LAU
School of **Arts and Sciences**

Department of Computer Science & Mathematics

MTH207 – Discrete Mathematics
Spring , 2016
Exam 1
(February 19, 2016)

Name: Answer Key ID:

Duration: **55** minutes

- Answer the questions in the space provided for each problem.
- If more space or scratch is needed, you may use the back pages.
- Only scientific calculators are permissible.
- The exam has 6 pages consisting of 12 exercises.

Grades:

1. 8%	
2. 8%	
3. 12%	
4. 4%	
5. 6%	
6. 9%	

7. 9%	
8. 6%	
9. 6%	
10. 10%	
11. 10%	
12. 12%	
Total 100%	

1. Let **n** be a positive integer. Prove that if **7n+4** is even then **n** is even.

$p \longrightarrow q$

Proof by contraposition

(1) We need to show $\neg q \to \neg p$
That is to show if n is odd then $7n+4$ is odd

suppose n odd (1) $\Rightarrow n = 2K+1$ (1)

$7n+4 = 7(2K+1)+4$ (1)
$= 14k + 11$ (1)
$= \underline{14k+10} + 1$ (1)
$= 2(7k+5) + 1$ (1)
$= 2Q+1$
odd (1)

Done!

8%

OR

Proof by contradiction

suppose it is false (1)

$p \to q$ False $\Rightarrow$ p True, q False

Suppose $7n+4$ even and n odd (1)

n odd $\Rightarrow n = 2K+1$ (1)

$7n+4 = 7(2k+1)+4$ (1)
$= 14k + 11$ (1)
$= \underline{14k+10} + 1$ (1)
$= 2(7k+5)+1$ (1)
$= 2Q+1$
odd

contradiction (1)

hence $(7n+4 \text{ even}) \longrightarrow (n \text{ even})$

2. Prove that $\sqrt{6}$ is irrational.

Proof by contradiction

(1) Suppose it is false
That is, suppose $\sqrt{6}$ is rational

Then $\sqrt{6} = \frac{m}{n}$ (1 1/2) where m and $n \neq 0$ are integers and the ratio $\frac{m}{n}$ is in its simplest form. (1 1/2)

(1/2) $6 = \frac{m^2}{n^2}$

(1/2) $m^2 = 6n^2 \Rightarrow m^2$ is divisible by 6 (1/2) $\Rightarrow$ $\boxed{m \text{ is divisible by } 6}$

$\Rightarrow m = 6K$

(1/2) $(6K)^2 = 6n^2$

$36K^2 = 6n^2$

(1/2) $n^2 = 6K^2 \Rightarrow n^2$ is divisible by 6 (1/2) $\rightarrow$ $\boxed{n \text{ is divisible by } 6}$ (1/2)

since both m and n are (1/2) divisible by 6, the ratio $\frac{m}{n}$ is not in its simplest form (1/2) $\longrightarrow$ contradiction (1/2)

Hence, $\sqrt{6}$ is irrational. (1/2)

8%

3. a. Show that the conditional statement $(p \wedge q) \longrightarrow (p \longrightarrow q)$ is a tautology **WITHOUT** using truth table.

$\neg(p \wedge q) \vee (p \to q)$ (1 1/2)

$(\neg p \vee \neg q)$ (1) $\vee (\neg p \vee q)$ (1 1/2)

$\neg p \vee \neg q \vee \cancel{\neg p} \vee q$ (1/2)

$\neg p \vee \underline{\neg q \vee q}$ (1/2)

$\neg p \vee T$ (1/2)

True (1/2)

4%

b. Construct the **truth table** for the statement $(p \wedge q) \vee (p \longrightarrow q)$.

4%

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \vee (p \to q)$
T	T	T	T	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T
1		1	1	1

c. If you know that the proposition **(p ^ q) → r** is false, what can you say about the truth value of the proposition **(p ^ ¬r) → (p ˅ q)** ?

4%

$(p \wedge q) \longrightarrow r$ False $\Rightarrow \left[\begin{array}{l} p \wedge q \text{ True } (\tfrac{1}{2}) \Rightarrow \left[\begin{array}{l} p \text{ true } (\tfrac{1}{2}) \\ q \text{ true } (\tfrac{1}{2}) \end{array}\right. \\ r \text{ false } (\tfrac{1}{2}) \end{array}\right.$

$(p \wedge \neg r) \longrightarrow (p \vee q)$

$(T \wedge \neg F) \longrightarrow (T \vee T)$ (½)

$(T \wedge T) \longrightarrow T$ (½)

$T \longrightarrow T$ (½)

True (½)

4. Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent **WITHOUT** using truth table.

4%

$\neg(p \vee (\neg p \wedge q))$

$\equiv \neg p \wedge \neg(\neg p \wedge q)$ (1)

$\equiv \neg p \wedge (p \vee \neg q)$ (1)

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ (1)

$\equiv F \vee (\neg p \wedge \neg q)$ (1)

$\equiv \neg p \wedge \neg q$

6%

5. Let P(x,y) be the statement " x has sent y an email message", where the domain of *x* and y consists of all students in your class. Express each of these quantifications in **English**.
   a. $\exists x \exists y P(x,y)$
   3 There exists a student in your class who has sent an email message to another student in your class.
   b. $\exists y \forall x P(x,y)$
   3 There is a student in your class who ~~has received~~ has been sent an email message from all students in your class.

