1. **(24%)** In each case, classify the differential equations below as separable, linear, or exact. In each case, solve the ode; in case of an IVP, find the particular solution satisfying the initial condition:

   a. $\left(\cos t \sin t - ty^2\right) + y\left(1 - t^2\right)\dfrac{dy}{dt} = 0$

b. $\frac{dy}{dt} = \frac{t}{y - t^2 y}; y(0) = -4$

c. $\frac{dy}{dt} + 2ty = f(t), y(0) = 2; f(t) = \begin{cases} t; 0 \le t < 1 \\ 0; t \ge 1 \end{cases}$

(Remark: The solution to this IVP should be **continuous**)

2. **(10%)** Find an integrating factor to the following ode that turns it into an exact one. **<u>Do not solve the ode</u>.**

$$6ty+\left(9t^2+4y\right)\frac{dy}{dt}=0$$

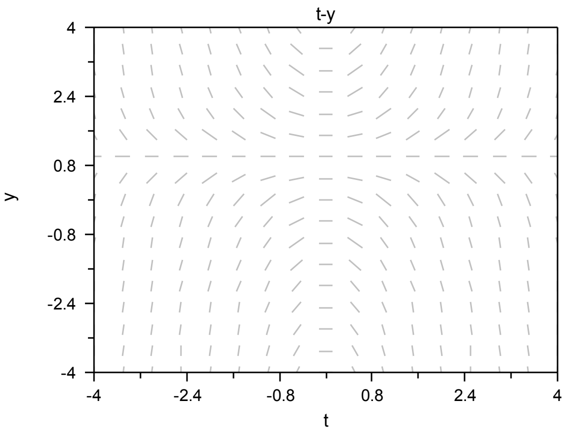
3. **(15%)** A 30-gallon tank initially contains 15 gallons of saltwater containing 6 pounds of salt. Suppose saltwater containing 1 pound of salt per gallon is pumped into the top of the tank at the rate of 2 gallons per minute, while the well-mixed solution leaves the bottom of the tank at the rate of 1 gallon per minute. How much salt is in the tank when the tank is full?

4. Consider the differential equation $\frac{dy}{dt} = -\frac{y}{t} + \frac{t-1}{2y}$. This equation is neither linear nor separable; however it can be turned into one or the other as follows:
   a. **(4%)** Define a new variable $u = y^2$. Find an expression relating $\frac{dy}{dt}$ and $\frac{du}{dt}$.

   b. **(6%)** Show that the given differential equation takes the new form $\frac{du}{dt} = -\frac{2}{t}u + (t-1)$.

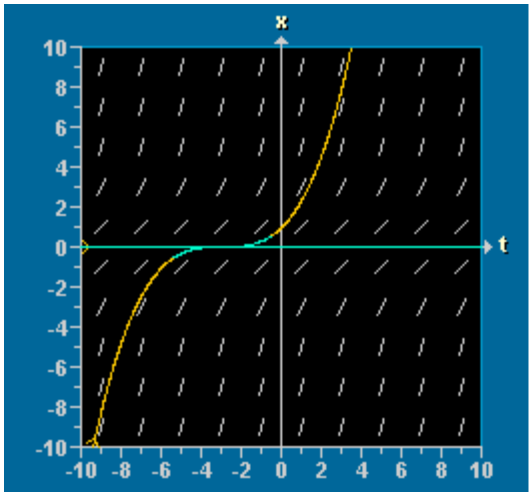
   c. **(6%)** Solve the new ode and conclude the family of solutions of the original one.

5. **(10%) Without solving,** verify that the direction field below corresponds to the ode $\frac{dy}{dt} = t(y-1)$. Also plot few solutions to this ode.



6. **(15%)**
   a. Do the phase line of the differential equation $\frac{dy}{dt} = \frac{(1-y)(2y+1)}{y^2}$
   b. Classify its equilibrium points.
   c. Sketch 4 solutions satisfying the following 4 initial conditions respectively: $y(0) = 2; y(0) = 0.5; y(0) = -0.75; y(0) = -1$.

7. **(10%)** Below are two intersecting solutions for the ode $\frac{dx}{dt} = x^{2/3}; x(0) = x_0$. Justify why the intersection at the point $(0, x_0)$ does not contradict the uniqueness theorem for initial value problems.



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