Lebanese American University
Department of Civil Engineering
STATICS – CIE 200 - Beirut
TEST 1 – Fall 2011

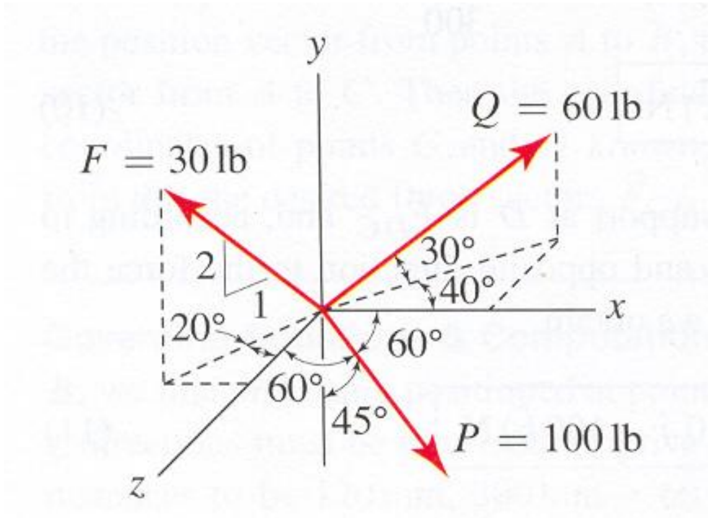
Date: ***November 4, 2011, 06:00 p.m.***
Duration: ***90 minutes***

Name	***SOLUTION***
ID #	

NOTES	***Show all calculations, and indicate the proper units***
	All problem solutions must include an FBD
	Closed book and notes
	Assume any missing information that is necessary
	Questions have weights as indicated
	Do not unstaple the exam booklet
	Exam booklet consists of 11 pages

Problem I (25%)

Determine the resultant force **R** of this system of forces in Cartesian form and compute the coordinate direction angles that **R** forms with the x, y, and z axes.



$\xrightarrow{\oplus}$ $R_x = \Sigma F_x = F_x + Q_x + P_x = -\frac{1}{\sqrt{5}} \times 30 \sin 20 + 60 \cos 30 \cos 40 + 100 \cos 60$

$= 85.21$ lb

$\overset{\oplus}{\uparrow}$ $R_y = \Sigma F_y = F_y + Q_y + P_y = 30 \times \frac{2}{\sqrt{5}} + 60 \sin 30 - 100 \cos 45$

$= -13.877$ lb

$\oplus\swarrow$ $R_z = \Sigma F_z = F_z + Q_z + P_z = \frac{1}{\sqrt{5}} \times 30 \cos 20 - 60 \cos 30 \sin 40 + 100 \cos 60$

$= 29.2084$ lb

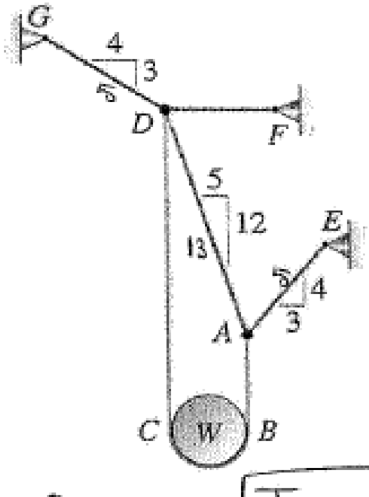
$\cos\alpha = \frac{R_x}{R} = \frac{85.21}{\sqrt{(85.21)^2 + (-13.877)^2 + (29.2084)^2}} \Rightarrow \boxed{\alpha = 20.78^\circ}$

$\cos\beta = \frac{R_y}{R} \Rightarrow \boxed{\beta = 98.76^\circ}$

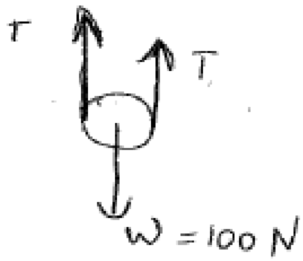
$\cos\gamma = \frac{R_z}{R} \Rightarrow \boxed{\gamma = 71.31^\circ}$

Problem II (25%)

Determine the tension force inside each of the five cables (GD, FD, AD, AE, and ABCD) of the system shown below such that the system remains in equilibrium. W is equal to 100 N. (Show FBDs)