9%

6. Let L(x,y) be the statement "x loves y", where the domain consists of all people in the world. Use quantifiers to express each of these statements.
   a. Everybody loves Tom. $\forall x \; L(x, Tom)$ 1 1 1
   b. No one loves everybody. $\neg \exists x \; \forall y \; L(x,y)$ 1 1 1
   $\forall x \forall y \neg L(x,y)$ $1\frac{1}{2}$
   c. Everybody is loved by somebody. $\forall y \; \exists x \; L(x,y)$ 1 1 1

9%

7. Consider the proposition: **If I stay up late, then I will sleep until noon.**
   p q
   a. Write down its contrapositive.
   3 If I will not sleep until noon, then I will not stay up late.
   $\neg q \longrightarrow \neg p$
   b. Write down its converse.
   3 If I will sleep until noon, then I stay up late.
   $q \longrightarrow p$
   c. Specify the necessary condition in the statement.
   3 I will sleep until noon
   $p \longrightarrow q$ sufficient necessary

6%

8. Find the negations of the statements.(**Don't** use the phrase "it is not the case that ...)
   a. Every student in this class likes swimming.
   Not all ... like ... 2
   Some students in this class dont like swimming. $1\frac{1}{2}$ $1\frac{1}{2}$
   b. Some students in this class do not understand methods of proof ☹.
   All students in this class understand methods of proof ☺. 1 1 1

6%

9. Write each of these statements in the form “if p, then q” in **English**.
   a. $\underbrace{\text{I go to the beach}}_{q}$ whenever $\underbrace{\text{it is a sunny summer day}}_{p}$.

(3) If $\underbrace{\text{it is a sunny summer day}}_{p}$, then $\underbrace{\text{I go to the beach}}_{q}$.

   b. $\underbrace{\text{You get promoted}}_{p}$ only if $\underbrace{\text{you have connections}}_{q}$.

(3) If $\underbrace{\text{you get promoted}}_{p}$, then $\underbrace{\text{you have connections}}_{q}$

10. Use mathematical induction to prove that $n \leq 2^n$ for all positive integers.

10%

$P(n): \quad n \leq 2^n \qquad n = 1, 2, 3 \ldots$

Basis step (1) $\quad P(1): \quad 1 \leq 2^1$ (1)
True

Inductive step (1)

(1) Assume it is true for some K.
That is, $P(K)$ is true $\Rightarrow \quad K \leq 2^K$

(1) We need to show that it is true for $K+1$
That is $\quad P(K+1): \quad K+1 \leq 2^{K+1}$ is True ?

$K \leq 2^K$

(1/2) $K+1 \leq 2^K + 1$

(1/2) $1 \leq 2^K$ for $K \geq 1$ integer

(1/2) $K+1 \leq 2^K + 2^K$

(1/2) $K+1 \leq 2^K \cdot 2$

(1/2) $K+1 \leq 2^{K+1}$

(1/2) True for $K+1$

$P(K) \longrightarrow P(K+1)$

(1) Conclusion

(1) By M.I

$P(n): n \leq 2^n$ is true for all positive integers n.

10%

11.Use mathematical induction to prove that

$$P(n)\ :\ 1+4+4^2+\ \dots+4^n=\frac{4^{n+1}-1}{3} \qquad n=0,1,2,\dots$$

where n is a non-negative integer.

Basis step: (1) $P(0)$: $1=\frac{4-1}{3}$

$1=\frac{3}{3}$ (1)

$1=1$ ✓

$P(0)$ is true

(1) Inductive step

(1) suppose that it is true for some k

That is $P(k)$: $1+4+4^2+\dots+4^k=\frac{4^{k+1}-1}{3}$ is true

(1) we need to show that it is true for $k+1$

That is $P(k+1)$: $1+4+4^2+\dots+4^{k+1}=\frac{4^{k+2}-1}{3}$ is true?

$1+4+4^2+\dots+4^{k+1}$

(½) $=\underbrace{1+4+4^2+\dots+4^k}+4^{k+1}$

(½) $=\frac{4^{k+1}-1}{3}+4^{k+1}$

(½) $=\frac{4^{k+1}-1+3\cdot 4^{k+1}}{3}$

$=\frac{4^{k+1}(1+3)-1}{3}$

$=\frac{4^{k+1}\cdot 4-1}{3}$ (½)

$=\frac{4^{k+2}-1}{3}$ ✓ (½)

True for $k+1$ (½)

$P(k)\longrightarrow P(k+1)$ ✓

Conclusion (1)

By M.I. $P(n)$ is true for all non-negative integers n.

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12.a) Fill in the blanks with **True** or **False**. (Do not justify)

(1)	True $\{x\}\in\{\{x\}\}$		$\emptyset\in\{x\}$	False	(1)
(1)	True $\{2,\{2\}\}=\{2,2,\{2\}\}$		$\{\emptyset\}\subset\{\emptyset,\{\emptyset\}\}$	True	(1)
(1)	True $\{x\}\subseteq\{x\}$		$\{\{\emptyset\}\}\subset\{\emptyset,\{\emptyset\}\}$	True	(1)
(1)	True $\emptyset\subseteq\{x\}$		$0\in\emptyset$	False	(1)

4%

b) Fill in the blanks the correct value

The Cardinality of $P(P(\emptyset))=$ $2^{(2^0)}=2^1=2$ (2)

The Cardinality of $P(\{\emptyset,a,\{a\}\})=$ $2^3=8$ (2)