

1. Find the steady-state and the transient current in an RL-circuit where  $R = 30$  ohms,  $L = 10$  henrys,  $E = 10 e^{-t}$  Volts,  $I(0) = 0$ . (10 pts)
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2. Solve the following 1st order DE:

- a)  $(\cos x) y' - (\sin x) y = 1$  (10 pts)  
b)  $x y' = y + x \cos (y/x)$  (10 pts)
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3. Solve the following 2nd order DE:

- a)  $y'' + 2 y' + 5 y = 0$  (10 pts)  
b)  $y y'' + y'^2 = 0$  (10 pts)
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4. Find an integrating factor then solve the DE :

$$y^2 dx + (e^x - 2y) dy = 0. \quad (12 \text{ pts})$$

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5. The mass of a radioactive substance decreased from 4 grams to 1 gram in 10 days. Without computations,

- a) Find its half life.  
b) After how many days the mass becomes 0.25 grams. (10 pts)
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6. Consider the Riccati equation:  $x^2 y' + x y = y^2$

- a) Show that the equation can be reduced to a Bernoulli equation by applying the substitution  $z = y - 2x$   
b) Find the general solution. (15 pts)
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7. Consider the 1<sup>st</sup> order DE:  $4y' = 4 - x^2 - y^2$ . Draw the direction field. Include the isoclines corresponding to the following values of  $m$ :  $0$ ,  $1$ ,  $\frac{3}{4}$ ,  $-\frac{5}{4}$  and  $-3$ . (1 unit = 1 square). Sketch the solution passing through the  $O(0,0)$ . (13 pts)
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1. Find the charge and the current in an RC-circuit, where  $R = 4$  ohms  
 $C = 0.5$ farad,  $E = 12 e^{-t}$  volts, and  $I(0) = 0$ .
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(10 pts)

2. A cup of water of temperature  $5^\circ\text{C}$  is placed in a room of constant temperature  $30^\circ\text{C}$ . Its temperature doubled in 3 hours. How long will it take the water to reach  $20^\circ\text{C}$ .
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(10 pts)

