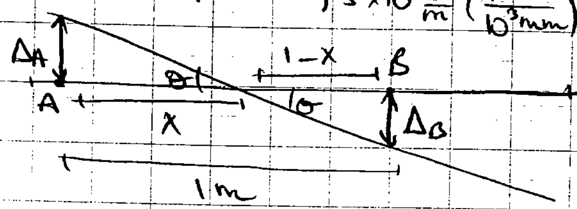
$\Rightarrow F_B = (392.4)(4) = 1569.6\text{ N}$.

$\Sigma M_B = 0 = (1)(F_A) + (3)(392.4)$

$\Rightarrow F_A = (392.4)(3) = -1177.20\text{ N}$

$$\Delta_A \uparrow = \frac{F_A}{K} = \frac{1177.20\text{ N}}{15.10^3 \cdot \left(\frac{1}{10^3}\frac{m}{mm}\right)} = 78.48\text{ mm}$$

$$\Delta_B \downarrow = \frac{F_B}{K} = \frac{1569.6\text{ N}}{15 \times 10^3 \frac{N}{m}\left(\frac{1m}{10^3 mm}\right)} = 106.64\text{ mm}.$$



$$\frac{\Delta_B}{1-X} = \frac{\Delta_A}{X} \Leftrightarrow \Delta_A - X\Delta_A = X\Delta_B$$

$$\Leftrightarrow X = \frac{\Delta_A}{\Delta_A + \Delta_B} = 0.43\text{ m}.$$

$$\theta = \tan^{-1}\left(\frac{\Delta_A}{X}\right) = \tan^{-1}\left(\frac{78.48\text{ mm}}{0.43 \times \frac{1000\text{ mm}}{1\text{ m}}}\right) = 10.36^\circ$$

5-61) $\theta = 0^\circ \Leftrightarrow \Delta_S = 0$

$\theta = 15^\circ \Leftrightarrow$ Equilibrium in 2-D $\begin{cases} \Sigma \vec{F} = 0 \\ \Sigma F\uparrow = 0 \\ \Sigma M = 0 \end{cases}$

$F = ?$

$V_A, H_A = ?$

$\Sigma M_A = 0 = (F)(400) - (F_{BC})(300\cos 45)$ —— ①

$\Delta_B = 300(\sin 60 - \sin 45) = 47.68\text{ mm}$.

$\Rightarrow F_{BC} = K\Delta = (2 \times 1000)\left(\frac{1}{1000\text{ mm}}\right)(47.68) = 95.36\text{ N}$

in ① $\Rightarrow F = \frac{(F_{BC})(300\cos 45)}{400} = \frac{(95.36)(300\cos 45)}{400} = 50.57\text{ N}$

$\Sigma \vec{F}_{\rightarrow} = 0 = -F_{BC} + H_A - F \sin 15$

$\Rightarrow H_A = F_{BC} + F \sin 15 = 95.36 + 50.57 \sin 15 = 108.45 N$

$\Sigma F \uparrow = 0 = V_A - F \cos 15 \Rightarrow V_A = F \cos 15 = 50.57 \cos 15 = 48.85$

Check: $\Sigma M_B = (48.85)(300 \sin 45) + (108.45)(300 \cos 45) - (50.57)(\cos 15)(400 \cos 15 + 300 \sin 45) - (50.57 \sin 15)(400 \sin 15 + 300 \cos 45)$

$= 1.90$ ✓

5-63] $M = 85 \, kg \Rightarrow W = Mg = (85)(9.81) = 833.85 N$

$\vec{R}_A = R_A \vec{k}$

$\vec{R}_C = R_C \vec{k}$ $\quad A(0.7, 0.2, 0)$

$\vec{R}_B = R_B \vec{k}$ $\quad B(0, 0.2, 0)$

$\vec{W}_G = -W \vec{k}$ $\quad C(0.35, 1.3, 0)$

$G(0.3, 0.45, 0)$

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma F_z = 0 = R_A + R_B + R_C - W$

$\Sigma \vec{M}_A = \vec{0} = \vec{r}_{AG} \times \vec{W}_G + \vec{r}_{AB} \times \vec{R}_B + \vec{r}_{AC} \times \vec{R}_C$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.4 & 0.25 & 0 \\ 0 & 0 & -833.85 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.7 & 0 & 0 \\ 0 & 0 & R_B \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.35 & 1.1 & 0 \\ 0 & 0 & R_C \end{vmatrix}$$

$= (0.25)(-833.85)\vec{i} - (-0.4)(-833.85)\vec{j} - (-0.7)(R_B)\vec{j} + (1.1)(R_C)\vec{i} - (-0.35)(R_C)\vec{j}$

$= 0\vec{i} + 0\vec{j} + 0\vec{k}$

$(0.25)(-833.85) + (1.1)(R_C) = 0 \Rightarrow R_C = 189.51 N$

$(-0.4)(-833.85) + (0.7)(R_B) + (-0.35)(R_C) = 0$

$\Rightarrow R_B = 381.73 N.$

in $\Sigma F_z = 0 \Rightarrow R_A = W - R_B - R_C$
$= 833.85 - 381.73 - 189.51$
$R_A = 262.61\ N.$

5-67)

$\vec{W}_A = (-225\ kN)\ \vec{k}$ A(2.1, 0, 0)
$\vec{W}_B = (-40\ kN)\ \vec{k}$ B(1.2, -1.8, 0)
$\vec{W}_C = (-30\ kN)\ \vec{k}$ C(1.2, 2.4, 0)
$\vec{R}_D = R_D \vec{k}$ D(0, -4.2, 0)
$\vec{R}_F = R_F \vec{k}$ E(0, 4.2, 0)
$\vec{R}_E = R_E \vec{k}$ F(8.1, 0, 0)

$\Sigma F_z = 0 = R_D + R_E + R_F - 225 - 40 - 30$ --- (1)

$\Sigma \vec{M}_o = \vec{0} = \vec{r}_{OA} \times \vec{W}_A + \vec{r}_{OB} \times \vec{W}_B + \vec{r}_{OC} \times \vec{W}_C + \vec{r}_{OD} \times \vec{R}_D +$
$\vec{r}_{OE} \times \vec{R}_E + \vec{r}_{OF} \times \vec{R}_F$.
$= (225)(2.1)\vec{j} + (40)(1.2)\vec{j} + (1.8)(40)\vec{i} + (30)(1.2)$
$-(30)(2.4)\vec{i} - 4.2 R_D \vec{j} + 4.2 R_E \vec{i} - 8.1 R_F \vec{j}$.

HW: 5-64 / 5-74 / 5-76.

5-73)

$\vec{R}_A = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$ $\vec{F}_B = (-200\vec{k})\ N$
$\vec{R}_B = R_B \vec{k}$ $\vec{F}_P = (-350\vec{k})\ N$
$\vec{T}_{CD} = T_{CD} \vec{k}$ $\vec{F}_Q = (-200\vec{k})\ N$

C(0, 0, 0) P(2, 0, 0)
A(3, 0, 0) Q(3cos60, 3sin60, 0)
B(0, 3, 0)

$\Sigma F_x = 0 = A_x$
$\Sigma F_y = 0 = A_y$
$\Sigma F_z = 0 = A_z + T_{CD} + R_B - 200 - 350 - 200$ --- (1)

$\Sigma \vec{M}_C = \vec{0} = \vec{r}_{CA} \times \vec{R}_A + \vec{r}_{CB} \times (\vec{R}_B + \vec{F}_B) + \vec{r}_{CP} \times \vec{F}_P + \vec{r}_{CQ} \times \vec{F}_Q$
$= (3\vec{i}) \times (A_z \vec{k}) + (3\vec{j}) \times [(R_B - 200)\vec{k}] + (2\vec{i}) \times (-350\vec{k}) +$
$(3\cos 60\,\vec{i} + 3\sin 60\,\vec{j}) \times (-200\vec{k})$
$= -3A_z\vec{j} + 3R_B\vec{i} - 600\vec{i} + 700\vec{j} + 600\cos 60\,\vec{j} - 600\sin 6$

$$\vec{0} = [3R_B - 600 - 600\sin 60]\vec{i} + [-3A_z + 700 + 600\cos 60]\vec{j}$$

$$3R_B - 600 - 600\sin 60 = 0 \Rightarrow R_B = \frac{600 + 600\sin 60}{3} = 373.21\,N$$

$$-3A_z + 700 + 600\cos 60 = 0 \Rightarrow A_z = \frac{700 + 600\cos 60}{3} = 333.33$$

In Eq. ① $\Rightarrow T_{CD} = 200 + 200 + 350 - 373.21 - 333.33 = 43.46\,N$

5_79)

$\vec{R_A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$

$T_B = ? \Rightarrow \vec{T_B} = (-T\vec{k})$

$\vec{F_C} = (-5\sin 30 - 5\sin 30)\vec{j} + (5\cos 30 - 5\cos 30)\vec{i}$

$= (-10\sin 30)\vec{j}$

$\vec{F_C} = (-5\vec{j})\,KN$

A(0, 0, 0)
B(0, 1.5, 0)
C(0, 0, 5)

$\Sigma F_x = 0 = A_x$

$\Sigma F_y = 0 = A_y - 5 \Rightarrow A_y = 5\,KN$.

$\Sigma F_z = 0 = A_z - T$

$\Sigma \vec{M_A} = \vec{0} = \vec{r_{AC}} \times \vec{F_C} + \vec{r_{AB}} \times \vec{T_B}$

$= (5\vec{k}) \times (-5\vec{j}) + (1.5\vec{j}) \times (-T\vec{k})$

$= 25\vec{i} - 1.5T\vec{i}$

$\vec{0} = (25 - 1.5T)\vec{i}$

$25 - 1.5T = 0 \Rightarrow T = \frac{25}{1.5} = 16.67\,KN$, $A_z = 16.67\,KN$

Check:

$\Sigma \vec{M_C} = \vec{r_{CA}} \times \vec{R_A} + \vec{r_{CB}} \times \vec{T_B}$

$= (5\vec{k}) \times (5\vec{j} + \cancel{A_z}\,16.67\vec{k}) + (1.5\vec{j} - 5\vec{k}) \times (-16.67\vec{k})$

$= 25\vec{i} - (1.5)(16.67)\vec{i}$

$= 0\vec{i}$.

5_83

$\vec{T}_{BC} = T_{BC}\left(\frac{\vec{r}_{BC}}{r_{BC}}\right)$, $\vec{F}_D = (-4.5\vec{k})$ KN

$\vec{R}_A = A_x\vec{i} + A_y\vec{j}$

$\vec{M}_A = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$

A(0, 0, 0)
B (3.6, 1.2, 0)
D (3.6, 0, 0)
C (0, 2.4, 1.8)

$$\vec{T}_{BC} = T_{BC}\left[\frac{-3.6\vec{i} + 1.2\vec{j} + 1.8\vec{k}}{\cancel{\sqrt{(-3.6)^2 + (1.2)^2 + (1.8)^2}}^{\,4.2}}\right]$$

$$= T_{BC}\left(\frac{-6\vec{i} + 2\vec{j} + 3\vec{k}}{7}\right)$$

$\Sigma F_x = 0 = A_x - \frac{6}{7}T_{BC}$

$\Sigma F_y = 0 = A_y + \frac{2}{7}T_{BC}$

$\Sigma F_z = 0 = \frac{3}{7}T_{BC} - 4.5 \Rightarrow T_{BC} = 4.5 \times \frac{7}{3} = 10.5$.

Additionel ($\vec{M}_A = ?$)

$\Sigma\vec{M}_A = \vec{0} = \vec{M}_{R_A} + \vec{r}_{AB} \times \vec{T}_{BC} + \vec{r}_{AD} \times \vec{F}_D$

$$\vec{0} = (M_x\vec{i} + M_y\vec{j} + M_z\vec{k}) + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.6 & 1.2 & 0 \\ -9 & 3 & 4.5 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.6 & 0 & 0 \\ 0 & 0 & -4.5 \end{vmatrix}$$

$$= M_x\vec{i} + M_y\vec{j} + M_z\vec{k} + (1.2)(4.5)\vec{i} - [(3.6)(4.5)]\vec{j} + [(3.6)(3) - (-9)(1.2)]\vec{k} - (3.6)(-4.5)\vec{j}$$

$$\vec{0} = [M_x + (1.2)(4.5)]\vec{i} + [M_y \cancel{-(3.6)(4.5)} \cancel{+(3.6)(4.5)}]\vec{j} + [M_z + (3.6)(3) + (9)(1.2)]\vec{k}$$

$M_x = -(1.2)(4.5) = -5.4$ KN.m

$M_y = 0$

$M_z = -(3.6)(3) - (9)(1.2) = -21.6$ KN.m

$\vec{R}_A = (9\vec{i} - 3\vec{j})$ KN

$\vec{M}_{R_A} = (-5.4\vec{i} - 21.6\vec{k})$ KN.m.

5-91| $\vec{F} = (-6\cos 30\,\vec{i} - 6\sin 30\,\vec{j})\ \text{KN}$

$\Sigma \vec{F} = 0 = H_B - 6\cos 30$ ——— ①

$\Sigma F\uparrow = 0 = R_A - 10 + V_B - 6\sin 30$ ——— ②

$\Sigma M_B = 0 = (R_A)(0.6 + 0.6 + 1.2\cos 60) - (10)(0.6 + 1.2\cos 60) - (6)(0.4)$

$\Rightarrow R_A = \dfrac{(10)(0.6 + 1.2\cos 60) + (6)(0.4)}{(1.2 + 1.2\cos 60)}$

$R_A = 8\ \text{KN}$

① $\Rightarrow H_B = 6\cos 30 = 5.2\ \text{KN}$

② $\Rightarrow V_B = 10 + 6\sin 30 - 8 = 5\ \text{KN}$

5-93|

$\Sigma M_B = 0 = (R_A)(2.4 + 1.8) - (25)(2.4 + 1.8) - (35)(1.8) + (10)(1.8) - (2.5)(1.8)$

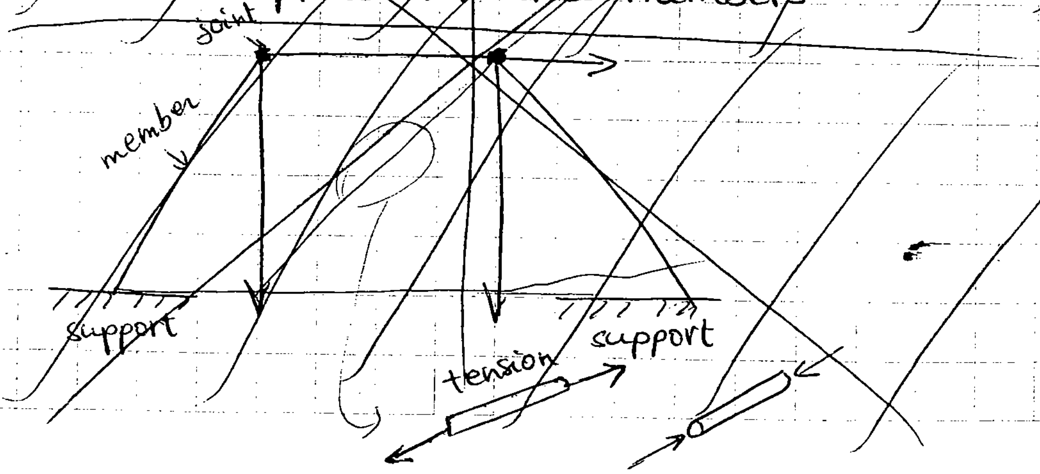
$\Rightarrow R_A = 36.79\ \text{KN}$

$\Sigma F\uparrow = 0 = V_B + 36.79 - 25 - 35 - 50 - 10$

$\Rightarrow V_B = 83.21\ \text{KN}$

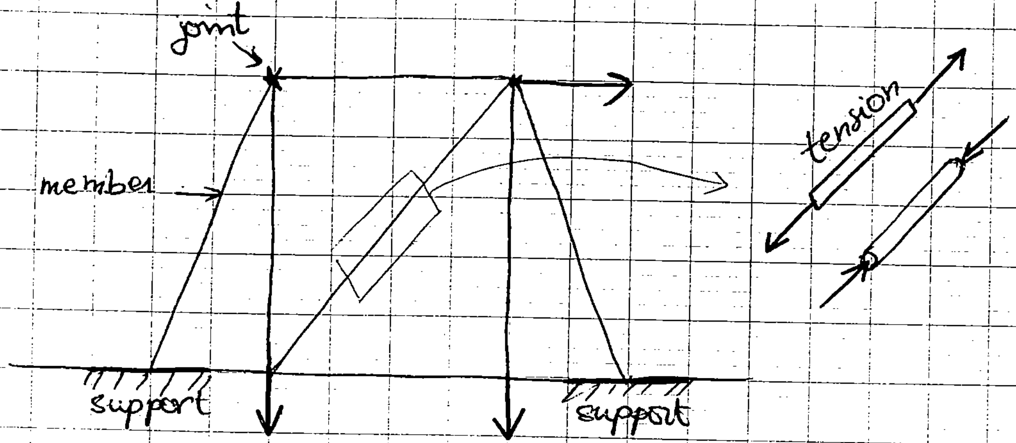
Truss Analysis

A truss is a structural system that is made of members that are pin connected. These members can handle only axial loads and these loads cause either tension or compression in the members.



