Lebanese American University **Diff.Equations**

Fall 2008

Byblos **Final Exam**

2 Hours

Name :

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1. Solve, by 2 different methods, the 2nd order LDE :

$$y'' + 4y' + 4y = e^{-2x}$$

a) using the method of variation of parameters.

b) using the method of undetermined coefficients. **(15 pts)**

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2. Solve the 2nd order LDE : $x^2 y'' + 3x y' + 5y = 2 + x^2$ **(10 pts)**

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3. Consider the linear system $\begin{cases} x' = x - y \\ y' = x + 3y \end{cases}$

a) Find the general solution to the system.

b) Determine the type and stability of the critical point O.

c) Draw the phase portrait. **(15 pts)**

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4. Using the improved-Euler method, solve numerically the IVP

$\begin{cases} y' = x(x - y) \\ y(3) = 2 \end{cases}$. **Do 2 steps with $h = 0.5$** **(10 pts)**

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5. Using power series, solve the LDE $(x^2 + x)y'' + y' + y = 0$. **(10 pts)**

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6. You parked your car in a garage that has a constant temperature

of 30 °C. Two hours later, the engine temperature was 40°C. At the time $t = 3$ hours, its temperature decreased to 35°C. What was the initial temperature of the engine when you parked the car. (10 pts)

Turn Over

Formulas:

1) $\mathcal{L}\{y'\} = s\,\mathcal{L}\{y\} - y(0)$

2) $\mathcal{L}\{y''\} = s^2\,\mathcal{L}\{y\} - s\,y(0) - y'(0)$

3) $\mathcal{L}\left\{e^{a\,t}\,f(t)\right\} = F(s-a)$

4) $\mathcal{L}\left\{\int_0^t f(u)\,du\right\} = \frac{1}{s}\,\mathcal{L}\{f(t)\}$

5) $\mathcal{L}\{U(t-a)\,f(t)\} = e^{-a\,s}\,\mathcal{L}\{f(t+a)\}$

6) $e^{-a\,s}\,\mathcal{L}\{f(t)\} = \mathcal{L}\{U(t-a)\,f(t-a)\}$

7) $\mathcal{L}\{t\,f(t)\} = -F'(s)$

8) $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{+\infty} F(s)\,ds$

9) $(f * g)(t) = \int_0^t f(t-u)\,g(u)\,du$

10) $\mathcal{L}\{f * g\} = \mathcal{L}\{f\}\ \mathcal{L}\{g\}$

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7. Using convolution product, solve the IVP: $\begin{cases} y'' + 4y = t \\ y(0) = 0\,,\ y'(0) = 0 \end{cases}$ **(10 pts)**

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8. Find the Laplace Transform of the function:

a) $f(t) = t\cosh 2t$ **b)** $f(t) = U(t-2)\sin t$ **(8 pts)**

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9. The currents i_1 and i_2 in an electric circuit satisfy the system

$$\begin{cases} i_1' + 2i_2 = 1 \\ i_2' - 2i_1 = -2t \end{cases} \quad \text{where } i_1(0) = 1\,,\ i_2(0) = 0$$

Find the currents using Laplace Transforms method (12 pts)
