Lebanese American University **Fall 2014**

Byblos

Discrete Structure I **Due:** None

HW #3

---

1. Do Ex: 10, 12, 14, 20, 26, 34 and 36 in the Textbook pages 280 and 281.

2. (a) Prove, using course values induction, that any positive integer can be written as product of prime numbers.
   (b) Prove that if $n \in \mathbb{N}$ such that n^2 is divisible by a prime number p, then n is divisible by p.
   (c) Prove that if $p \geq 2$ is a prime number, then $\sqrt{p}$ is irrational.
   (d) Prove that if $p \geq 2$ and $q \geq 2$ are two distinct prime numbers, then $\sqrt{p} + \sqrt{q}$ and $\sqrt{pq}$ are irrational.
   (e) Prove that if $p \geq 2$, $q \geq 2$ and $r \geq 2$ are three prime numbers such that $p + q = r$, then $\sqrt{p} + \sqrt{q} + \sqrt{r}$ is irrational.
   (f) Now let $p \geq 2$, $q \geq 2$ and $r \geq 2$ be three prime numbers.
      i. Prove that $\sqrt{pqr}$ is irrational.
      ii. Deduce that $\sqrt{p} + \sqrt{q} + \sqrt{r}$ is irrational.

3. We consider the following program (input is n and output is r):

```
r:=0;
if n>0 then
   begin r:=1;
         i:=1;
         while i<n do
            begin i:=i+1;
                  r:=r+3*i*i-3*i+1;
            end;
   end;
```

   (a) Find the output of the program for $n = 1$, $n = 2$ and $n = 3$.
   (b) Which function computes the program?
   (c) Write the correctness proof of the program

4. (a) Prove by induction that for $n \geq 0$, $n(n+1)(2n+1)$ is divisible by 6.
   (b) Prove by induction that the sum of the square of the first n positive integers is $\dfrac{n(n+1)(2n+1)}{6}$, that is, $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

(c) Now we consider the following program (input is n and output is r):

```
r:=0;
if n > 0 then
   begin r := 6;
         i := 1;
         while i < n do
            begin i := i + 1;
                  r := r + 6*i*i;
            end;
   end;
```

i. Which function calculate the program?

ii. Prove the correctness of the program.