Lebanese American University

Byblos

Diff.Equations
Spring 2005
Test # 2
85 minutes

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1. Solve the following LDEC:
   a) $y'' - y = xe^{x}$ Undetermined Coefficients (15 pts)
   b) $y'' - 4y' + 4y = \frac{e^{2x}}{x^2}$ Variation of parameters (15 pts)

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2. Solve the Cauchy-Euler DE: $x^2 y'' - 4xy' + 4y = x^3$ (10 pts)

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3. Solve the HLDEC : $y^{(4)} - 16y = x^3 - 16$ (10 pts)

Formulas:

1) $\pounds\{y'\} = s\pounds\{y\} - y(0)$

2) $\pounds\{y''\} = s^2\pounds\{y\} - s\,y(0) - y'(0)$

3) $\pounds\{e^{at} f(t)\} = F(s-a)$

4) $\pounds\left\{\int_0^t f(x)\,dx\right\} = \frac{1}{s}\pounds\{f(t)\}$

5) $\pounds\{U(t-a)\,f(t)\} = e^{-as}\pounds\{f(t+a)\}$

6) $e^{-as}\pounds\{f(t)\} = \pounds\{U(t-a)\,f(t-a)\}$

7) $\pounds\{t\,f(t)\} = -F'(s)$

8) $\pounds\left\{\frac{f}{t}\right\} = \int_s^{+\infty} F(s)\,ds$

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4. Using differentiation or integration, find the inverse Laplace Transform of
   a) $F(s) = \tan^{-1} s$ b) $F(s) = \frac{2s+2}{(s^2+2s+5)^2}$ (15 pts)

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5. Using Laplace transforms, solve the initial value problems
   $y'' + y = \int_0^t y(t-u)\sin u\,du, \; y(0) = 0, \; y'(0) = 1$ (15 pts)

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6. Using Laplace transforms, find the current i(t) in an RL-circuit where $i(0) = 0$, $R = 2\,\Omega$, $L = 1$ henry and $E = \begin{cases} e^{-2t} & \text{if } t < 1 \\ 0 & \text{if } t > 1 \end{cases}$ (15 pts)

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7. Find the Laplace Transform of $f(t) = \frac{\sin t}{t}$ (5 pts)

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Lebanese American University — Byblos

Diff.Equations
Fall 2004
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85 minutes

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1. Find the current in an RLC-circuit, where $R = 2\ \Omega$, $L = 1$ Henry, $C = 1$ farad, $E = 4\,e^{-t}$ volts, Initial values: $I(0) = 1$ and $Q(0) = 1$. Use the method of undetermined coefficients. (15 pts)

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2. Solve the 10^{th} order HLDEC : $(D^2+4)^3\,(D^2-2)^2\,y = 0$ (7 pts)

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3. Solve the Cauchy-Euler DE: $x^3\,y''' + 2\,x^2\,y'' - 4\,x\,y' + 4\,y = x^3$ (13 pts)

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4. Solve the following LDE: $y'' - 2\,y' + 2\,y = \frac{e^x}{\cos x}$ (15 pts)

Formulas:

1) $\mathcal{L}\{y'\} = s\,\mathcal{L}\{y\} - y(0)$
2) $\mathcal{L}\{y''\} = s^2\,\mathcal{L}\{y\} - s\,y(0) - y'(0)$
3) $\mathcal{L}\{e^{at}\,f(t)\} = F(s-a)$
4) $\mathcal{L}\left\{\int_0^t f(x)\,dx\right\} = \frac{1}{s}\,\mathcal{L}\{f(t)\}$
5) $\mathcal{L}\{U(t-a)\,f(t)\} = e^{-as}\,\mathcal{L}\{f(t+a)\}$
6) $e^{-as}\,\mathcal{L}\{f(t)\} = \mathcal{L}\{U(t-a)\,f(t-a)\}$
7) $\mathcal{L}\{t\,f(t)\} = -F'(s)$
8) $\mathcal{L}\left\{\frac{f}{t}\right\} = \int_s^{+\infty} F(s)\,ds$