Chap. 6 : Truss Analysis.

A Truss is a structural system that is made of members that are pin connected. These members can handle only axial loads and these loads cause either tension or compression in the members.



In the analysis process of trusses, the first thing that usually is done, is to determine the reactions at the supports. These are two methods of analysis :

1- Method of joints :

That you will establish equilibrium at each joint in order to determine the member (bar) forces. Thus you will have two equations of equilibrium ie $\Sigma \vec{F} = 0$ and $\Sigma F \uparrow = 0$. This means that you should normally start the analysis at a joint where you have only two unknown member forces. This method is usually used when all the Bar forces are required.

Tension = pulling at joint.
Compression = pushing at joint.

6_1)

$\sum F^{\rightarrow} = 0 = 600 + 900 + H_A + H_B$ ——— ①

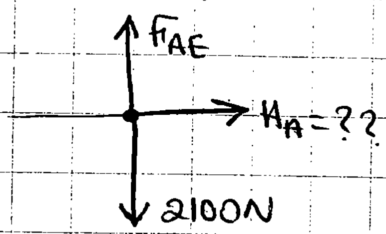
$\sum F\uparrow = 0 = V_A + V_B$ ——— ②

$\sum M_A \circlearrowright = 0 = (600)(4) + (900)(2) - (V_B)(2)$

$\Rightarrow V_B = \dfrac{(600)(4) + (900)(2)}{2} = 2100\,N$

in ② $\Rightarrow V_A = -2100\,N$

joint A:

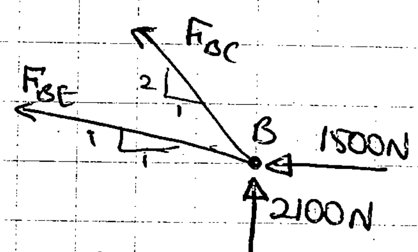


$\sum F\uparrow = 0 = -2100 + F_{AE} \Rightarrow F_{AE} = 2100\,N$ (T)

$\sum F^{\rightarrow} = 0 = H_A$

in ① $\Rightarrow H_B = -1500\,N$

joint B:



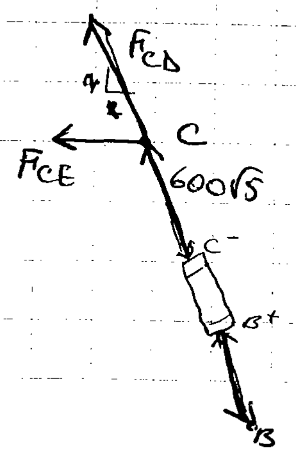
$$\left.\begin{aligned} \sum F^{\rightarrow} &= 0 = -1500 - \frac{1}{\sqrt{5}} F_{BC} - \frac{1}{\sqrt{2}} F_{BE} \\ \sum F\uparrow &= 0 = 2100 + \frac{2}{\sqrt{5}} F_{BC} + \frac{1}{\sqrt{2}} F_{BE} \end{aligned}\right\} \oplus$$

$600 + \frac{1}{\sqrt{5}} F_{BC} = 0 \Rightarrow F_{BC} = (-600\sqrt{5})\,N$ (C)

in $\sum F^{\rightarrow} = 0 \Rightarrow -1500 - \frac{1}{\sqrt{5}}(-600\sqrt{5}) - \frac{1}{\sqrt{2}} F_{BE} = 0$

$F_{BE} = (-900\sqrt{2})\,N$ (C)

joint C:



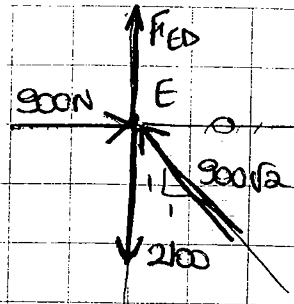
$\sum F\uparrow = 0 = \frac{2}{\sqrt{5}}(600\sqrt{5}) + \frac{2}{\sqrt{5}} F_{CD}$

$\Rightarrow F_{CD} = -600\sqrt{5}$ (C)

$\sum F^{\rightarrow} = 0 = -F_{CE} - \frac{1}{\sqrt{5}}(600\sqrt{5}) - \frac{1}{\sqrt{5}} F_{CD}$

$\Rightarrow F_{CE} = -\frac{1}{\sqrt{5}}(600\sqrt{5}) - \frac{1}{\sqrt{5}}(-600\sqrt{5})$
$= 0$

~~check~~ joint E:

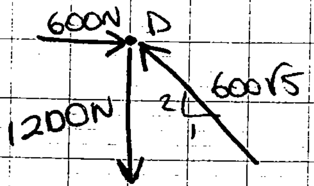


$\Sigma \vec{F} = 900 - \frac{1}{\sqrt{2}}(900\sqrt{2}) = 0$

$\Sigma F\uparrow = 0 = F_{ED} - 2100 + \frac{1}{\sqrt{2}}(900\sqrt{2})$

$\Rightarrow F_{ED} = 2100 - 900 = 1200\ N$ (

joint D: check

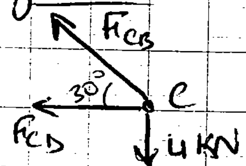


$\Sigma \vec{F} = 600 - \frac{1}{\sqrt{5}}(600\sqrt{5}) = 0$ ✓

$\Sigma F\uparrow = -1200 + \frac{2}{\sqrt{5}}(600\sqrt{5}) = 0$ ✓

6-7) $P_1 = P_2 = 4KN$

joint C



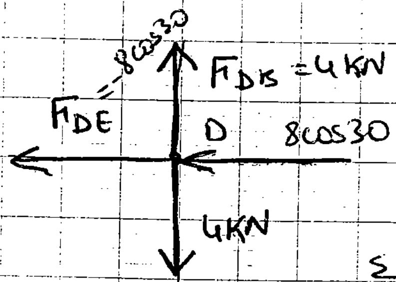
$\Sigma F\uparrow = 0 = F_{CB} \sin 30 - 4 \Rightarrow F_{CB} = \frac{4}{\sin 30} = 8\ KN$ (

$\Sigma \vec{F} = 0 = -F_{CB}\cos 30 - F_{CD} \Rightarrow F_{CD} = -F_{CB}\cos 30$

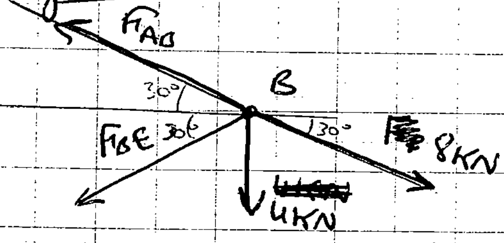
$= -8\cos 30$

$= -6.93\ KN$ (

joint D



joint B



$\Sigma F\uparrow = 0 = (F_{BA}\sin 30 - F_{BE}\sin 30 - 8\sin 30 - 4)\cos 30$

$+ \quad \Sigma \vec{F} = 0 = (-F_{BA}\cos 30 - F_{BE}\cos 30 + 8\cos 30)\sin 30$

$-2F_{BE}\sin 30\cos 30 - 4\cos 30 = 0$

$\Rightarrow F_{BE} = \frac{-2}{\sin 30} = -4\ KN$ (C)

$\Rightarrow F_{BA} = \frac{-4\sin 30 + 8\sin 30 - 4}{\sin 30} = -4\ KN$ (C)

whole structure

$\Sigma M_A \curvearrowright = 0 = (4)(3) + (4)(6) - (H_E)(6\tan 30)$

$\Rightarrow H_E = \frac{6}{\tan 30} = \frac{6\cos 30}{\sin 30} = 10.39\ KN = 12\cos 30$

joint E : Check

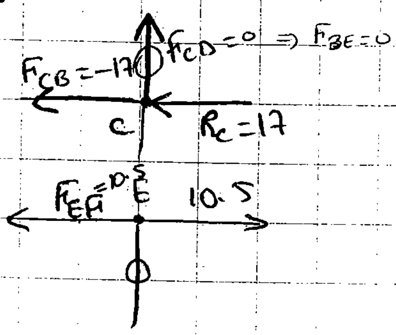
$\sum \vec{F} = 10.39 - 4\cos 30 - 8\cos 30 = 0$

Hw (6-2 / 6-4 / 6-12 / 6-16 / 6-24)

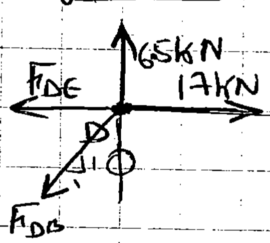
6-8 $P = 4\ KN$

$\sum M_D = 0 = (4)(3) + (2.5)(2) - (R_c)(1) \Rightarrow R_c = 17KN$

joint C:



joint D



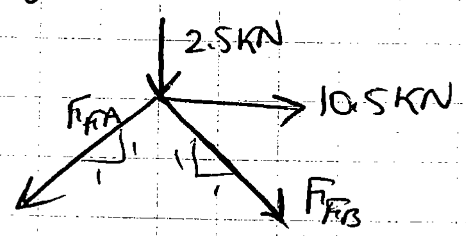
$\sum F\uparrow = 0 = 6.5 - \frac{1}{\sqrt{2}} F_{DB}$

$F_{DB} = (6.5)\sqrt{2}\ KN\ (T)$

$\sum \vec{F} = 0 = -F_{DE} + 17 - \frac{1}{\sqrt{2}} F_{DB}$

$\Rightarrow F_{DE} = 17 - \frac{1}{\sqrt{2}}(6.5\sqrt{2}) = 10.5\ KN$

joint F



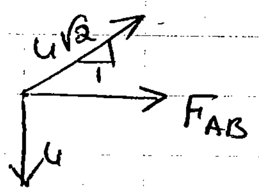
$$\left.\begin{array}{l} \sum F\uparrow = 0 = -\frac{1}{\sqrt{2}} F_{FA} - \frac{1}{\sqrt{2}} F_{FB} - 2.5 \\ \sum \vec{F} = 0 = -\frac{1}{\sqrt{2}} F_{FA} + \frac{1}{\sqrt{2}} F_{FB} + 10.5 \end{array}\right\} (+)$$

$-\frac{2}{\sqrt{2}} F_{FA} + 8 = 0$

$\Rightarrow F_{FA} = 4\sqrt{2}\ KN\ (T)$

$\sum \vec{F} = 0 \Rightarrow F_{FB} = \left[-\frac{1}{\sqrt{2}}(4\sqrt{2}) - 2.5\right]\sqrt{2} = -6.5\sqrt{2}\ KN\ (C)$

joint A



$\sum F\uparrow = \frac{1}{\sqrt{2}}(4\sqrt{2}) - 4 = 0$

$\sum \vec{F} = 0 = \frac{1}{\sqrt{2}}(4\sqrt{2}) + F_{AB}$

$\Rightarrow F_{AB} = -4\ kN\ (C)$

Super check joint B

6-14) $P = 12.5\,KN$ (symetrical)

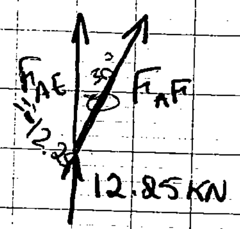
$\Sigma M_A = 0 = (12.5)(2) + (6)(4) - (R_B)(4)$

$\Rightarrow R_B = 12.25\,KN$

$\Sigma M_B = 0 = (4)(V_A) - (6)(4) - (12.5)(2)$

$\Rightarrow V_A = 12.25\,KN$

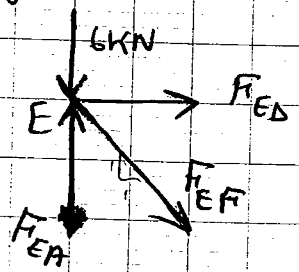
joint A :



$\Sigma \vec{F} = 0 = F_{AF} \sin 30 \Rightarrow F_{AF} = 0$

$F_{AE} = -12.25\,KN$ (~~compr~~ (C))

joint E :

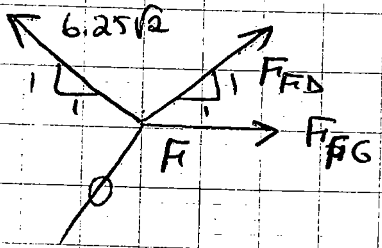


$\Sigma F\uparrow = 0 = 12.25 - 6 - \frac{1}{\sqrt{2}} F_{EF}$

$\Rightarrow F_{EF} = 6.25\sqrt{2}\,KN$ (T)

$\Sigma \vec{F} = 0 = F_{ED} + \frac{1}{\sqrt{2}} F_{EF} \Rightarrow F_{ED} = -6.25\,KN$ (C)

joint F :



$\Sigma F\uparrow = 0 = \frac{1}{\sqrt{2}} F_{FD} + \frac{1}{\sqrt{2}}(6.25\sqrt{2})$

$\Rightarrow F_{FD} = -6.25\sqrt{2}\,KN$ (C)

$\Sigma \vec{F} = 0 = -\frac{1}{\sqrt{2}}(6.25\sqrt{2}) + \frac{1}{\sqrt{2}} F_{FD} + F_{FG}$

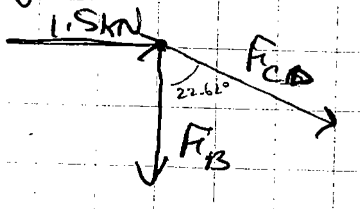
$\Rightarrow F_{FG} = 6.25 - \frac{1}{\sqrt{2}}(-6.25\sqrt{2})$

$= 12.5\,KN$ (T)

super check : joint D

6-26) $\tan^{-1}\left(\frac{1.5}{3.9}\right) = 22.62°$

joint C

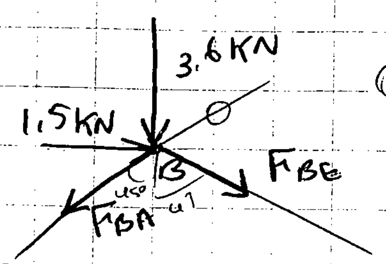


$\Sigma \vec{F} = 0 = 1.5 + F_{CD} \sin 22.62°$

$\Rightarrow F_{CD} = \frac{-1.5}{\sin 22.62} = -3.90$ (C)

$\Sigma F\uparrow = 0 = -F_{CB} - F_{CD} \cos 22.62°$

$\Rightarrow F_{CB} = -(-3.90)\cos 22.62 = 3.60$ (T joint B



$\sum \vec{F} = 0 = 1.5 - F_{BA} \sin 45 + F_{BE} \sin 45.26$

$\sum F\uparrow = 0 = -3.6 - F_{BA} \cos 45 - F_{BE} \cos 45.2$

$5.1 + F_{BE}(\sin 45.26 + \cos 45.26) =$

$F_{BE} = -\dfrac{5.1}{\sin 45.26 + \cos 45.26} = -3.6\ KN\ (C)$

$F_{BA} = \dfrac{1.5 - 3.61 \sin 45.26}{\sin 45} = -1.51\ KN\ (C)$

~~check~~

$\sum M_A = 0 = 1.5(3.9 + 3.9\cos 45.26) + (1.5)(3.9\cos 45.26)$
$-(V_E)(47.7 + 3.9 \cos 45.26)$
$\Rightarrow V_E = 2.55\ KN.$

2. Method of sections

Usually ~~a se~~ used when specific members forces are required.

