LAU
School of Arts and Sciences

Department of Computer Science & Mathematics

MTH207 – Discrete Mathematics
Spring 2015
Exam 1
(February 23, 2015)

Name: ID:

Duration: 50 minutes

Instructor: Silvana Nahlus

- Answer the questions in the space provided for each problem.
- If more space or scratch is needed, you may use the back pages.
- Only scientific calculators are permissible.
- The exam has 6 pages consisting of 11 exercises.

Grades:

1. 8%	
2. 8%	
3. 4%	
4. 24%	
5. 6%	
6. 8%	

7. 6%	
8. 9%	
9. 6%	
10. 6%	
11. 15%	
Total 100%	

1. Prove that if **n** is an integer and **3n+2** is even, then **n** is even.

2. Prove that if **x** is Irrational number, then **3x+2** is irrational.

3. Prove or Disprove that if a and b are rational numbers, then a^b is rational.

4. a. Show that the conditional statement $\neg(p \longrightarrow q) \longrightarrow \neg q$ is a tautology **WITHOUT** using truth table.

   b. Construct the **truth table** for the statement $[p \wedge (p \longrightarrow q)] \longrightarrow q)$.

   c. If you know that the proposition $[p \wedge \neg(q \vee \neg s)] \longrightarrow (t \wedge p)$ is false, what can you say about the truth value of the proposition
   $t \vee (q \vee s)$?

5. Let C(x,y) be the statement “student x is enrolled in class y”, where the domain of *x* consists of all students in your class and for *y* consists of all classes being given at your school. Express each of these quantifications in **English**.
   a. $\exists x C(x, MTH201) \land C(x, MTH207)$

   b. $\exists y \forall x C(x, y)$

6. Let M(x,y) be the statement “x has sent y an email message”, where the domain consists of all students in your class. Use quantifiers to express each of these statements.
   a. Fares has never sent an email message to Karim .

   b. No one in your class has sent an email message to Hadi.

   c. Everybody sent an email message to himself.

   d. There is someone who sent an email message to no one besides himself.

7. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.
   a. $\forall x \exists y (y^2 = x)$

   b. $\forall x \forall y (x^2 \neq y^3)$

8. Consider the proposition: **If you don't study, you won't pass your exams**.
   a. Write down its contrapositive.

   b. Write down its inverse.

   c. Specify the sufficient and necessary conditions in the statement.

9. Express the negation of these propositions using quantifiers, and then express the negation in English.(**Don't** use the phrase "it is not the case that …)
   a. Some drivers do not obey the speed limit.

   b. No one can keep a secret.

10. Write each of these statements in the form "if p, then q" in **English**.
   a. Dany gets caught whenever he cheats.

   b. It is necessary to have a password to log on to the server.

   c. That you get the job implies that you had the best credentials.

11. a. Fill in the blanks with True or False. (Do not justify)

	$\emptyset \in \{0\}$
	$\{0\} \in \{0\}$
	$\{\emptyset\} \in \{\{\emptyset\}\}$
	$\{\emptyset\} \subseteq \{\emptyset\}$
	If x is a positive rational number, then $\sqrt{x}$ is irrational.
	$\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
	$\{x\} \in \{\{x\}\}$
	The Cardinality of $P(P(\{\emptyset\})) = 2$
	If $A \subseteq B$, then $P(A) \subseteq P(B)$
	$A \times \emptyset = A$