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5. Find the inverse Laplace Transform of

a) $F(s) = \frac{1}{s^2+4\,s+6}$ b) $F(s) = \frac{4}{s^4+4\,s^2}$ (10 pts)

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6. Using Laplace transforms, solve the initial value problems

a) $y'' + 3\,y' + 2\,y = U(t-2)$, $y(0) = 1$, $y'(0) = 0$ (15 pts)

b) $t\,y' + y = e^t$, $y(0) = 1$ (15 pts)

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7. Find the Laplace Transform of $f(t) = \begin{cases} \cos t & \text{if } 0 < t < \pi/2 \\ (t-\pi/2)^{1/2} & \text{if } t > \pi/2 \end{cases}$

Given: $\Gamma\left[\frac{1}{2}\right] = \sqrt{\pi}$. (10 pts)

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Diff.Equations
Spring 2004
Test # 2
80 minutes

Name:

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1. Solve the 7th order HLDEC : $(D+1)^3\,(D^2-2\,D+5)^2\,y=0$ (5 pts)

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2. Solve the differential equation: $y''+2\,y'+(1+a)\,y=0$

   Hint: The form of the solution depends on the values of a. (10 pts)

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3. Solve the following differential equations:

   a) $y^{(4)}-y''=e^x$ (10 pts)

   b) $x^2\,y''-x\,y'+y=\frac{1}{x}\ \ (x>0)$ (10 pts)

   c) $y''+y=\tan x$ (10 pts)

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4. A mass $m=0.5\,\text{kg}$ is attached to the lower end of a spring with spring constant $k=2$. If we compress the spring 0.10 m and release it without initial velocity.

   a) Find the position of the mass at the time t.

   b) Find the period and the frequency. (10 pts)

Turn Over

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Formulas:

1) $\pounds\{y'\} = s\,\pounds\{y\} - y(0)$

2) $\pounds\{y''\} = s^2\,\pounds\{y\} - s\,y(0) - y'(0)$

3) $\pounds\{e^{at} f(t)\} = F(s-a)$

4) $\pounds\left\{\int_0^t f(x)\,dx\right\} = \frac{1}{s}\,\pounds\{f(t)\}$

5) $\pounds\{U(t-a)\,f(t)\} = e^{-as}\,\pounds\{f(t+a)\}$

6) $e^{-as}\,\pounds\{f(t)\} = \pounds\{U(t-a)\,f(t-a)\}$

7) $\pounds\{t\,f(t)\} = -F'(s)$

8) $\pounds\left\{\frac{f}{t}\right\} = \int_s^{+\infty} F(s)\,ds$

In the following problems, use the Laplace transform method.

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5. Solve the initial value problem $\begin{cases} y'' - 4y = \delta(t-5) \\ y(0) = 1,\ y'(0) = 2 \end{cases}$ (10 pts)

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6. Find the charge q(t) and the current $i(t)$ in an LC-circuit,

   where $L = 2$ Henrys, $C = \frac{1}{2}$ farad, and $E = 3\sin(2t)$ volts.

   Assume that the initial current and charge are 0. (10 pts)

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7. Find the Laplace Transform of $f(t) = \begin{cases} \sin t & \text{if } 0 < t < \pi/2 \\ 0 & \text{if } \frac{\pi}{2} < t < \pi \\ t\sqrt{t-\pi} & \text{if } t > \pi \end{cases}$

   Given: $\Gamma\left[\frac{1}{2}\right] = \sqrt{\pi}$. (10 pts)

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8. Find the inverse Laplace Transform of $F(s) = \frac{2s}{(s^2+9)^2}$ (7 pts)

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9. Find the Laplace Transform of:

   a) $f(t) = \frac{\sin t}{t}$ b) $f(t) = \int_0^t \frac{\sin x}{x}\,dt$ (8 pts)

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