Looking a section you will have three equation of equilibrium ($\sum \vec{F} = 0$, $\sum F\uparrow = 0$ and $\sum M = 0$)

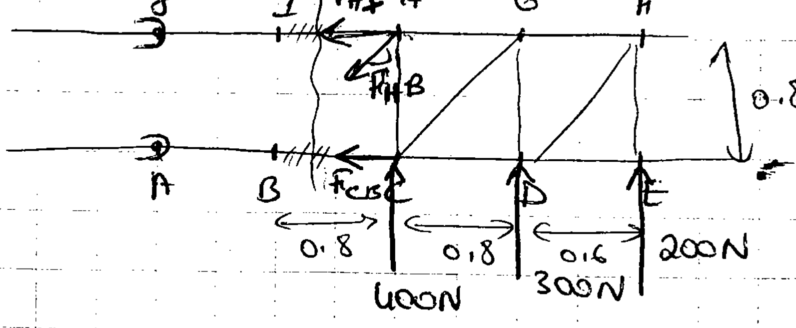
Thus you need to select a section where there is only 3 unknown members forces.

6-31) $\sum M_A = 0 = (H_J)(0.8) - (400)(0.8) - (400)(1.6) - (300)(2.4) - (200)(3.2)$

$\Rightarrow H_J = 2850\ N$

$\vec{F}_{JI} = -2850\ N$

section a-a

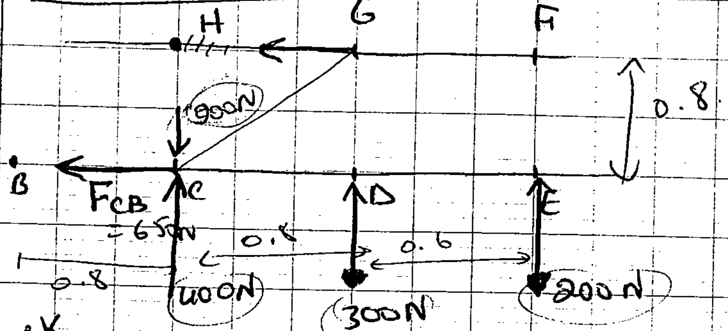


$$\Sigma M_H = 0 = (F_{CB})(0.8) - (300)(0.8) - (200)(0.8+0.6)$$

$$\Rightarrow F_{CB} = 650 \text{ N } (T)$$

$$\Sigma F\uparrow = 0 = -\frac{1}{\sqrt{2}} F_{HB} + 400 + 300 + 200 \Rightarrow F_{HB} = 900\sqrt{2} \text{ N } ($$

section b_b

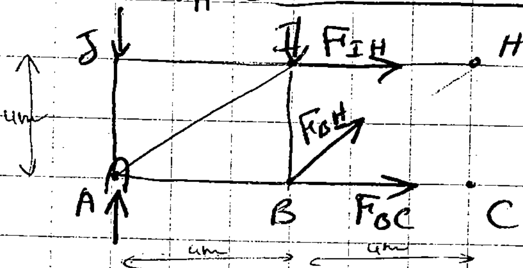


Check

$$\Sigma M_G = (650)(0.8) - (900)(0.8) + (400)(0.8) - (200)(0.6)$$
$$= 0$$

6_33) $R_E = \dfrac{(20)(4) + (20)(8) + (40)(12)}{16} = 45 \text{ KN}$

$$V_A = \frac{(4)(4) + (20)(8) + (20)(12) + (30)(16)}{16} = 65 \text{ KN}$$



$$\Sigma F\uparrow = 0 = \frac{1}{\sqrt{2}} F_{BH} + 65 - 30 - 20$$

$$\Rightarrow F_{BH} = -15\sqrt{2} \text{ KN } (C)$$

$$\Sigma M_H = 0 = (65)(8) - (30)(8) - (F_{BC})($$
$$-(20)(4)$$

$$\Rightarrow F_{BC} = 50 \text{ KN } (T)$$

$$\Sigma M_B = 0 = (65)(4) - (30)(4) + (F_{IH})(4)$$

$$\Rightarrow F_{IH} = -35 \text{ KN } (C)$$

super check:

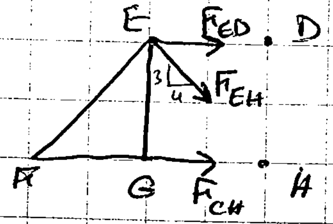
$$\Sigma \vec{F} = -35 - \frac{1}{\sqrt{2}}(15\sqrt{2}) + 50 = 0$$

6-39) $H_A = 70\,KN\ (= 30 + 40)$

$$V_A = \frac{(40)(2) + (50)(4) + (40)(1.5) - (70)(3)}{4} = 32.5\ KN$$

$$R_F = \frac{(40)(1.5) + (30)(3) + (40)(2)}{4} = 57.5$$

$V_A + R_F = 90 = (40+50)\,KN$



$\Sigma F \uparrow = 0 = 57.5 - 40 - \frac{3}{5} F_{EH}$

$\Rightarrow F_{EH} = \frac{5}{3}(17.5) = 29.17\ KN\ (T)$

$\Sigma M_E = 0 = (57.5)(2) - (F_{GH})(1.5)$

$\Rightarrow F_{GH} = 76.67\ KN\ (T)$

$\Sigma M_H = 0 = (57.5)(4) - (40)(2) + (F_{ED})(1.5)$

$\Rightarrow F_{ED} = -100\ KN\ (C)$

Super check:

$\Sigma \vec{F} = -100 + (29.17)\left(\frac{4}{5}\right) + 76.67 = 0.006$

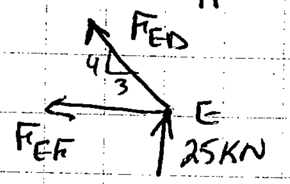
BC = ?
GF = ?
ED = ?

6-51)

$$R_E = \frac{(40)(1.5) + (20)(4.5)}{5} = 25\ KN$$

$$V_A = \frac{(40)(4.5) + (20)(1.5)}{5} = 35\ KN$$

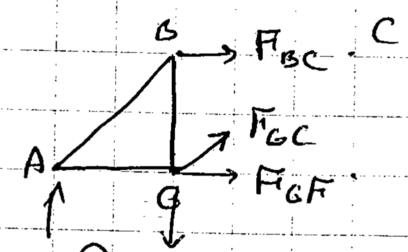
$\oplus = 60\ kN$



$\Sigma F \uparrow = 0 = \frac{4}{5} F_{ED} + 25 \Rightarrow F_{ED} = -\frac{5}{4}(25)$

$= -31.25\,K$

(C)



$\Sigma M_C = 0 = (35)(3) - (40)(1.5) - (F_{GF})(2)$

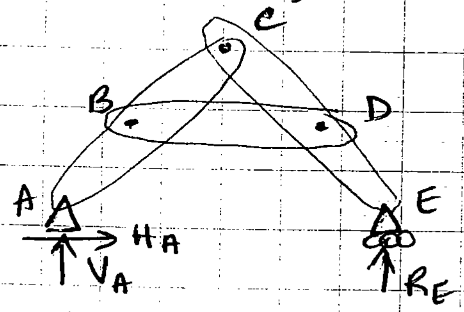
$\Rightarrow F_{GF} = -$

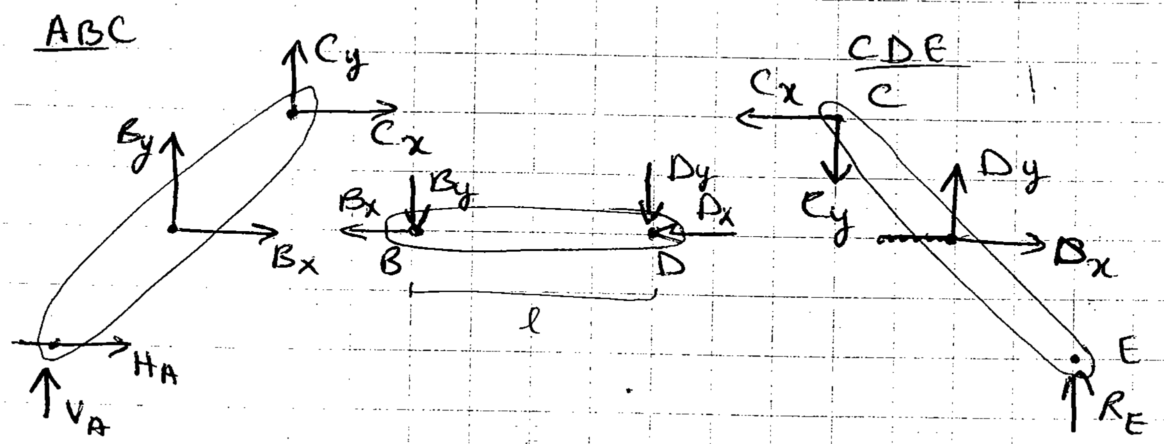
Hw (6-34 / 6-40 / 6-44 / 6-76 / 6-78)

Frames

Frames are structures made of two force members that are pin connected.

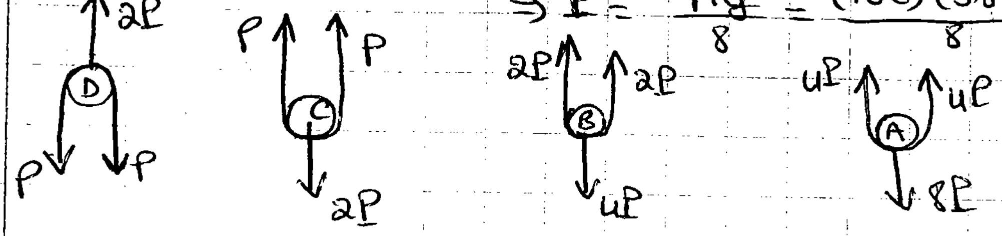
That is whenever you un-pin a member, you usually expose two forces at each pin (that does not mean that force on member can be axial). Thus you will have 3 eq of equilibrium for each section or member of the frame that is $\Sigma \vec{F} = 0$, $\Sigma F \uparrow = 0$, $\Sigma M = 0$



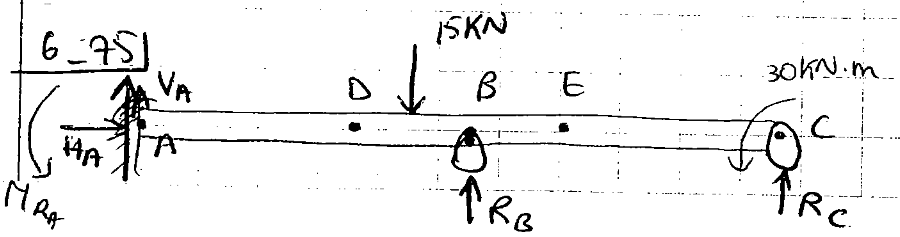


6_67) $M = 100\ Kg \rightarrow W = Mg = 8P$

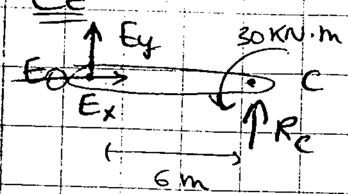
$$\Rightarrow P = \frac{Mg}{8} = \frac{(100)(9.81)}{8} = 122.63\ N$$



6_75)

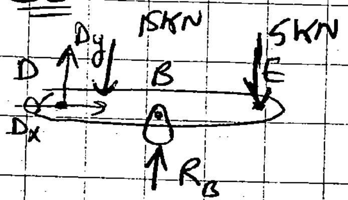


CE



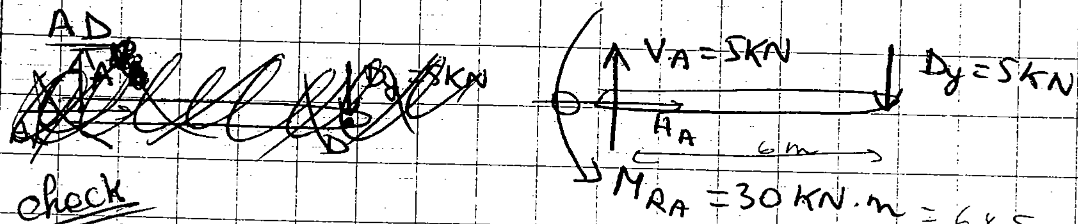
$$\Sigma M_E = 0 = 30 + 6R_C$$
$$\Rightarrow R_C = -5\,KN$$
$$E_y = 5\,KN$$

DE



$$\Sigma M_D = 0 = (15)(2) + (5)(6) - 4R_B$$
$$\Rightarrow R_B = 15\,KN.$$
$$D_y = 5\,KN.$$

