Math 218 Quiz 2 Fall 2010

Name: ____________________

Student ID: ____________________

Instructions:

1. This exam has 5 pages. Please make sure you have all pages.
2. The point value of each problem occurs to the left of the problem.
3. **You must show correct work to receive credit.** Correct answers with inconsistent work or with no justification will not be given credit.
4. Only non-graphing and non-programmable calculators are allowed.
5. **Turn off and put away all cell phones.**

Page	Points	Points Possible
2		11
3		12
4		9
5		8
Total		40

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1. (4 pts) For which values of k do the vectors $\begin{bmatrix} -1 \\ 1 \\ k \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} k \\ 2 \\ 2 \end{bmatrix}$ form a basis for $\mathbb{R}^3$?

$$c_1\begin{bmatrix} -1 \\ 1 \\ K \end{bmatrix} + c_2\begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} + c_3\begin{bmatrix} K \\ 2 \\ 2 \end{bmatrix} = 0$$

$$\left[\begin{array}{ccc|c} -1 & 0 & K & 0 \\ 1 & 1 & 2 & 0 \\ K & 3 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} -1 & 0 & K & 0 \\ 0 & 1 & 2+K & 0 \\ 0 & 3 & 2+K^2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} -1 & 0 & K & 0 \\ 0 & 1 & 2+K & 0 \\ 0 & 0 & 2+K^2-6-6K & 0 \end{array}\right]$$

$2+K^2-6-6K \neq 0$

$K^2-6K-4 \neq 0$

$$\begin{vmatrix} -1 & 0 & K \\ 1 & 1 & 2 \\ K & 3 & 2 \end{vmatrix} = 0 + 1\begin{vmatrix} -1 & K \\ K & 2 \end{vmatrix} - 3\begin{vmatrix} -1 & K \\ 1 & 2 \end{vmatrix}$$

$= 1(-2-K^2) + 3(-2-K)$

$= -2-K^2+6+3K$

$= -4-K^2+3K$

2. Let $t : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 6x+3y \\ 4x+2y+z \\ 4z \end{bmatrix}$.

(a) (4 pts) Find a basis for $R(T)$.

$$\begin{bmatrix} 6 & 3 & 0 \\ 4 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\beta = \left\{ \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix} \right.$

(b) (3 pts) Is T an isomorphism?

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3. Let $T : \mathcal{P}_2 \to \mathcal{P}_1$ be given by $T(p(x)) = xp(1) + p(0)$.

(a) (4 pts) Show that T is a linear transformation.

$T\big(p(x) + c\,q(x)\big) = x\big(p(1) + c\,q(1)\big) + p(0) + q(0)$

$= x\,p(1) + c\,x\,q(1) + p(0) + q(0)$

$= x\,p(1) + p(0) + c\left[x\,q(1) + q(0)\right]$

$= T(p(x)) + c\,T(q(x))$.

(b) (4 pts) Find ker(T).

(c) (4 pts) Find $R(T)$.

$R(T) = \{ T(p(x)) : p(x) \in P_2 \}$

$= \{ x\,p(1) + p(0) : p(x) \in P_2 \}$

Let $p(x) = ax^2 + bx + c$

$= \{ x(a+b+c) + c : a, b, c \in \mathbb{R} \}$

$= \{ ax + bx + c(x+1) : a, b, c \in \mathbb{R} \}$

$= \{ x(a+b) + c(x+1) : a, b$

$N(T)$ is a subspace of V

$R(T)$ is a subspace of W

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4. (4 pts) Let $T : V \to W$ be a linear transformation. Determine whether $\{\mathbf{v} \in V \mid T(\mathbf{v}) \neq \mathbf{0}\}$ is a subspace of V.

$T(\alpha) = 0_W$

Then it is not a subspace of V

$cW_1 + W_2 = 2cu_1 + 3cv_1 + 2u_2 + 3v_2 = 2(cu_1 + u_2) + 3(cv_1 + v_2)$

$2cu_1 + 3cv_1 + 2u_2 + 3v_2$

$2(cu_1 + u_2) + 3(cv_1 + v_2)$

5. (5 pts) Suppose S and T are subspaces of a vector space V. Define $W = \{2\mathbf{u} + 3\mathbf{v} \mid \mathbf{u} \in S,\ \mathbf{v} \in T\}$. Determine whether W is a subspace of V.

S & $T \subset V$

$W = \{2u + 3v \mid u \in S, v \in T\}$

~~let~~ $w_1, w_2 \in V$

$cw_1 + w_2 = c(2u_1 + 3v_1) + 2u_2$

$W \neq \emptyset$ since if $u = 0$ and $v = 0$ then $0 \in W$

let $u_1, u_2 \in S$ and $v_1, v_2 \in T$ and $c \in \mathbb{R}$.

then $2u_1 + 3v_1 \in W$

and $2u_2 + 3v_2 \in W$

Prove: $c(2u_1 + 3v_1) + (2u_2 + 3v_2) \in W$

$= 2cu_1 + 3cv_1 + 2u_2 + 3cu_2$

$= c(u_1 + u_2) + 3c$

$c(2u_1 + 3v_1) + (2u_2 + 3v_2) = 2cu_1 + 3cv_1 + 2u_2 + 3v_2$

$= 2cu_1 + 2u_2 + 3cv_1 + 3v_2$

$= 2(cu_1 + u_2) + 3(cv_1 + v_2)$

~~hence~~ since $cu_1 + u_2 \in S$ and $cv_1 + v_2 \in T$

then ~~$2(cu_1 + u_2) + 3(cv_1 + v_2) \in W$ hence W is a su~~

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6. (3 pts) Find a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ and a linearly independent subset $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ of $\mathbb{R}^3$ such that $\{T(\mathbf{v_1}), T(\mathbf{v_2}), T(\mathbf{v_3})\}$ is linearly dependent.

Sub $\{e_1, e_2, e_3\}$ is a linea inde sub of $\mathbb{R}^3$

Suppose $T(v) = 0$, then $T(e_1) = T(e_2) = T(e_3) = 0$

so $\{0\} = \{T(v_1), T(v_2), T(v_3)\}$

ad $\{0\}$ is l.d $\{e_1, e_2, e_3\}$

7. (5 pts) Let $T : V \to W$ be a linear transformation and let $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ be a subset of V. Show that if T is one-to-one and $\{T(\mathbf{v_1}), T(\mathbf{v_2}), T(\mathbf{v_3})\}$ is linearly dependent then $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is also linearly dependent.

$N(T) = \{0\}$.

$c_1 T(v_1) + c_2 T(v_2) + c_3 T(v_3) = 0$

$T(c v_1 + c_2 v_2 + c_3 v_3) = 0$

~~but~~ hence $c_1 v_1 + c_2 \cdots \in N(T)$

but $N(T) = \{0\}$

we $c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

but c_1 or c_2 or $c_3 \neq 0$

so $\{v_1, v_2, v_3\}$