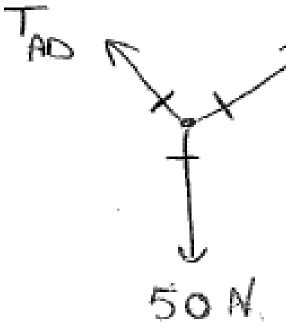


FBD @ W:



$$\overset{+}{\uparrow} \Sigma F_y = 2T - 100 = 0 \Longrightarrow \boxed{T = 50\,N}$$

FBD @ A:



$$\overset{+}{\rightarrow} \Sigma F_x = T_{AE} \times \frac{3}{5} - T_{AD} \times \frac{5}{13} = 0 \quad (1)$$

$$\overset{+}{\uparrow} \Sigma F_y = T_{AE} \times \frac{4}{5} + T_{AD} \times \frac{12}{13} - 50 = 0 \quad (2)$$

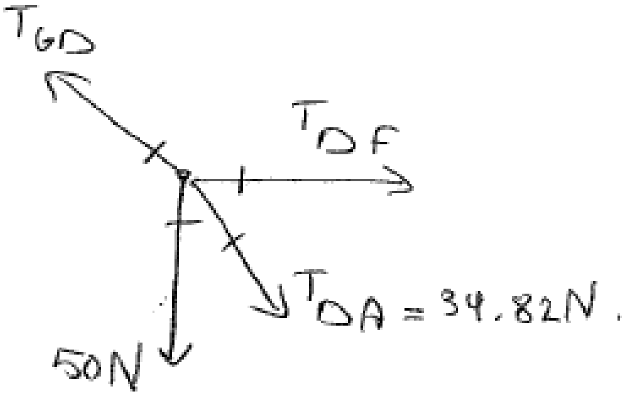
$$(1) \xRightarrow{\times 12} \frac{36}{5} T_{AE} - \cancel{\frac{12}{13} \times 5}\, T_{AD} = 0$$

$$(2) \xRightarrow{\times 5} 4\, T_{AE} + \cancel{5 \times \frac{12}{13}} \times T_{AD} = 250$$

$$(1) + (2) \Longrightarrow 11.2\, T_{AE} = 250 \Longrightarrow \boxed{T_{AE} = 22.32\,N}$$

$$\boxed{T_{AD} = \frac{13}{5} \times \frac{3}{5} \times T_{AE} = 34.82\,N}$$

FBD @ D:



$\overset{+}{\rightarrow} \Sigma F_x = T_{DF} + T_{DA} \times \frac{5}{13} - T_{GD} \times \frac{4}{5} = 0$

$+\uparrow \Sigma F_y = T_{GD} \times \frac{3}{5} - T_{DA} \times \frac{12}{13} - 50 = 0$

$\Rightarrow \boxed{T_{GD} = \frac{5}{3}\left(\frac{12}{13} \times 34.82 + 50\right) = 136.9\ N}$

$T_{DF} = \frac{4}{5} \times 136.9 - \frac{5}{13} \times 34.82$

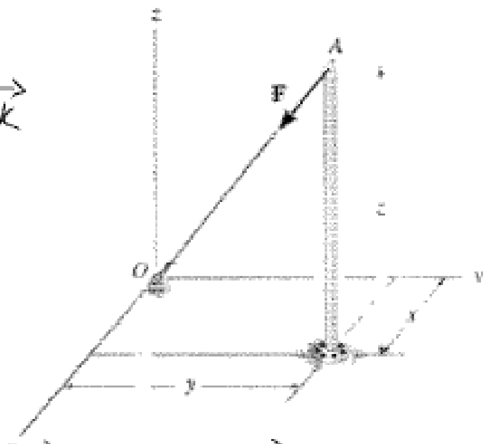
$= 109.52 - 13.39$

$\boxed{T_{DF} = 96.13\ N}$

Problem III (20%)

The cable AO exerts a force on the top of the pole of **F= {-60i-45j-40k} N**. If the cable has a length of 68m, determine the height **z** of the pole and the location **(x, y)** of its base.