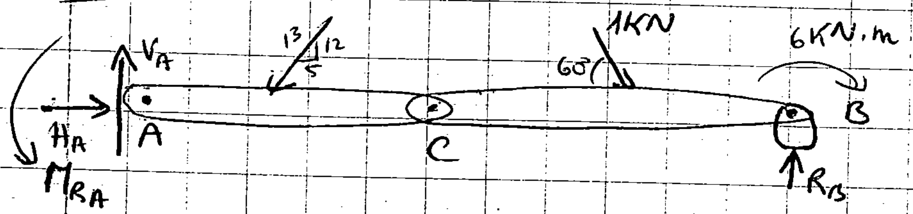
AD



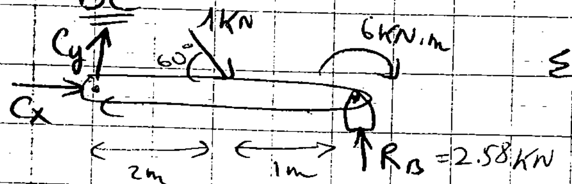
check

$$\Sigma M_C = -30 + (5)(18) - (15)(10) + (15)(8) - 30 = 0$$

6-77)



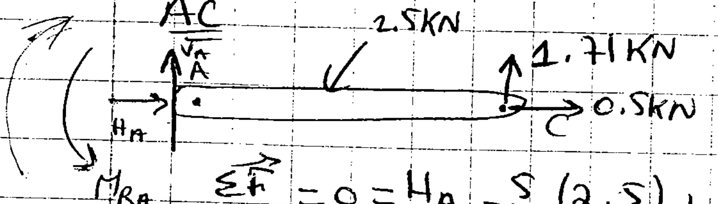
BC



$$\Sigma M_C = 0 = (1\sin 60)(2) + 6 - 3R_B$$
$$\Rightarrow R_B = 2.58\,KN$$
$$\cancel{C_y = 1\sin 60}$$

$$\Sigma F\uparrow = 0 = C_y - 1\sin 60 + 2.58 \Rightarrow C_y = -1.71\,KN$$
$$\Sigma \overrightarrow{F} = 0 = C_x + 1\cos 60 \Rightarrow C_x = -\tfrac{1}{2} = -0.5\,KN.$$

AC



$$\Sigma \overrightarrow{F} = 0 = H_A - \frac{5}{13}(2.5) + \frac{1}{2} \Rightarrow H_A = 0.46\,KN$$
$$\Sigma F\uparrow = 0 = V_A - \frac{12}{13}(2.5) + 1.71 \Rightarrow V_A = 0.60\,KN$$
$$\Sigma M_C = 0 = -M_{RA} + (0.60)(2) - \left(\frac{12}{13}\right)(2.5)(1) \Rightarrow M_{RA} = -0.80\,KN.m$$

whole structure $\Sigma M_B = 0.8 + (0.6)(5) + 6 - \left(\frac{12}{13}\right)(2.5)(4) - (1\sin 60)(1) = -0.296 \cong 0$

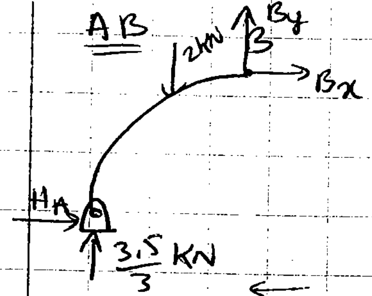
6_83)

$\Sigma F^{\rightarrow} = 0 = H_A + H_c + 1.5$

$\Sigma F\uparrow = 0 = V_A + V_C - 2 \Rightarrow V_A = 2 - \frac{2.5}{3} = \frac{3.5}{3}$ kN

$\Sigma M_A = (2)(0.5) + (1.5)(1) - (V_C)(3) \Rightarrow V_C = \frac{2.5}{3}$ kN

AB



$\Sigma M_B = 0 = \left(\frac{3.5}{3}\right)(1.5) - (2)(1) - (H_A)(1.5$

$\Rightarrow H_A = -\frac{1}{6}$ kN

$\overleftarrow{H_A} = 1/6$ kN

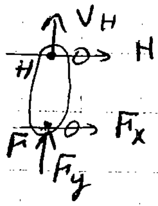
$\overleftarrow{H_c} = +1.5 - \frac{1}{6} = \frac{8}{6} = \frac{4}{3}$ kN

whole structure

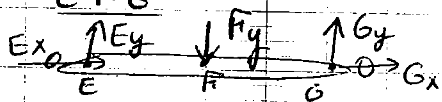
$\Sigma M_B = \left(\frac{3.5}{3}\right)(1.5) + \left(\frac{1}{6}\right)(1.5) - (2)(1) - (1.5)(0.5) - \left(\frac{2.5}{3}\right)(1.5$
$+ \left(\frac{4}{3}\right)(1.5) = 0$

6_85) $x = 450$mm $M_S = ?$ $M_L = 90$kg

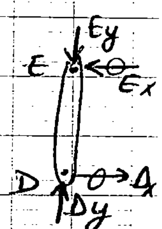
HF



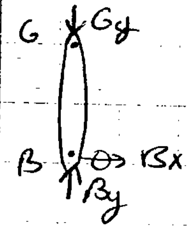
EFG



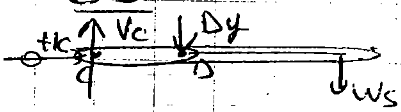
ED



GB

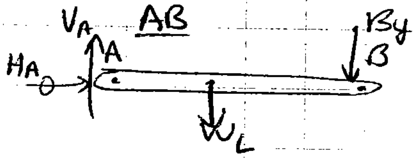


CD



$\Sigma M_C = 0 = 100(D_y) + (950)(W_s)$

AB



$\Sigma M_A = 0 = (150)(W_L) + (500)(B_y$

$B_y = \left(-\frac{150}{500}\right)W_L = -0.3W_L$

EFG $\rightarrow$ $\Sigma M_F = 0 = (E_y)(250) + (0.3W_L)(150)$ $\rightarrow G_y = -0.3W_L$

$\Rightarrow E_y = -(0.3W_L)\left(\frac{150}{250}\right) = -0.18W_L$

ED $\rightarrow$ $E_y = -0.18W_L$, $D_y = -0.18W_L$

GB $\rightarrow$ $G_y = -0.3W_L$, $B_y = -0.3W_L$

$D_y = -0.18 W_L$

CD $\rightarrow$ $\Sigma M_C = 0 = 950 W_S - (100)(0.18 W_L)$

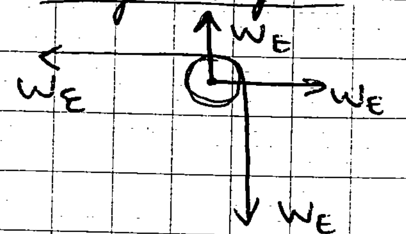
$\Rightarrow W_S = \frac{(100)(0.18)}{950} W_L = 0.019 W_L$

$\Rightarrow M_S = \frac{W_S}{g} = \frac{0.019 W_L}{g} = 0.019 M_L = 0.019(90)$

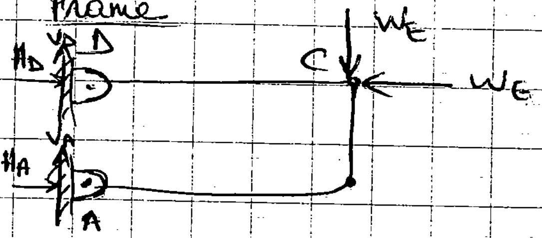
$M_S = 1.71$ Kg

6-88 $M_E = 100$ Kg $\Rightarrow W_E = M_E g = (100)(9.81) = 981 N$

Pulley Analysis



Frame



$\Sigma M_D = 0 = (W_E)(1.2) - (H_A)(0.6)$

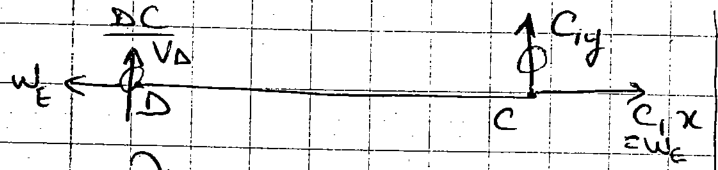
$\Rightarrow H_A = 2 W_E$

$\Sigma M_A = 0 = (W_E)(1.2) - (W_E)(0.6) + (H_D)(0.6)$

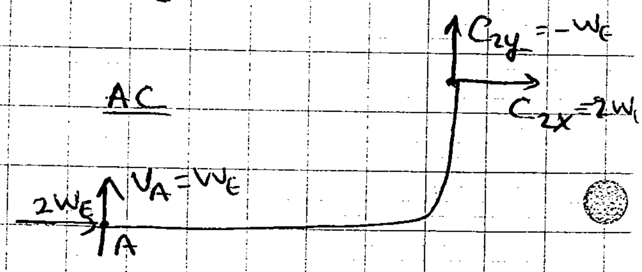
$\Rightarrow H_D = -\frac{0.6 W_E}{0.6} = -W_E$

$\Sigma F\uparrow = 0 = V_A + V_D - W_E$

DC



AC

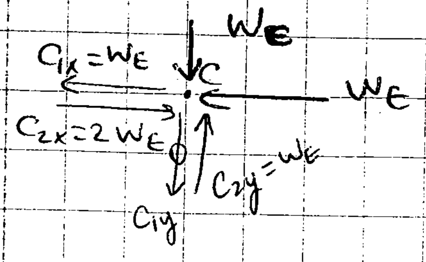


$\Sigma M_C = 0 = (V_D)(1.2) \Rightarrow V_D = 0$

$\Sigma M_C = 0 = (1.2)(V_A) - (2W_E)(0.6)$

$\Rightarrow V_A = W_E$

joint C



6_89

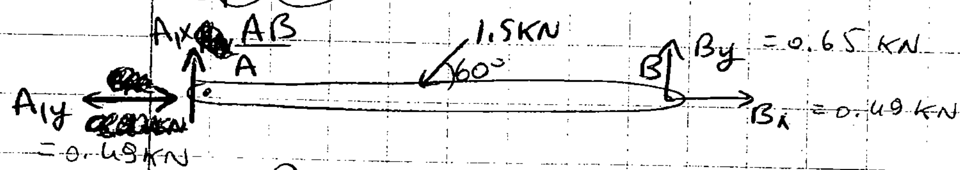
$\Sigma M_E = 0 = (H_A)(1.2) - (1.5\cos 60)(1.2) + (1.5\sin 60)(0.9) + (2.5)(1$

$\Rightarrow H_A = -3.97\ KN$, $H_E = 3.97 + 1.5\cos 60$

$H_E = 4.72\ KN$

~~EDC~~

AB



$\Sigma M_A = 0 = (1.5\sin 60)(0.9) - (1.8)(B_y) \Rightarrow B_y = 0.65\ KN$

$\rightarrow A_{1x} = 0.65\ KN$

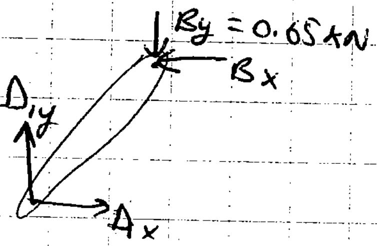
BD

$\Sigma M_D = 0 = (0.65)(0.9) - (1.2)(B_x)$

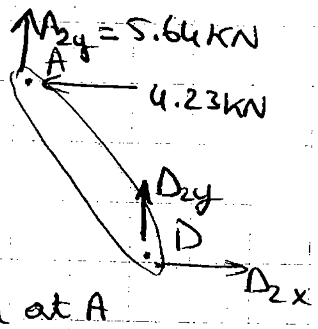
$\Rightarrow B_x = 0.49\ KN$

$A_{1y} = 0.49\ KN$

$A_{2x} = 4.72 - 0.49 = 4.23\ KN$



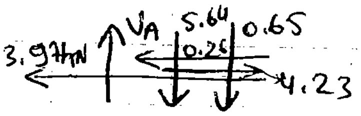
AD



$\Sigma M_D = 0 = (A_{2y})(0.9) - (4.23)(1.2)$

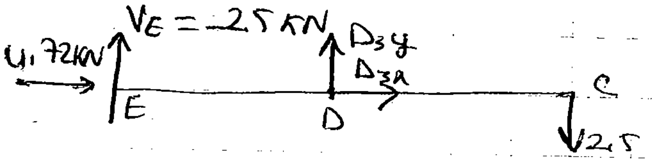
$\Rightarrow A_{2y} = 5.64\ KN.$

Pin at A



$\Rightarrow V_A = 6.29\ KN.$

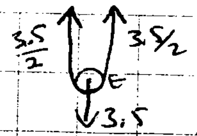
Check EDC



$V_A = 2.5 + 1.5\sin 60 + 2.5$

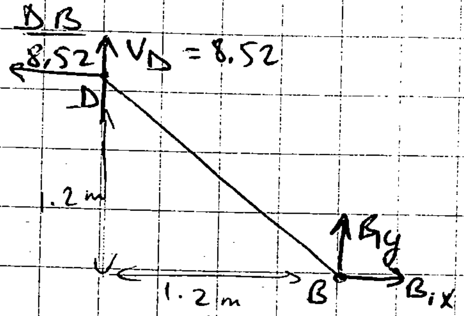
$V_A = 6.30\ KN \simeq 6.29$

6-92) $T_W = \frac{3.5}{2}$ kN



$\Sigma M_A = 0 = (1.2)(H_D) + (3.5)(2.4) + \left[\left(\frac{3.5}{2}\right)\cos 60\right](1.2\tan 60)$

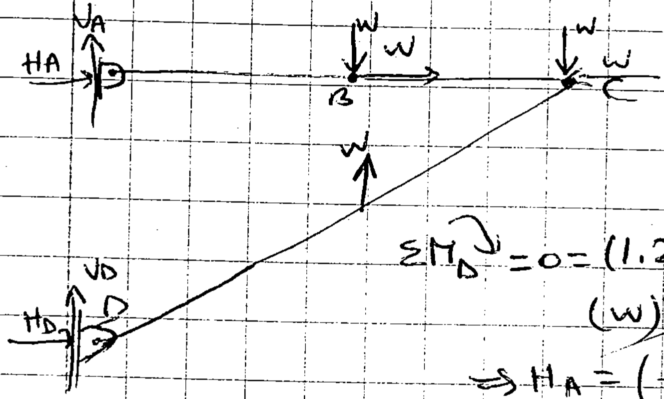
$\Rightarrow H_D = -8.52$ kN



$\overrightarrow{H_A} = 8.52 + \left(\frac{3.5}{2}\right)\cos 60 = 9.40$

$V_A \uparrow = 3.5 + \left(\frac{3.5}{2}\right)\sin 60 - 8.52 = 3.50$

6-101) $M = 50$ kg $\Rightarrow W = Mg = (50)(9.81) = 490.5$ N

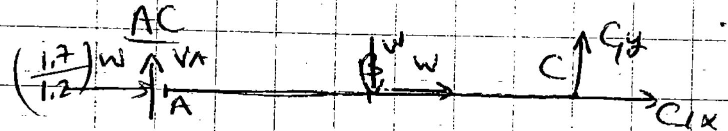


$\Sigma M_D = 0 = (1.2)(H_A) + (W)(0.8) + (W)(1.6) + \cancel{(W)(1.2)} - \cancel{(W)(1.2)} - (W)(0.7)$

