Lebanese American University **Diff. Equations**

Fall 2010

Byblos **Final Exam**

2 Hours

Name :

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1. Consider the linear system $\begin{cases} x' = 4x + y \\ y' = x + 4y \end{cases}$

a) Find the general solution to the system, and

determine the type and stability of the critical point O.

b)

Draw the phase portrait. **(15 pts)**

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2. Determine the type of the critical points of the

nonlinear system $\begin{cases} x' = 2x - y \\ y' = (y-x)^2 + y + x \end{cases}$ (12 pts)

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3. Using the undetermined coeficients method, solve
   the 2nd order LDE : $y'' - 3y' + 2y = x\,e^{2x}$ (12 pts)

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4. Solve the LDE: $x^2 y'' + x\,y' - y = x^2 \sin x$ where $x > 0$ (12 pts)

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5. Solve, by 2 methods, the IVP : $y' = \frac{x^2}{y} - \frac{y}{x}$, $y(1) = 3$

   1st method : Change the Bernoulli equation to a LDE,
   then solve.

   2nd method : Use the Improved-Euler's method.
   Perform 2 steps with $h = 0.2$ (15 pts)

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Turn Over

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6. Using Laplace transforms, solve the IVP:

$$\begin{cases} y'' + y' = r(t) \\ y(0) = 0,\ y'(0) = 2 \end{cases} \quad \text{where } r(t) = \begin{cases} 1 & \text{if } t < 2 \\ t & \text{if } t > 2 \end{cases}$$ **(14 pts)**

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7. A mass of 2 Kg is attached to the lower end of a spring with spring constant $k = 6$. We pull down the mass 0.1 m and release it an upward initial velocity of 4 m/sec. Using Laplace transforms, find the position of the mass at any time t. **(8 pts)**

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8. Using Laplace transforms, solve the system of LDE:

$$\begin{cases} x' = x - 8y \\ y' = x - 3y \end{cases} \quad \text{where } x(0) = 2,\ y(0) = 1$$ **(12 pts)**

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Formulas:

(1) $\pounds\{y'\} = s\,\pounds\{y\} - y(0)$

(2) $\pounds\{y''\} = s^2\,\pounds\{y\} - s\,y(0) - y'(0)$

(3) $\pounds\left\{e^{at} f(t)\right\} = F(s-a)$

(4) $\pounds\left\{\int_0^t f(x)\,dx\right\} = \frac{1}{s}\,\pounds\{f(t)\}$

(5) $\pounds\{U(t-a)\,f(t)\} = e^{-as}\,\pounds\{f(t+a)\}$

(6) $e^{-as}\,\pounds\{f(t)\} = \pounds\{U(t-a)\,f(t-a)\}$

(7) $\pounds\{t\,f(t)\} = -F'(s)$

(8) $\pounds\left\{\frac{f}{t}\right\} = \int_s^{+\infty} F(s)\,ds$

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