3. Solve the following differential equations:

- a)  $(2xy^2 + y) dx + (x^2y - 1) dy = 0$  (10 pts)  
b)  $y = xy' + y'^2$ . Clairaut equation. Hint: Differentiate then solve. (8 pts)  
c)  $xy' = y + x \sec(y/x)$  (8 pts)  
d)  $y' + \frac{1}{x}y = y^2 - \frac{1}{x^2}$ . Riccati. Hint: Verify that  $y = \frac{1}{x}$  is a solution then apply a change of variables. (10 pts)
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4. A mass of 1 Kg is attached to the lower end of a spring with spring constant  $k$ . We push the mass 0.2 meter upward and release it without initial velocity. Find in terms of  $k$  the motion of the mass. Assume that the mass returns to its initial position after 3 seconds. Find the spring constant  $k$ .
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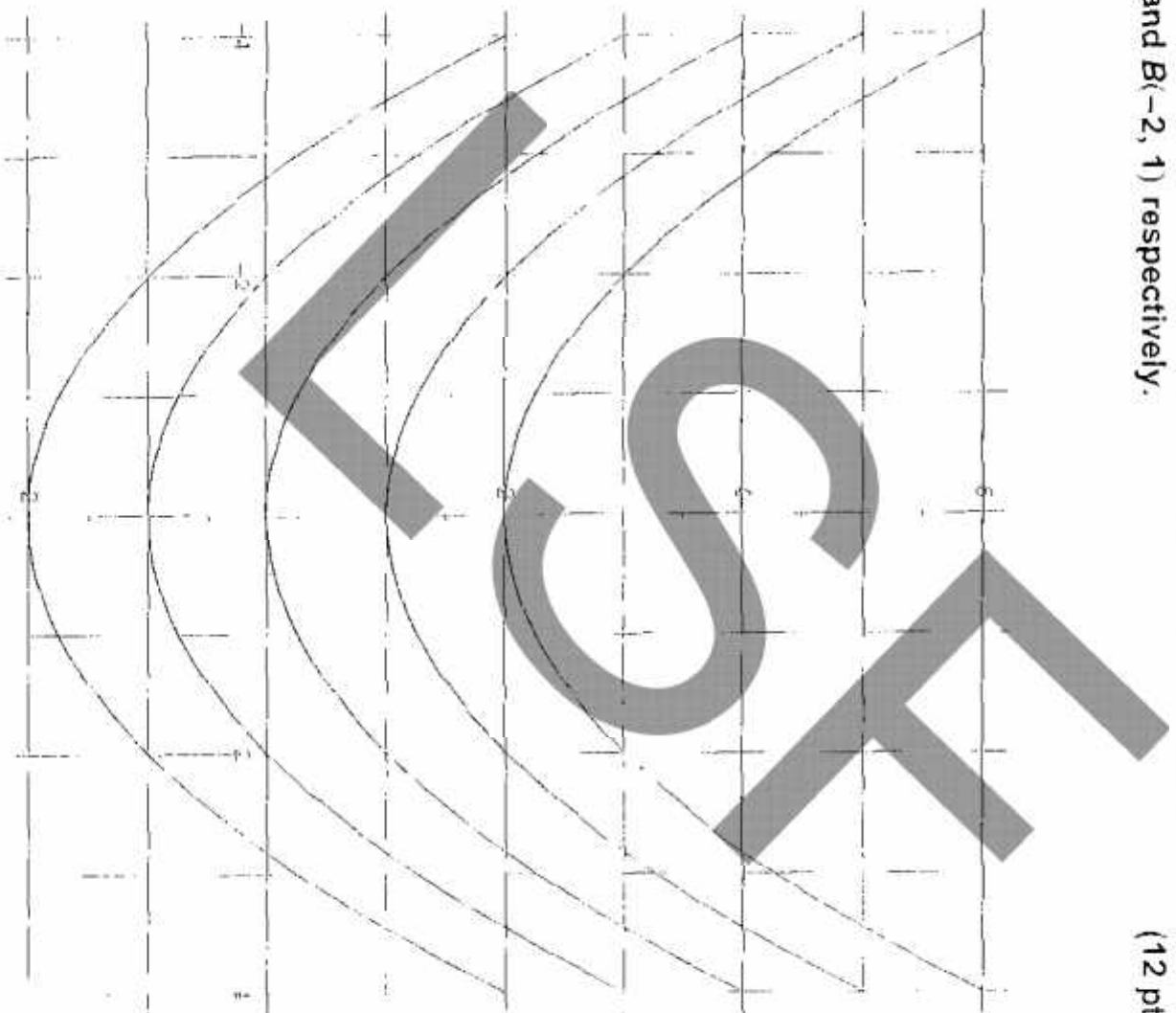
(10 pts)

5. Solve the following 2<sup>nd</sup> order differential equations:

- a)  $y'' + 3y' + 4y = 0$  (6 pts)  
b)  $y'' + 6y' + 9y = 0$  (6 pts)  
c)  $xy'' - 2y' + (x + \frac{2}{x})y = 0$ . (10 pts)  
Verify that  $y_1 = x \cos x$  is a solution then solve.
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Turn over

6. Consider the 2<sup>nd</sup> order DE:  $4y' + 4y = x^2$
- a) Find the region where the slope  $y'$  is positive.
  - b) Show that the isoclines are parabolas.
  - c) Draw the direction field. Include the isoclines corresponding to the following values of  $m$ :  $-2, -1, 0, 1$  and  $2$ .
  - d) Sketch 2 solutions passing through the points  $A(0,2)$  and  $B(-2, 1)$  respectively.
- (12 pts)