$\Rightarrow H_A = \left(\frac{1.7}{1.2}\right)W, \quad H_D = \left(\frac{1.7}{1.2}\right)W$

$\Sigma F \uparrow = 0 = V_A + V_D - W - W + W$

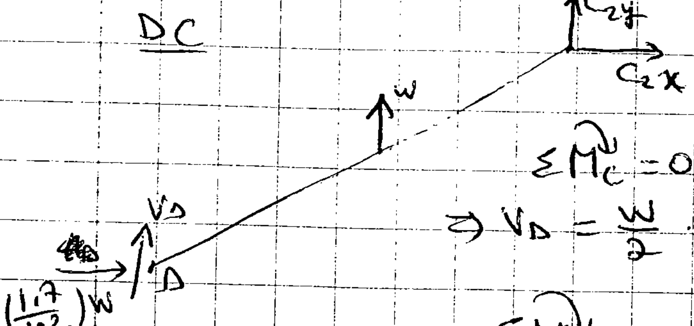
$\Rightarrow V_A + V_D = W$



$\Sigma M_C = 0 = 1.6V_A - (W)(0.8)$

$\Rightarrow V_A = \frac{W}{2}, \quad C_{1y} = \frac{W}{2}$

$\Sigma \vec{F} = 0 = \left(\frac{1.7}{1.2}\right)W + W + C_{1x} \Rightarrow C_{1x} = -\left(\frac{2.9}{1.2}\right)W$

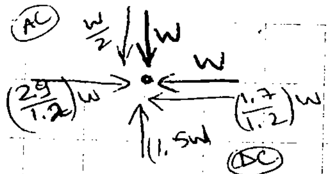


$\Sigma M_C = 0 = (V_D)(1.6) + (W)(0.9) - \left[\left(\frac{1.7}{1.2}\right)W\right](1.2)$

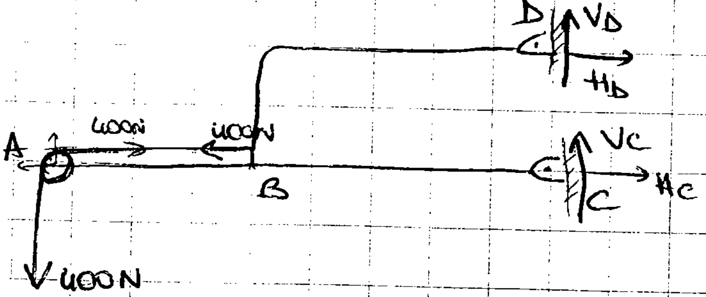
$\Rightarrow V_D = \frac{W}{2}$

~~$\Sigma M_B = 0 =$~~

joint c



6_119

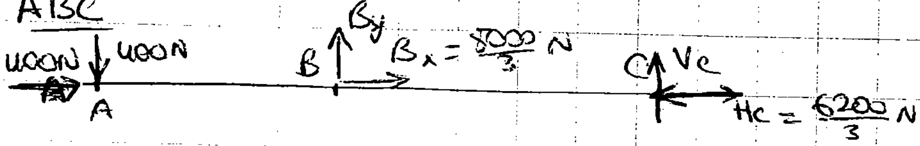


$\Sigma M_C = 0 = (H_D)(0.9) - (400)(0.15) - 400(1.8+2.7)$

$\Rightarrow H_D = \frac{6200}{3}$ N, $H_C = -\frac{6200}{3}$ N

$\Sigma F\uparrow = 0 = V_D + V_C - 400$

ABC

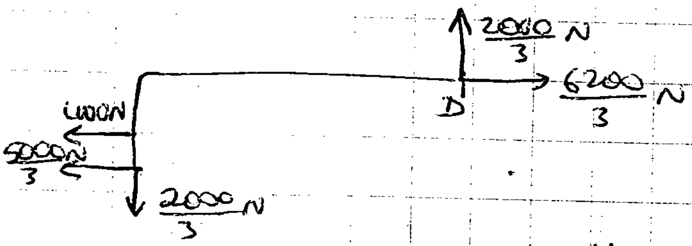


$\Sigma M_B = 0 = (-400)(1.8) - (V_C)(2.7)$

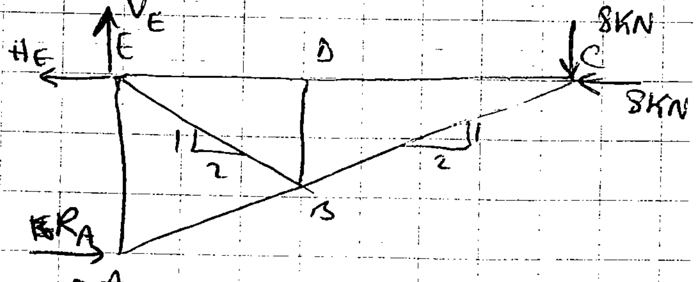
$\Rightarrow V_C = -\frac{(400)(1.8)}{2.7} = -\frac{800}{3}$ N $\rightarrow V_D = \frac{2000}{3}$ N

extra

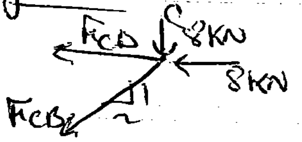
BD



6_129

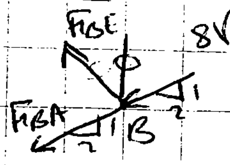


joint C



$\Sigma F\uparrow = 0 = -\frac{1}{\sqrt{5}} F_{CB} - 8 \Rightarrow F_{CB} = -8\sqrt{5}$ KN (C)

$\Sigma \vec{F} = 0 = -F_{CD} - \frac{2}{\sqrt{5}} F_{CB} - 8 \Rightarrow F_{CD} = -\frac{2}{\sqrt{5}}(-8\sqrt{5}) - 8 = 8$ KN (T) joint B



$\sum \vec{F} = 0 = -\frac{2}{\sqrt5}F_{BA} - \frac{2}{\sqrt5}F_{BE} - \frac{2}{\sqrt5}(8\sqrt5)$ (1)

(×2) $\sum F\uparrow = 0 = -\frac{1}{\sqrt5}F_{BA} + \frac{1}{\sqrt5}F_{BE} - \frac{1}{\sqrt5}(8\sqrt5)$

$-\frac{4}{\sqrt5}F_{BA} - \frac{4}{\sqrt5}(8\sqrt5) = 0$

$F_{BA} = -8\sqrt5$ kN (C)

in $\sum F\uparrow = 0 \Rightarrow -\frac{1}{\cancel{\sqrt5}}(-8\cancel{\sqrt5}) + \frac{1}{\sqrt5}F_{BE} - \frac{1}{\cancel{\sqrt5}}(8\cancel{\sqrt5}) = 0$

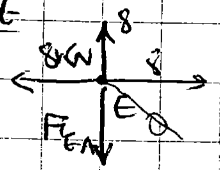
$F_{BE} = 0$

$\sum M_E = 0 = (8)(6) - 3R_A \Rightarrow R_A = 16$ kN

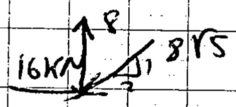
$H_E = 8$ kN

$V_E = 8$ kN

joint E



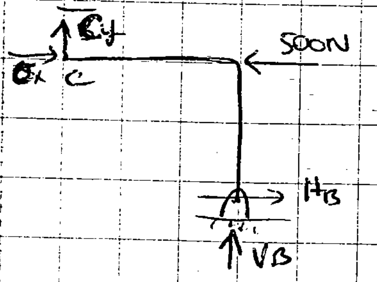
super check joint A :



6-133) $\sum M_A = 0 = (400)(1.5)(\frac{1.5}{2}) - (500)(1.5\sin 60) - (V_B)(1+1.5\cos 60) + (H_B)(1.5\sin 60 - 1)$

$\Rightarrow -1.25V_B + 0.30H_B = 199.52$ (1)

BC

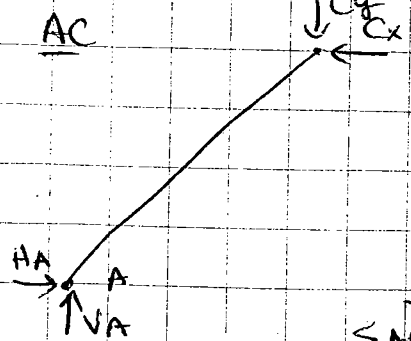


$\sum M_C = 0 = (V_B)(1) + (H_B)(1)$

$\Rightarrow V_B = -H_B$ (2)

(2) in (1) $\Rightarrow V_B = \frac{-199.52}{(1.25+0.30)} = -97.33$ N

AC



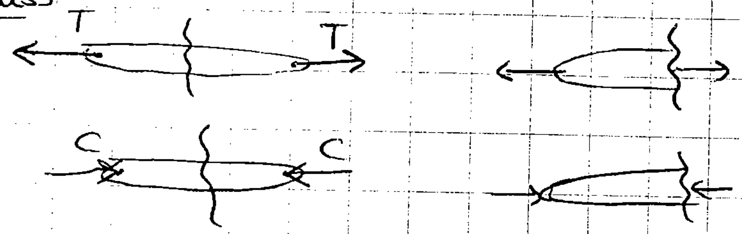
$\sum M_C = 0 = (1.5\cos 60)(V_A) - (H_A)(1.5\sin 60) - (400)(1.5)(\frac{1.5}{2})$

$\Rightarrow 0.75V_A - 1.30H_A = 450$

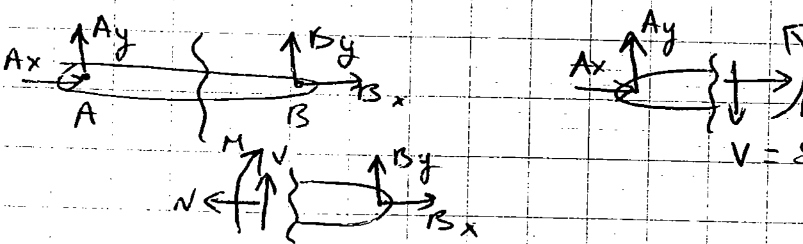
$\sum M_B = 0 = \ldots$

7_ Internal Forces

Truss



Frame (Beam)



N = normal ($\perp$ to cross-section)
M = moment
V = Shear (// cross-section)

7_1 $H_A = 0$

$$\Sigma M_A = 0 = (40)(2) - 60 - 6R_B$$

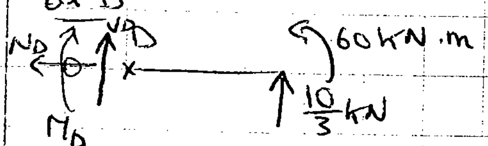
$$\Rightarrow R_B = \frac{20}{6} = \frac{10}{3}\ \text{KN}.$$

$$\Sigma M_B = 0 = 6V_A - (40)(4) - 60$$

$$\Rightarrow V_A = \frac{220}{6} = \frac{110}{3}\ \text{KN}$$

$\oplus = \frac{120}{3} = 40$ ✓

at D

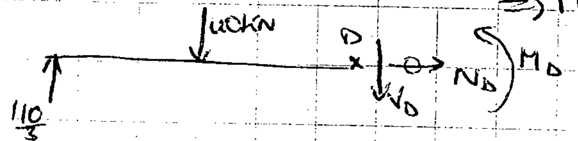


$$V_D = -\frac{10}{3}\ \text{KN}$$

$$\Sigma M_{cut} = 0 = M_D - 60 - \left(\frac{10}{3}\right)(2)$$

$$\Rightarrow M_D = \frac{200}{3}\ \text{KN.m}.$$

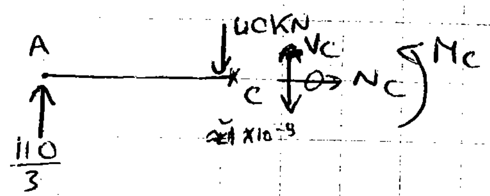
check :



$$\Sigma F\uparrow = 0 = \frac{110}{3} - 40 - V_D \Rightarrow V_D = -\frac{10}{3}\ \text{KN}$$

$$\Sigma M_{cut} = 0 = \left(\frac{110}{3}\right)(4) - (40)(2) - M_D \Rightarrow M_D = \frac{200}{3}\ \text{KN.m}$$

at C

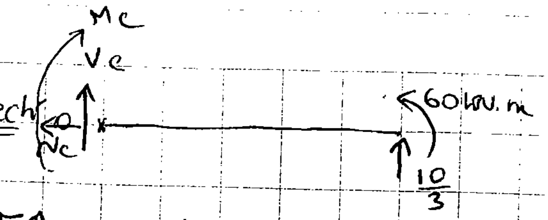


$$\Sigma F\uparrow = 0 = \frac{110}{3} - 40 - V_C$$

$$\Rightarrow V_C = -\frac{10}{3}\ \text{KN}.$$

$$\Sigma M_{cut} = 0 = \left(\frac{110}{3}\right)(2) - \cancel{(40)(+)}^{0} - M_C \Rightarrow M_C = \frac{220}{3}\ \text{KN.m}$$

check



$\Sigma F\uparrow = 0 = V_c + \frac{10}{3} \Rightarrow V_c = -\frac{10}{3}$ kN.

$\Sigma M_{cut} = 0 = -M_c + 60 + (\frac{10}{3})(4) \Rightarrow M_c = 60 + \frac{40}{3} = \frac{220}{3}$ kN.m

7-3| $\vec{R}_B = -R_B \cos 60 \vec{i} + R_B \sin 60 \vec{j}$.

$\Sigma M_A = 0 = [(10)(4)](\frac{4}{2}) + 2.5 - 4(R_B \sin 60)$

$\Rightarrow R_B = \frac{(10)(4)(4/2) + 2.5}{4 \sin 60} = 23.82$ kN.

$\Sigma M_B = 0 = -(10)(4)(\frac{4}{2}) + 2.5 + 4V_A$

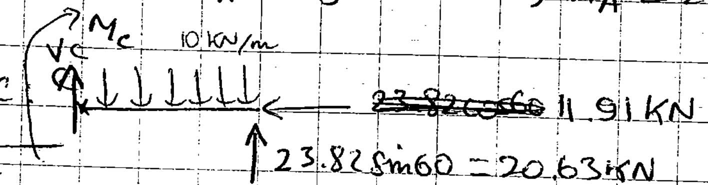
$\Rightarrow V_A = \frac{(10)(4)(4/2) - 2.5}{4} = 19.38$ kN.

check

$\Sigma F\uparrow = 23.82 \sin 60 + 19.38 - (4)(10) = 0.0087 \approx 0$

$\Sigma \vec{F} = 0 = H_A - R_B \cos 60 \Rightarrow H_A = 23.82 \cos 60 = 11.91$ kN

at C



$\Sigma \vec{F} = 0 = -N_c - 11.91 \Rightarrow N_c = -11.91$ kN.