$$\vec{F} = -60\vec{i} - 45\vec{j} - 40\vec{k}$$



$A(x, y, z)$

$$\vec{F} = F \cdot \vec{u}_{AO}$$

$$\vec{u}_{AO} = \frac{\vec{r}_{AO}}{r_{AO}} = \frac{-x\vec{i} - y\vec{j} - z\vec{k}}{68}$$

$$F = \sqrt{60^2 + 45^2 + 40^2} = 85\,N.$$

$$\vec{F} = 85\left(-\frac{x}{68}\right)\vec{i} - \frac{85}{68}y\vec{j} - \frac{85}{68}z\vec{k}$$

$$= -60\vec{i} - 45\vec{j} - 40\vec{k}$$

$$\frac{85}{68}x = 60 \Rightarrow \boxed{x = 48\,m.}$$

$$\frac{85}{68}y = 45 \Rightarrow \boxed{y = 36\,m.}$$

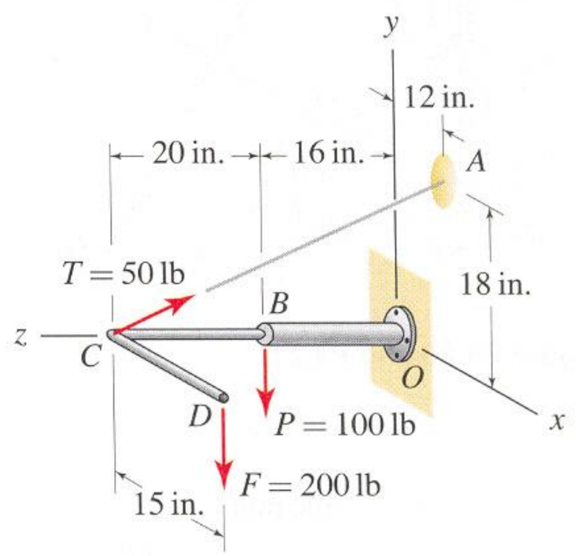
Page 5 of 10

$$\frac{85}{68}z = 40 \Rightarrow \boxed{z = 32\,m.}$$

Problem IV (30%)

Structure OBCD is built in at point O and supports a 50 lb cable force at point C and 100 and 200 lb vertical forces at points B and D, respectively.

1) Determine the resultant moment of these forces around point O in Cartesian form;
2) Determine the magnitude of the component of this moment around OA axis.



A (12, 18, 0)
C (0, 0, 36)
B (0, 0, 16)
D (15, 0, 36)

$$\vec{M}_R = \vec{M}_P + \vec{M}_F + \vec{M}_T$$

$$= \vec{r}_{OB} \times \vec{P} + \vec{r}_{OD} \times \vec{F}$$

$$+ \vec{r}_{OC} \times \vec{T}$$

$$\vec{T} = T.\,\vec{u}_{CA} = 50\left(\frac{12\vec{i} + 18\vec{j} - 36\vec{k}}{\sqrt{(12)^2 + (18)^2 + (-36)^2}}\right) = 14.28\vec{i} + 21.43\vec{j} - 42.86\vec{k}$$

$$\vec{P} = -100\,\vec{j}$$

$$\vec{F} = -200\,\vec{j}$$

$$\vec{r}_{OB} = 16\,\vec{k}\ ,\quad \vec{r}_{OD} = 15\vec{i} + 36\vec{k}\ ,\quad \vec{r}_{OC} = 36\,\vec{k}$$

$$\vec{M}_R = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 16 \\ 0 & -100 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 15 & 0 & 36 \\ 0 & -200 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 36 \\ 14.28 & 21.43 & 42.86 \end{vmatrix}$$

$$\vec{M}_R = +1600\vec{i} + 7200\vec{i} - 3000\vec{k} - 771.48\vec{i} + 514.08\vec{j}$$

$$= 8028.52\,\vec{i} + 514.08\,\vec{j} - 3000\,\vec{k}$$

$$M_R = \sqrt{(8028.52)^2 + (514.08)^2 + (-3000)^2}$$

$$= 8585.62 \text{ lb.in.}$$

2°) $$\vec{u}_{OA} = \frac{\vec{r}_{OA}}{r_{OA}} = \frac{12\vec{i} + 18\vec{j}}{\sqrt{12^2 + 18^2}} = 0.55\,\vec{i} + 0.83\,\vec{j}$$

$$\Rightarrow M_{OA} = \vec{u}_{OA} \cdot \vec{M}_R = 0.55 \times 8028.52 + 0.83 \times 514.08$$

$$= 4842.37 \text{ lb.in.}$$