1. The mass of a radioactive substance at the time  $t = 1$  year is 5 grs. Its mass at the time  $t = 2$  years is 4 grs.
- Find the initial mass.
  - Find its half life.
- (10 pts)
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2. A small metal bar is dropped into a container of boiling water. Its temperature increased from  $20^{\circ}\text{C}$  to  $28^{\circ}\text{C}$  in 4 seconds. Find its temperature  $T(t)$ .
- (10 pts)
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3. Consider the 2<sup>nd</sup> order HLDE:  $x^2 y'' + 3x y' + y = 0$
- Verify that  $y_1 = \frac{1}{x}$  is a particular solution.
  - Find the general solution.
- (10 pts)
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4. Solve the following LDE:
- $x y' + y = \cos x$  (10 pts)
  - $y'' + (\tan x) y' = 0$  (10 pts)
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5. Solve the equation  $(x - y) dx + x dy = 0$ . (13 pts)
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6. Solve for  $z$  the equation  $\tan z = 2i$  (10 pts)
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7. Let  $z = -3 + 4i$ . Find  $e^z$ ,  $\log[z]$ ,  $\text{Log}[z]$  and the principal values of  $(-2)^z$  (15 pts)
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Turn Over

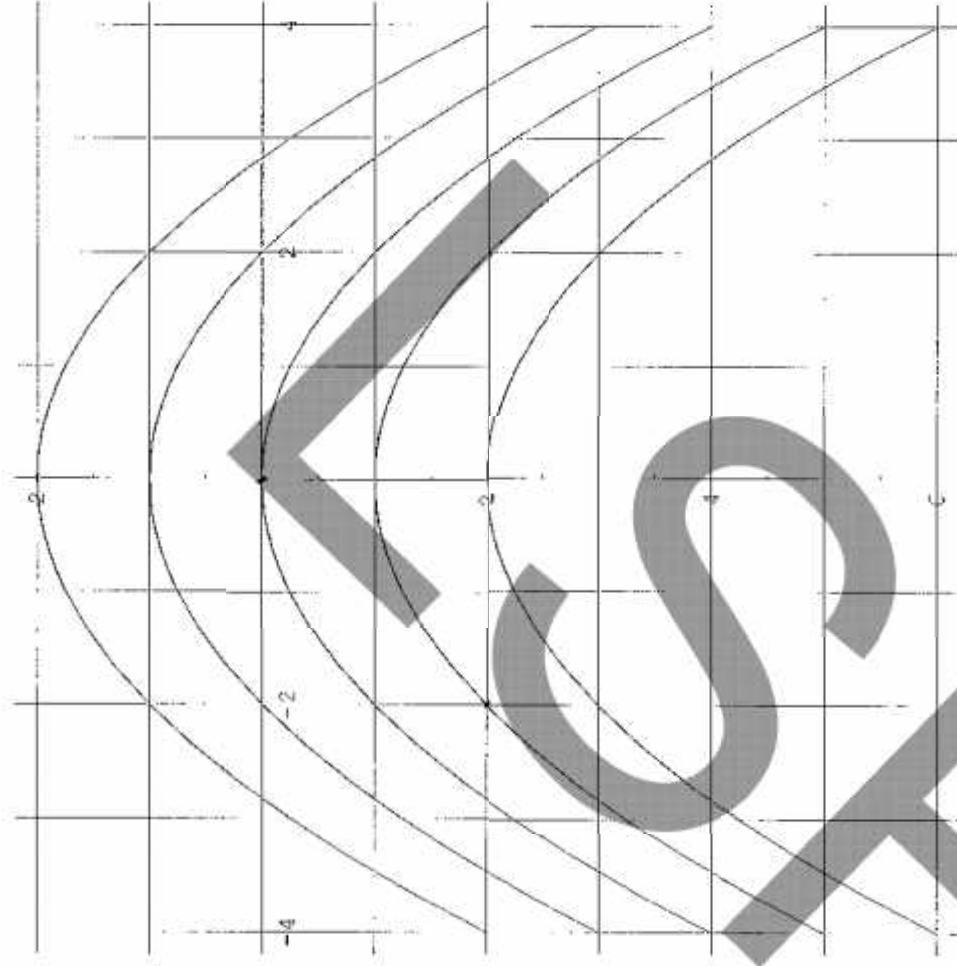
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8. Consider the 1<sup>st</sup> order DE:  $2y' = y + \frac{x^2}{4}$ .

b) Draw lineal elements on the isoclines  $y' = m$  corresponding to the following values of  $m$ :  $-1$ ,  $-0.5$ ,  $0$ ,  $0.5$  and  $1$ .

c) Sketch 2 solutions passing through the points  $A(2,-2)$  and  $B(0, 0)$  respectively.

(12 pts)



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