$\Sigma F\uparrow = 0 = V_c + 20.63 - (10)(2) \Rightarrow V_c = -20.63 + 20 = -0.63$ kN

$\Sigma M_{cut} = 0 = M_c + (10)(2)(\frac{2}{2}) - (20.63)(2)$

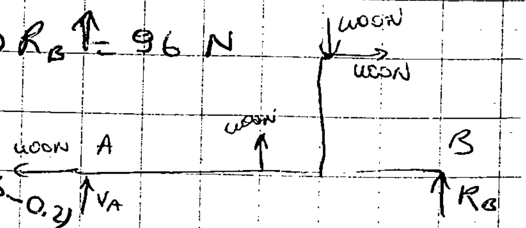
$\Rightarrow M_c = -(10)(2)(\frac{2}{2}) + (20.63)(2) = 21.26$ kN.m.

7-5| $\overleftarrow{H_A} = 400$N

$\Sigma M_A = 0 = (400)(1 + 0.2) - 5R_B \Rightarrow R_B\uparrow = 96$ N

same $\Rightarrow V_A\downarrow = 96$N

$\Sigma M_A = 0 = -(400)(3) + (400)(1) + (400)(3 - 0.2) - 5R_B$



at C $\Sigma \vec{F} = 0 = N_c - 400 \Rightarrow N_c = 400$N

$\Sigma F\uparrow = 0 = -V_c - 96 \Rightarrow V_c = -96$ N

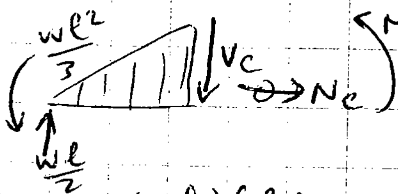
$\Sigma M_{cut} = 0 = -(96)(1.5) - M_c \Rightarrow M_c = -144$ N.m

7-7] $M_A = \left[\frac{1}{2}(w)(\ell)\right]\left[\frac{2}{3}(\ell)\right] = \frac{w\ell^2}{3}$

$V_A = \frac{w\ell}{2}$

$H_A = 0$

at C:



$\Sigma F \uparrow = 0 = \frac{w\ell}{2} - \frac{1}{2}\left(\frac{w}{2}\right)\left(\frac{\ell}{2}\right) - V_c$

$\Rightarrow V_c = \frac{w\ell}{2} - \frac{w\ell}{8} = \frac{3}{8} w\ell$

$\Sigma M_{cut} = 0 = \left(\frac{w\ell}{2}\right)\left(\frac{\ell}{2}\right) - \frac{w\ell^2}{3} - \left[\frac{1}{2}\left(\frac{w}{2}\right)\left(\frac{\ell}{2}\right)\right]\left[\frac{1}{3}\left(\frac{\ell}{2}\right)\right] - M_c$

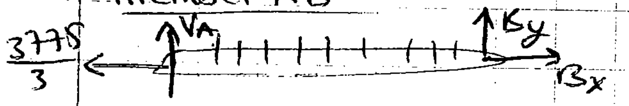
$\Rightarrow M_c = \frac{w\ell^2}{4} - \frac{w\ell^2}{3} - \frac{w\ell^2}{48} = -\frac{5}{48} w\ell^2$.

7-13] $\Sigma M_A = 0 = (250)(4)\left(\frac{4}{2}\right) + \left[\frac{1}{2}(1.5)(300)\right]\left[\frac{2}{3}(1.5)\right] - (1.5)(H_c)$

$\Rightarrow H_c = \frac{4450}{3}$ N

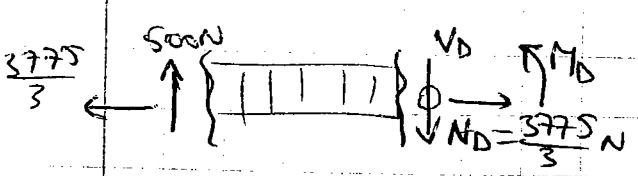
$\Sigma \vec{F} = 0 = H_A + \frac{4450}{3} - \frac{1}{2}(300)(1.5) \Rightarrow H_A = -\frac{3775}{3}$ N.

member AB



$\Sigma M_B = 0 = 4V_A - (250)(4)\left(\frac{4}{2}\right)$

$\Rightarrow V_A = 500$ N



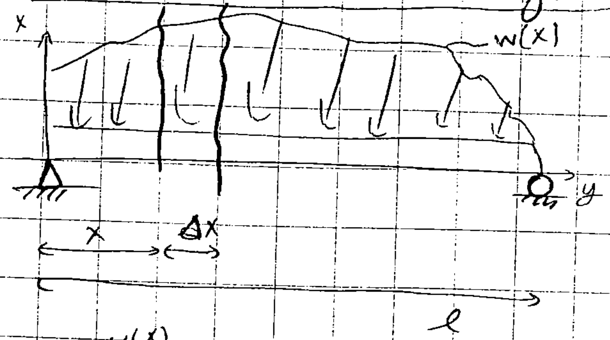
$\Sigma F \uparrow = 0 = 500 - (250)(2) - V_D$

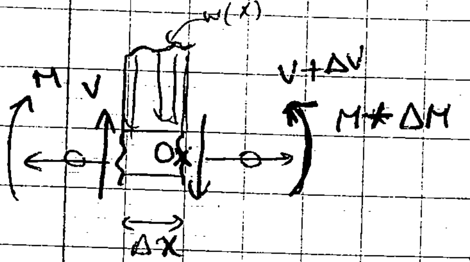
$\Rightarrow V_D = 0$.

$\Sigma M_{cut} = 0 = (500)(2) - (250)(2)\left(\frac{2}{2}\right) - M_D$

$\Rightarrow M_D = 500$ N.m

Moment and shear diagrams

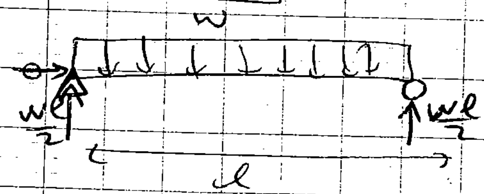


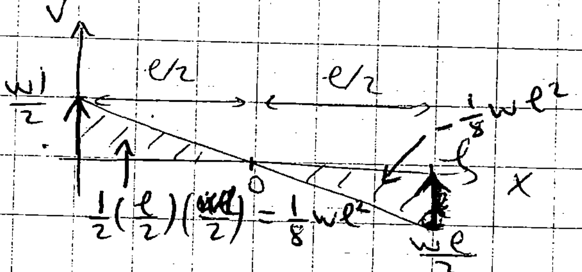


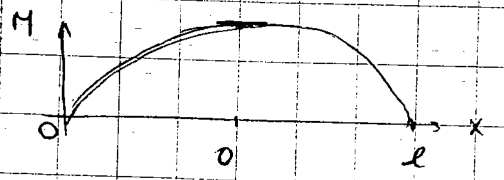
$$\Sigma F \uparrow = 0 = V - [w(x)][\Delta x] - (V + \Delta V)$$
$$\Rightarrow \Delta V = -[w(x)][\Delta x]$$
$$\Rightarrow \frac{\Delta V}{\Delta x} = -w(x)$$
as $\Delta x \to dx \Rightarrow \Delta V \to dV$
$$\boxed{\frac{dV}{dx} = -w(x)}$$

$$\Sigma M_O = 0 = M + (V)(\Delta x) - [w(x)][\Delta x]\left[\frac{\Delta x}{2}\right] - (M + \Delta M)$$
$$\Rightarrow \Delta M = [V][\Delta x] - \cancel{[w(x)][\Delta x]\left[\frac{\Delta x}{2}\right]} \to 0$$
$\Delta x <<<$ $\Rightarrow \Delta x^2 \simeq 0$
$$\Rightarrow \frac{\Delta M}{\Delta x} = V$$
as $\Delta x \to dx \Rightarrow \Delta M \to dM$
$$\boxed{\frac{dM}{dx} = V}$$

ex)



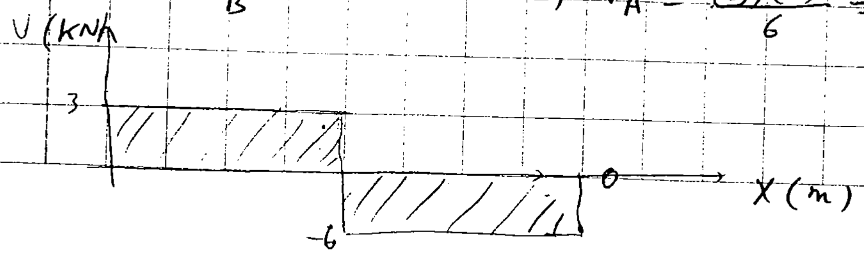


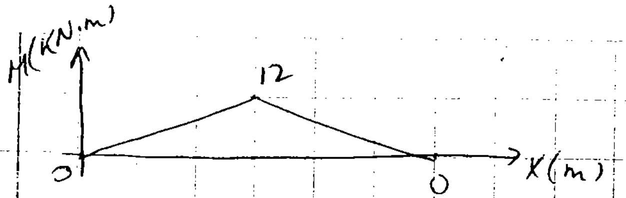


7-41) $H_A = 0$

$\Sigma M_A = 0 = \_\_\_\_ \Rightarrow R_B = \frac{(9)(4)}{6} = 6\,kN$

$\Sigma M_B = 0 = \_\_\_\_ \Rightarrow V_A = \frac{(9)(2)}{6} = 3\,kN$

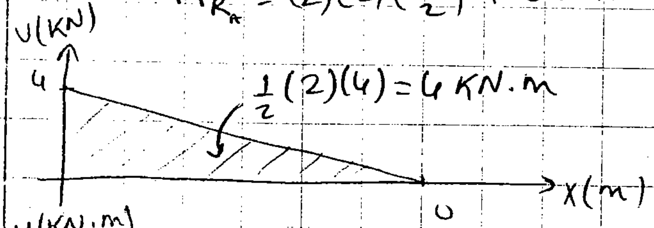


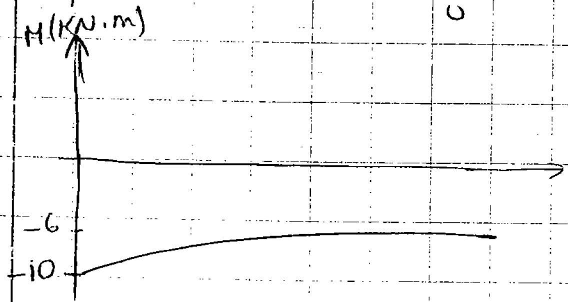


7-43) $H_A = 0$

$V_A \uparrow = (2)(2) = 4$

$M_{R_A} = (2)(2)(\frac{2}{2}) + 6 = 10\ KN.m$

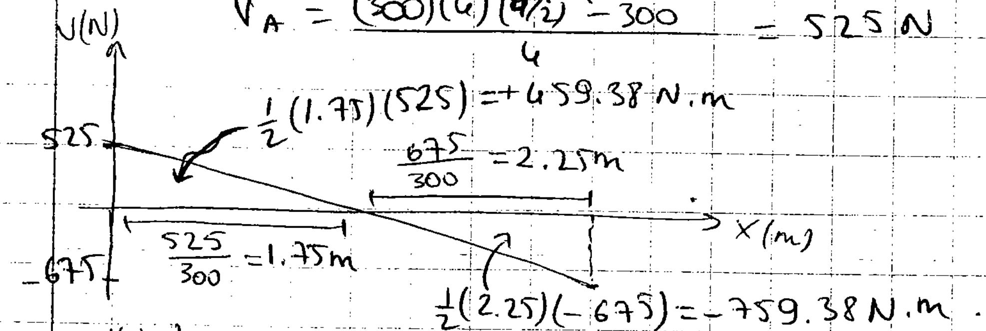


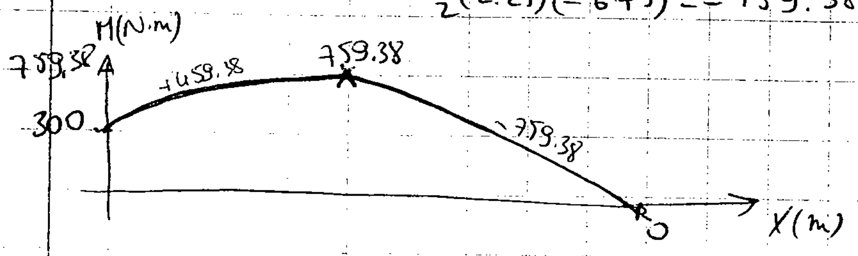


4-47) $H_A = 0$,

$R_B = \frac{(300)(4)(4/2) + 300}{} = 675\ N$

$V_A = \frac{(300)(4)(4/2) - 300}{4} = 525\ N$

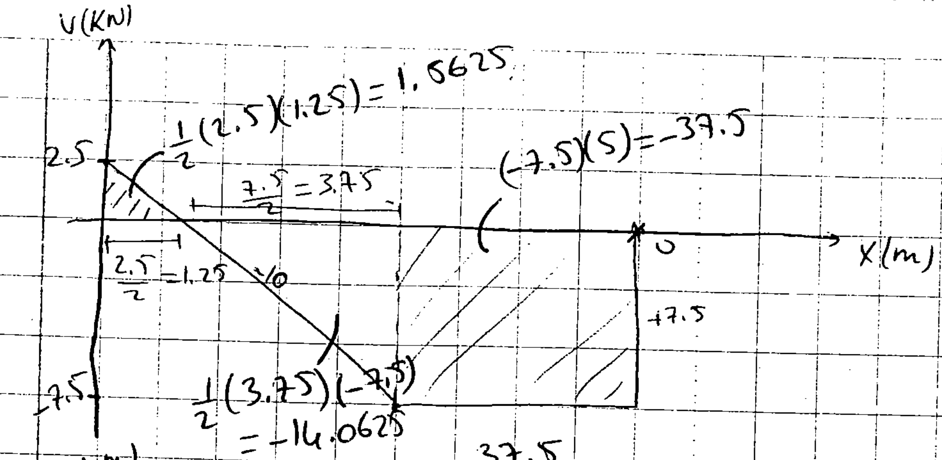


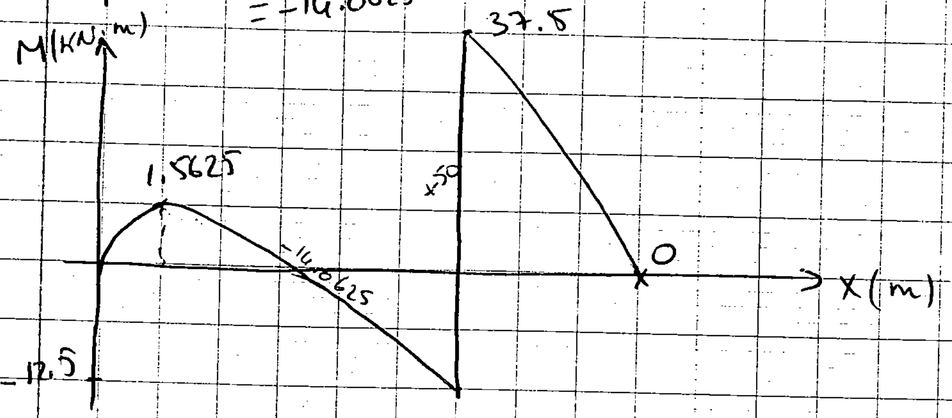


7-49) $H_A = 0$

$\Sigma M_A = 0 = \_\_\_ \Rightarrow R_C = \frac{(2)(5)(5/2) + 50}{} = 7.5\ KN$

$\Sigma M_C = 0 = \_\_\_ \Rightarrow V_A = \frac{(2)(5)(5/2 + 5) - 50}{10} = 2.5\ KN$





