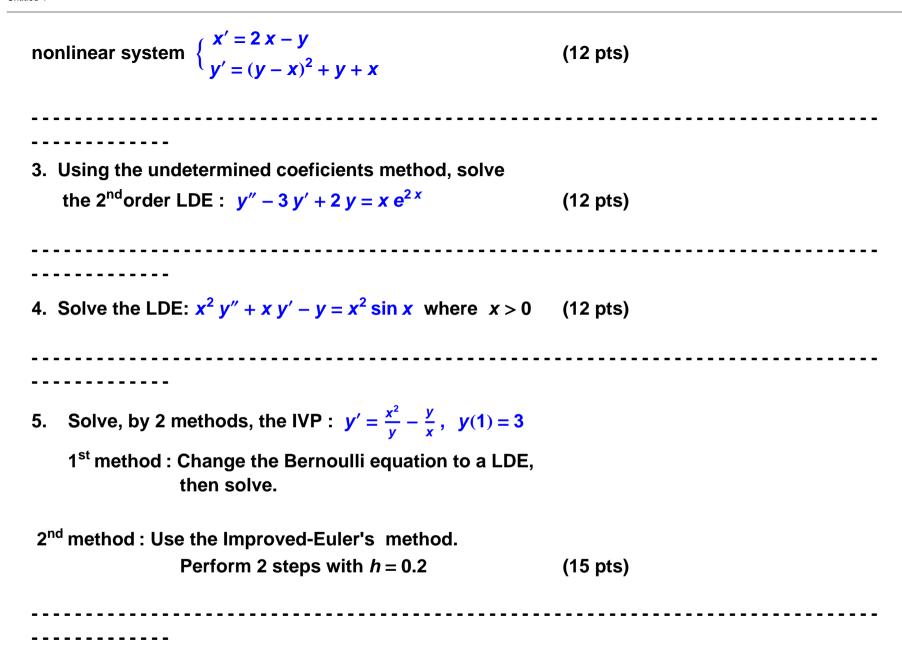
Lebanese American University	Diff. Equations	
	Fall 2010	
Byblos	Final Exam	
	2 Hours	
Name :		
		
1. Consider the linear system $\begin{cases} x' = 4 \\ y' = x \end{cases}$	(+ <i>y</i> + 4 <i>y</i>	
a) Find the general solution to the	system, and	
determine the type and stability of the b)	critical point O.	
Draw the phase portrait.	(15 pts)	
2. Determine the type of the critical p	oints of the	

Untitled-1



Turn Over

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6. Using Laplace transforms, solve the IVP:

$$\begin{cases} y'' + y' = r(t) \\ y(0) = 0, \ y'(0) = 2 \end{cases} \text{ where } r(t) = \begin{cases} 1 \text{ if } t < 2 \\ t \text{ if } t > 2 \end{cases}$$
 (14 pts)

7. A mass of 2 Kg is attached to the lower end of a spring with

spring constant k = 6. We pull down the mass 0.1 m and release it an upward initial velocity of 4 m/sec.

Using Laplace transforms, find the position of the

mass at any time t. (8 pts)

8. Using Laplace transforms, solve the system of LDE:

$$\begin{cases} x' = x - 8y \\ y' = x - 3y \end{cases} \text{ where } x(0) = 2, \ y(0) = 1$$
 (12 pts)

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Formulas:

(1) £
$$\{y'\}$$
 = s£ $\{y\}$ – $y(0)$

(2) £
$$\{y''\} = s^2 £\{y\} - s y(0) - y'(0)$$

(3) £
$$\{e^{at} f(t)\} = F(s-a)$$

(4)
$$\mathcal{E}\left\{\int_0^t f(x) dx\right\} = \frac{1}{s} \mathcal{E}\{f(t)\}$$

(5) £
$$\{U(t-a) f(t)\} = e^{-as}$$
£ $\{f(t+a)\}$

(5) £
$$\{U(t-a) f(t)\} = e^{-as} £\{f(t+a)\}$$
 (6) $e^{-as} £\{f(t)\} = £\{U(t-a) f(t-a)\}$

(7) £
$$\{t f(t)\} = -F'(s)$$

(8) £
$$\left\{\frac{f}{t}\right\} = \int_{s}^{+\infty} F(s) ds$$