7-51 $H_B = 0$

$\Sigma M_A = 0 = \_\_\_ \Rightarrow V_B = \frac{(1.5)(3)\left[\frac{2}{3}(3)\right]}{} = 1.5 \text{ KN}$

$\Sigma M_B = 0 = \_\_\_ \Rightarrow R_A = \frac{(1.5)(3)^3\left[\frac{1}{3}(3)\right]}{3} = 0.75 \text{ KN}$

check: $\Sigma F\uparrow = 0.75 + 1.5 - \frac{1}{2}(1.5)(3) = 0$

$w(x) = ax + b = \frac{1.5}{3}x = \frac{x}{2}$, $V = \int -w(x)\,dx$

$= \int -\left(\frac{x}{2}\right)dx$

$= -\frac{x^2}{4} + C_1$

at $x = 0 \Rightarrow V = 0.75 = C_1$

$M = \int V\,dx$

$= \int\left(-\frac{x^2}{4} + 0.75\right)dx$

$M = -\frac{x^3}{12} + 0.75x + \cancel{C_2}^{\,0}$

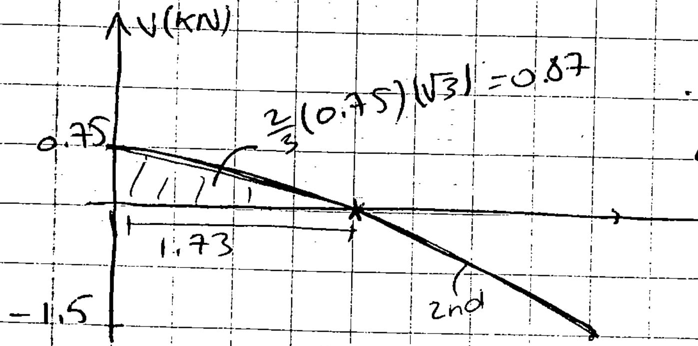
at $x = 0 \Rightarrow M = 0 = C_2$

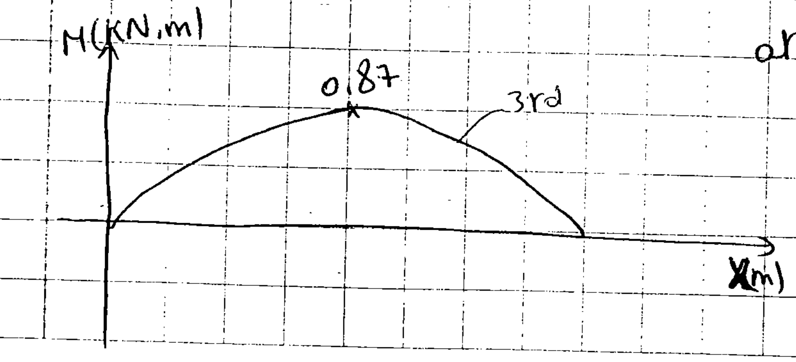
$M(x=3) = -\frac{3^3}{12} + (0.75)$ =

$V = 0 \Rightarrow -\frac{x^2}{4} + 0.75 = 0$

$\Rightarrow x = \sqrt{(0.75)(4)} = \sqrt{3}$

$= 1.73\text{m}$





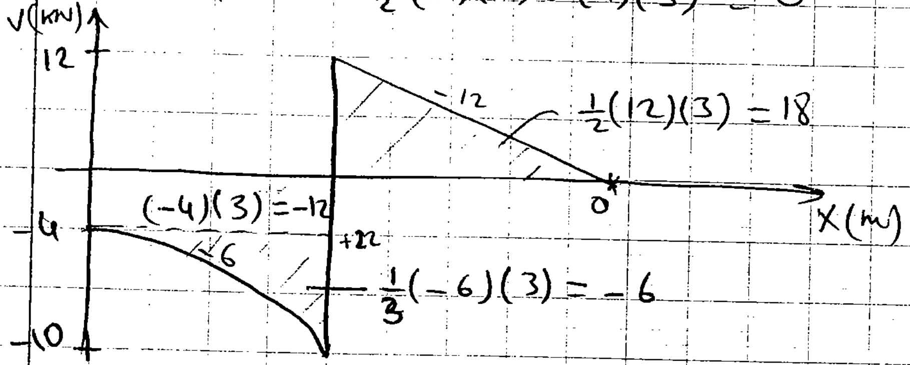
7-57) $H_A = 0$

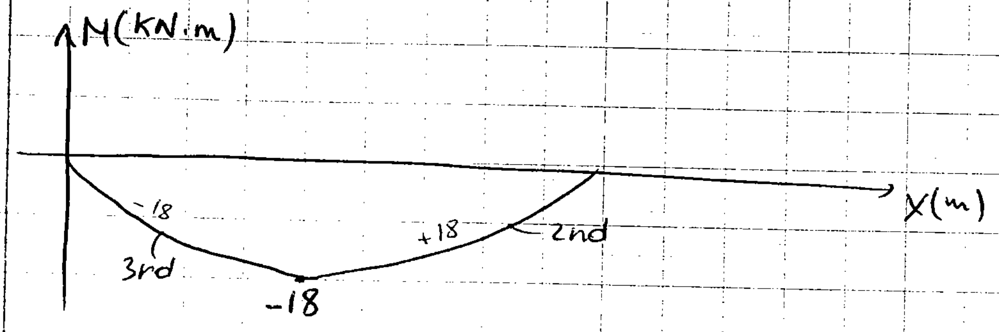
$\Sigma M_A = 0 = \longrightarrow R_B = \dfrac{[\frac{1}{2}(3)(4)][\frac{2}{3}(3)] + [(4)(3)][3+\frac{3}{2}}{3}$

$= 22 kN$

$\Sigma M_B = 0 = \longrightarrow V_A = \dfrac{[\frac{1}{2}(4)(3)][\frac{1}{3}(3)] - (4)(3)(\frac{3}{2})}{3} = -$

check $\Sigma F\uparrow = -4 + 22 - \frac{1}{2}(3)(4) - (4)(3) = 0$

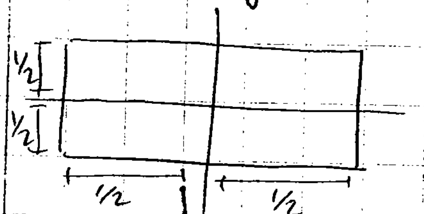


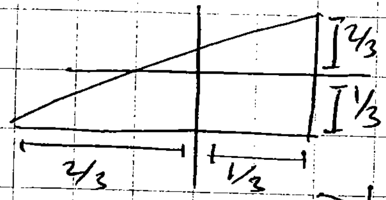


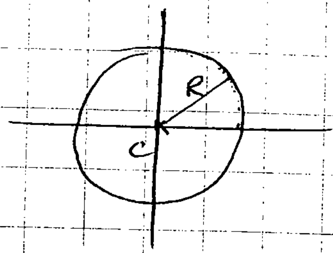
Hw: 7-10/7-16/7-24/7-50/7-55

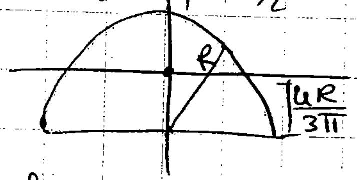
9-10 - Centroid and moment of inertia

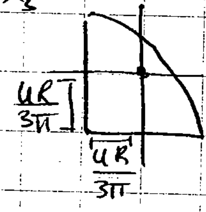
centroid = geometric center of a body.

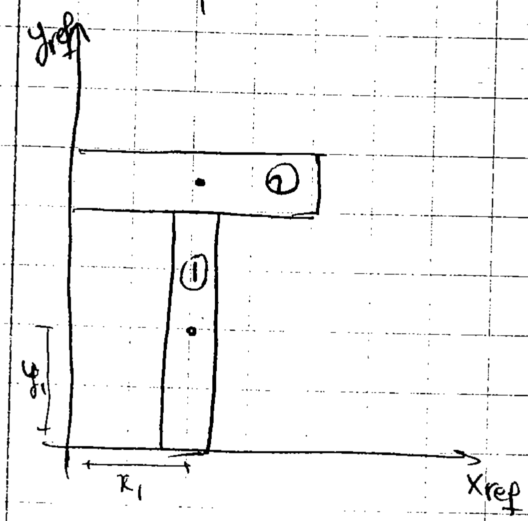












$Q_y = \Sigma y_i A_i$ = Moments of area

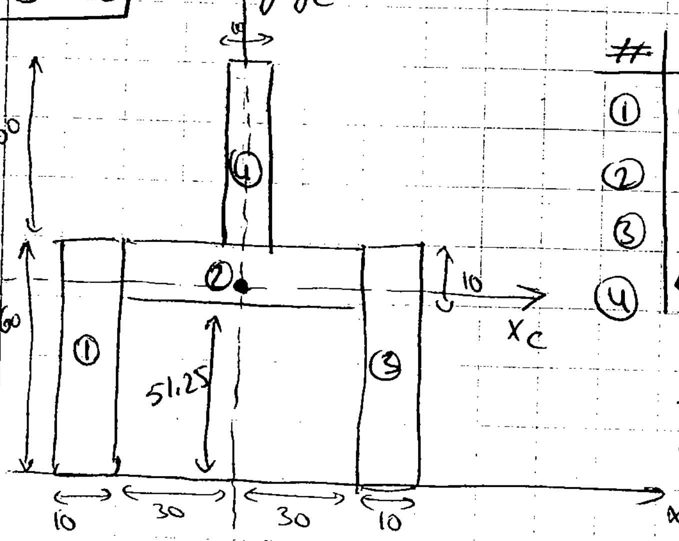
$\bar{y} = y_c$ = location of y centroidal axis wrt REF X

$$= \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2}$$

$\bar{X} = X_c$ = location of x centroidal axis wrt Ref y

$$= \frac{\Sigma X_i A_i}{\Sigma A_i} = \frac{x_1 A_1 + X_2 A_2}{A_1 + A_2}$$

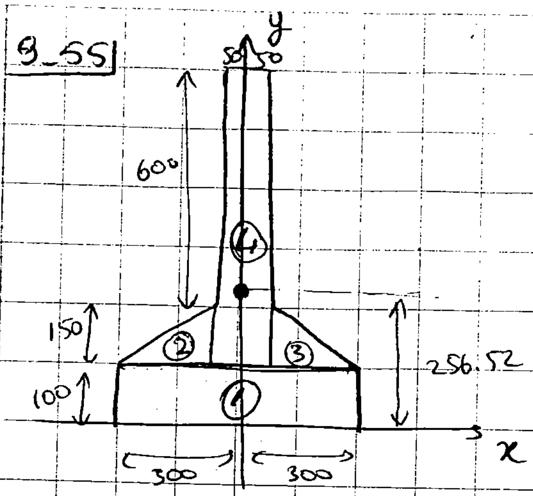
9-53



#	y_i	A_i	$y_i A_i$
①	$\frac{60}{2} = 30$	(10)(60)	(30)(10)(60)
②	$60 - \frac{10}{2} = 55$	(60)(10)	(55)(60)(10)
③	$\frac{60}{2} = 30$	(10)(60)	(30)(10)(60)
④	$60 + \frac{60}{2} = 90$	(60)(10)	(90)(60)(10)
		2400	123000

$$\bar{y} = \frac{123000}{2400} = 51.25$$

9_55



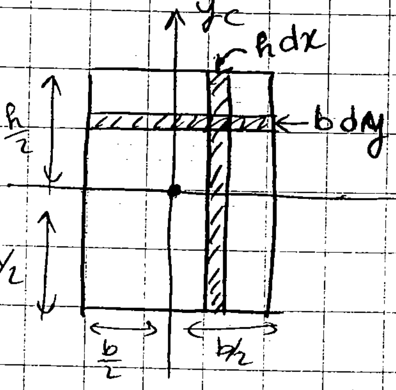
#	y_i	A_i	$y_i A_i$
①	$\frac{100}{2} = 50$	$(600)(100)$	$(50)(600)(100)$
②	$100 + \frac{150}{3} = 150$	$\frac{1}{2}(150)(250)$	$(150)\left[\frac{1}{2}(150)(250)\right]$
③	$100 + \frac{150}{3} = 150$	$\frac{1}{2}(150)(250)$	$(150)\left[\frac{1}{2}(150)(250)\right]$
④	$100 + \left(\frac{600+150}{2}\right) = 475$	$(600+150)(100)$	$(475)(750)(100)$
		$172\,500$	$4425 \cdot 10^6$

$$\bar{y} = \frac{4425 \cdot 10^6}{172\,500} = 256.52 \text{ mm}$$

Moment of inertia (second moment of area)

$I_x = \int y^2 \, dA$ = moment inertia about x-axis.

$I_y = \int x^2 \, dA$ = " " " y-axis.



$$I_{x_c} = \int_A y^2 \, dA = \int_{-h/2}^{h/2} b y^2 \, dy = b \frac{y^3}{3} \Big|_{-h/2}^{h/2}$$

$$= \frac{b}{3}\left[\frac{h^3}{8} - \left(-\frac{h^3}{8}\right)\right] = \frac{bh^3}{12}$$

$$I_{y_c} = \int_{-b/2}^{b/2} h x^2 \, dx = \frac{h}{3}\left[x^3\right]_{-b/2}^{b/2} = \frac{hb}{12}$$

$$* \; I_{Axis} = \frac{(\text{side } \parallel)(\text{side } \perp)^3}{12}$$

∥ Axis theorem

$I_{x_c} = \Sigma\left[I_{x_{0i}} + A_i (\Delta y_i)^2\right]$ where I_{x_c} = moment of inertia about x centroidal Axis.

$I_{x_{0i}}$ = moment of inertia about area A_i own x center.

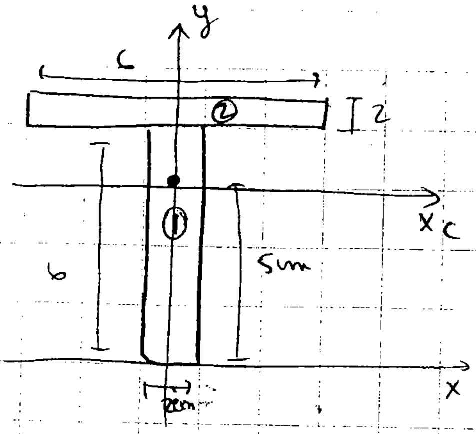
A_i = Area i

$\Delta y_i = |\bar{y} - y_i| = |y_c - y_i|$

$I_{y_c} = \Sigma\left[I_{y_{0i}} + A_i (\Delta x_i)^2\right]$, $\Delta x_i = |\bar{x} - x_i| = |x_c - x_i|$

Ch 5.1

ex)



#	y_i	A_i	y_iA_i
①	$6/2 = 3$	$(6)(2)$	$(3)(6)(2)$
②	$6+\frac{2}{2}=7$	$(6)(2)$	$(7)(6)(2)$
		24	120

$$\bar{y} = \frac{120}{24} = 5 \text{ cm}.$$

#	I_{ox_i}	Δy_i	$A_i(\Delta y_i)^2$
①	$\frac{1}{12}(2)(6)^3$	$\lvert 5-3 \rvert = 2$	$(2)^2[(6)(2)]$
②	$\frac{1}{12}(6)(2)^3$	$\lvert 5-7 \rvert = 2$	$(2)^2[(6)(2)]$
	40 cm^4		96 cm^4

$$I_{x_c} = 40 + 96 = 136 \text{ cm}^4$$

10_32/10_33 : locate the centroide and determine the moment of inertia about the centroidal axis.

#	X_i	y_i	A_i	X_iA_i	y_iA_i
①	$\frac{2}{3}(300) = 200$	$\frac{1}{3}(200) = \frac{200}{3}$	$\frac{1}{2}(300)(200)$	$(200)[\frac{1}{2}(300)(200)]$	$(\frac{200}{3})[\frac{1}{2}(300)(200)]$
②	$300+150=450$	100	$(300)(200)$	$(450)[(300)(200)]$	$(100)[(300)(200)]$
-③	450	100	$-\pi(75)^2$	$(450)[-\pi(75)^2]$	$(100)[-\pi(75)^2]$
			72 328.54	25 047 843.60	6 232 854.13

$$\bar{X} = \frac{\Sigma X_iA_i}{\Sigma A_i} = \frac{25\,047\,843.60}{72\,328.54} = 346.31 \text{ mm}.$$

$$\bar{Y} = \frac{\Sigma y_iA_i}{\Sigma A_i} = \frac{6\,232\,854.13}{72\,328.54} = 86.17 \text{ mm}.$$

	I_{ox_i}	Δy_i	$A_i(\Delta y_i)^2$	I_{oy_i}	Δx_i	$A_i(\Delta x_i)^2$
①	$\frac{1}{36}(300)(200)^3$	$\lvert 86.17-\frac{200}{3} \rvert = 19.50$	$[\frac{1}{2}(300)(200)](19.50)^2$	$\frac{1}{36}(200)(300)^3$	$\lvert 346.31-200 \rvert = 146.31$	$[\frac{1}{2}(300)(200)](146.31)^2$
①	$\frac{1}{12}(300)(200)^3$	$\lvert 86.17-100 \rvert = 13.83$	$[(300)(200)](13.83)^2$	$\frac{1}{12}(200)(300)^3$	$\lvert 346.3-450 \rvert = 103.69$	$[(300)(200)](103.69)^2$
-③	$-\frac{\pi}{4}(75)^4$	$\lvert 86.17-100 \rvert = 13.83$	$[-\pi(75)^4](13.83)^2$	$-\frac{\pi}{4}(75)^4$	$\lvert 450-346.31 \rvert = 103.69$	$[-\pi(75)^2](103.69)^2$
	241,816,177.90 $241.82 \times 10^6 \text{ mm}^4$		$19.51 \times 10^6 \text{ mm}^4$	575.15×10^6		$1,097.30 \times 10^6$

$$I_{x_c} = \Sigma[I_{ox_i} + A_i(\Delta y_i)^2] = 241.82 \times 10^6 + 19.51 \times 10^6 = 261.33 \times 10^6 \text{ mm}^4$$

$I_{yc} = 575.15 \times 10^6 + 1{,}097.30 \times 10^6 = 1672.45 \times 10^6 \ mm^4$.

some questions for HWs ← **10_39 / 10_40** (moment of inertia about x, y centroidal axis)

$\bar{X} = 0$

#	y_i	A_i	$y_i A_i$	I_{ox_i}	Δy_i	$A_i (\Delta y_i)^2$
1	$\frac{100}{2} = 50$	$(100)(200)$	$(50)(100)(200)$	$\frac{1}{12}(200)(100)^3$	120	$[(100)(200)](120)^2$
2	$100 + \frac{300}{2} = 250$	$(100)(300)$	$(250)(100)(300)$	$\frac{1}{12}(100)(300)^3$	80	$[(100)(300)](80)^2$
		$50{,}000$	$8{,}500{,}000$	$241.67 \cdot 10^6$		480×10^6

$$\bar{y} = \frac{8{,}500{,}000}{50{,}000} = 170$$

$$I_{xc} = 241.67 \times 10^6 + 480 \times 10^6 = 721.67 \times 10^6 \ mm^4$$

#	I_{oy_i}	Δx_i	$A_i (\Delta x_i)^2$
1	$\frac{1}{12}(100)(200)^3$	0	0
2	$\frac{1}{12}(300)(100)^3$	0	0
	$91.67 \cdot 10^6 \ mm^4$		0

$I_{yc} = 91.67 \cdot 10^6 \ mm^4$.

Hw (10_27) (10_43, 10_44) (10_49, 10_50)

10_47 $\bar{X} = 0$

#	y_i	A_i	$y_i A_i$	I_{ox_i}	Δy_i	$A_i (\Delta y_i)^2$
1	$\frac{450}{2} = 225$	$(300)(450)$	$(225)[(300)(450)]$	$\frac{1}{12}(300)(450)^3$	108.71	$(300)(450)(108.71)^2$
−2	$50 + \frac{350}{2} = 225$	$-(200)(350)$	$(225)[-(200)(350)]$	$-\frac{1}{12}(200)(350)^3$	108.71	$-(200)(350)(108.71)^2$
3	$450 + \frac{1}{3}(240) = 530$	$\frac{1}{2}(300)(240)$	$(530)[\frac{1}{2}(300)(240)]$	$\frac{1}{36}(300)(240)^3$	196.29	$\frac{1}{2}(300)(240)(196.29)^2$
		101×10^3	33.71×10^5	$1{,}678.74 \times 10^6$		215.5×10^6

$$\bar{y} = \frac{33.71 \times 10^5}{101 \times 10^3} = 333.71 \ mm$$

$$I_{xc} = 1{,}678.74 \times 10^6 + 215.5 \times 10^6 = 1{,}894.24 \times 10^6 \ mm^4$$

#	I_{oy_i}	Δx_i	$A_i (\Delta x_i)^2$
1	$\frac{1}{12}(450)(300)^3$	0	0
−2	$-\frac{1}{12}(350)(200)^3$	0	0
3	$\frac{1}{36}(240)(300)^3$	0	0
	$959.17 \cdot 10^6$		

$I_{yc} = 959.17 \times 10^6 \ mm^4$.

10_53 $\bar{X} = 0$

#	y_i	A_i	$y_i A_i$	I_{ox_i}	Δy_i	$A_i(\Delta y_i)^2$
①	$\frac{65}{2} = 32.5$	$(140)(65)$	$(32.5)(140)(65)$	$\frac{1}{12}(140)(65)^3$	15	$(140)(65)(15)^2$
②	$\frac{60}{2} = 30$	$-(60)(130)$	$-(30)(60)(130)$	$-\frac{1}{12}(130)(60)^3$	17.5	$-(130)(60)(17.5)^2$
		1 300	61,750	$\cancel{2.66\times10^6}$ 8.64×10^5		$-341,250$

$$\bar{y} = \frac{61,750}{1,300} = 47.5 \text{ mm}$$

$$I_{x_c} = \cancel{2.66\times10^6}\; 8.64\times10^5 - 341,250 = 522,75 \text{ mm}^4$$

~~or~~

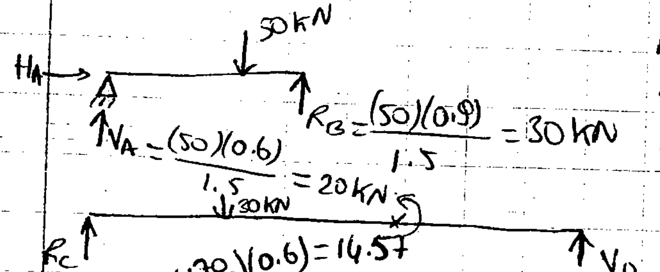
#	y_i	A_i	$y_i A_i$
①	$\frac{65}{2} = 32.5$	$(65)(5)$	$(32.5)(65)(5)$
②	$\frac{65}{2} = 32.5$	$(65)(5)$	$(32.5)(65)(5)$
③	$60 + \frac{5}{2} = 62.5$	$(5)(130)$	$(62.5)(5)(130)$

I_{oy_i}	Δx_i	$A_i(\Delta x_i)^2$
$\frac{1}{12}(65)(140)^3$	0	0
$-\frac{1}{12}(60)(130)^3$	0	0
$3.88\times10^6 \text{ mm}^4$		0

$$I_{y_c} = 3.88 \times 10^6 \text{ mm}^4$$

Review problems

7_115



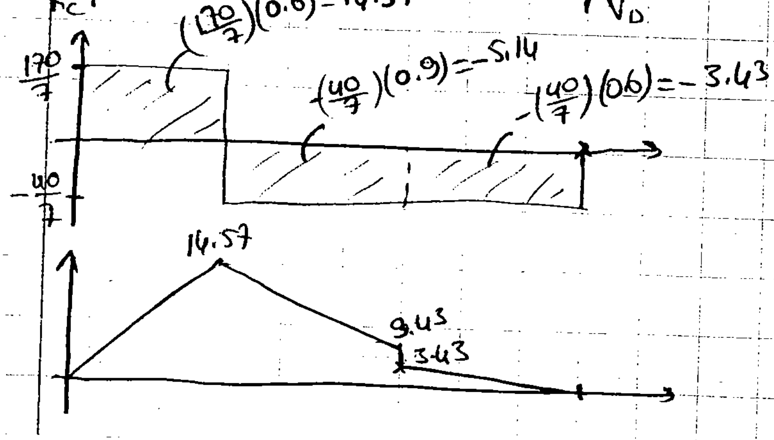
$$N_A = \frac{(50)(0.6)}{1.5} = 20 \text{ kN}$$

$$R_B = \frac{(50)(0.9)}{1.5} = 30 \text{ kN}$$

$$R_C = \frac{(30)(1.5)+6}{2.1} = \frac{170}{7} = 24.29$$

$$V_D = \frac{(30)(0.6)-6}{2.1} = 5.71 = \frac{40}{7}$$

$$H_D = 0$$

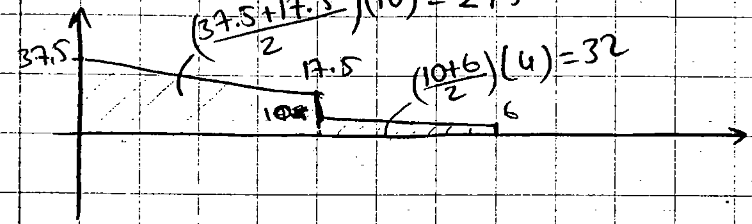


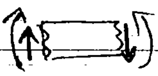
7-116

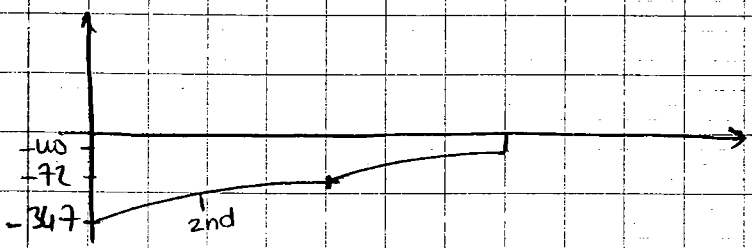
$V = (10)(2) + 7.5 + (4)(1) + 6 = 37.5\,kN$

$H = 0$

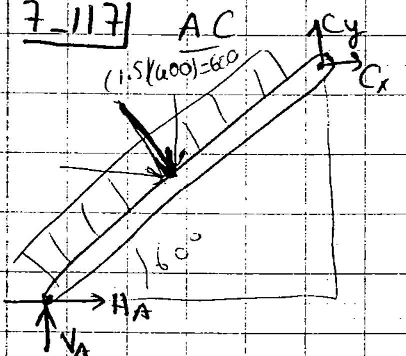
$M_R = (2)(10)\left(\frac{10}{2}\right) + (7.5)(10) + (1)(4)\left(\frac{4}{2}+10\right) + (6)(10+4) + 40 = 347\,kN.m$







7-117 AC



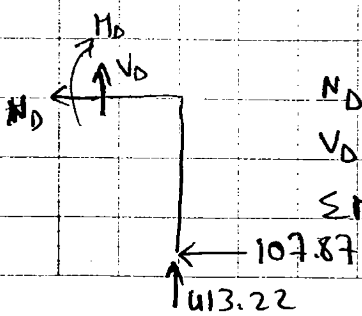
$\Sigma M_C = 0 = (400)(1.5)\left(\frac{1.5}{2}\right) - (V_A)(1.5\cos60) + (H_A)(1.5\sin60)$

whole structure: $\Sigma M_B = 0 = (H_A)(1.5\sin60 - 1) - (V_A)(1+1.5\cos60) - (600\cos60)(1+0.75\cos60)$
$- (600\sin60)[0.75\sin60 - (1.5\sin60 - 1)]$

$\Rightarrow 0.299 H_A - 1.75 V_A = \_\_\_\_\_$ ①

$V_A = -113.22\,N$, $H_B = 411.78 - 600\sin60 = -107.84$.

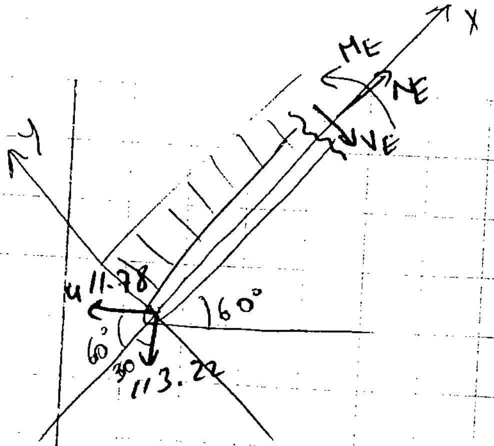
$H_A = -411.78\,N$, $V_B = 113.22 + 600\cos60 = 413.22\,N$



$N_D = -107.84\,N$

$V_D = -413.22\,N$

$\Sigma M_D = 0 = (413.22)(0.75) - (107.84)(1) - M_D \Rightarrow M_D = 202.08\,N.m$



$\Sigma \vec{F_x} = 0 = N_E - 113.22\cos 30 - 411.78\cos 60$

$\Rightarrow N_E = 303.94\ N$

$\Sigma F_y = 0 = -V_E - 113.22\sin 30 + 411.78\sin 60 - (40$

$\Rightarrow V_E = 0$

$\Sigma M_E = 0 = -M_E - (400)(0.75)(\frac{0.75}{2}) + (411.78\sin 60)(0.75) - (113.72\sin 30)(0.$

$\Rightarrow M_E = 337.50\ N.m$

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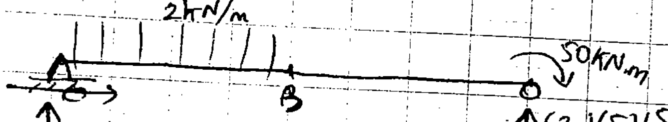

$V_A = \frac{(2)(5)(\frac{5}{2}+5) - 50}{10} = 2.5kN$

$R_C = \frac{(2)(5)(\frac{5}{2})+50}{10} = 7.5kN$