Lebanese American University **Fall 2010**

Byblos

Discrete Structure I **Date:** 11/11/2010
Test #1 **Duration:** 1h 30

---

1. Let φ be the following predicate logic formula:
$$[\exists t(\forall x \underline{P}(x,y) \rightarrow \underline{Q}(x,t))] \rightarrow [\exists y \underline{R}(y) \vee \underline{S}(y,t)]$$
   (a) List free and bound occurrences of variables in φ.
   (b) Give the parse tree of φ.
   (c) Find the prenex form of φ.

2. Let p, q, and r be the following propositions:

   p : You get an A on the final exam.
   q : You do every exercise in this book.
   r : You get an A in this class.

   (a) Write these propositions using p, q, and r and logical connectives:
      (i) You get an A in this class, but you do not do every exercise in this book.
      (ii) You get an A on the final, you do every exercise in this book, and you get an A in this class.
      (iii) To get an A in this class, it is necessary for you to get an A on the final.
      (iv) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
      (v) You get an A in this class if and only if you either do every exercise in this book or you get an A on the final.
   (b) State (as a sentence and formula) the converse, contrapositive, and inverse of the sentence (iii) of the preceding question.

3. Express the following in predicate logic:
   (a) Every even number is a sum of two odd numbers.
   (b) *Bonus question.* There is no greatest natural number.

4. Find a resolution and a formal proof for the following:
   (a) $\vdash \{p\,;\, p \longrightarrow q\,;\, (p \wedge q) \longrightarrow r\} \vdash r.$
   (b) $\{p \longrightarrow (q \vee r)\,;\, (q \wedge p) \longrightarrow s\,;\, \neg r\,;\, p\} \vdash s$
   (c) $\vdash ((p \vee (p \longrightarrow q)) \longrightarrow q) \longrightarrow q$

5. Find using Karnaugh maps the DNF form of the formula φ which has the following truth table:

$\longrightarrow$

p	q	r	φ
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

MARKS: 1. [20] **2**. [20] **3**. [10] **4**. [30] **5**. [15] **Bonus questions** [10]

Lebanese American University **Fall 2011**

Byblos

Discrete Structure I **Date:** 15/11/2011

Test 1 **Duration:** 1h 30

Name: **ID:**

---

1. **[20 pts]** Let φ be the following predicate logic formula:

$$\{\forall x \underline{P}(x, y) \rightarrow \exists t \underline{Q}(x, t)\} \wedge \{\neg[\exists y \underline{Q}(y, z) \rightarrow \exists z \underline{R}(z, t)]\}$$

   (a) List free and bound occurrences of variables in φ.

   (b) Give the parse tree of φ.

   (c) Find the prenex form of φ.

2. **[20 pts]** Let p and q be the following propositions:

p : It is November 22$^{\text{nd}}$.

q : School is closed.

Write as an english statement the following:

(a) $p \longrightarrow q$

(b) The inverse of $p \longrightarrow q$.

(c) The converse of $p \longrightarrow q$.

(d) The contrapositive of $p \longrightarrow q$.

(e) The contradiction of $p \longrightarrow q$.

3. [**20 pts**] Find using Karnaugh maps a DNF and a CNF for the formula φ which has the following truth table:

p	q	r	φ
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

4. [**15 pts**] Express the following in predicate logic:

   (a) The product of two negative integers is positive.

   (b) Nobody loves anybody.

5. **[35 pts]** Find a resolution and a formal proof for the following:

(a) $\{A \longrightarrow B\,;\, \neg A \longrightarrow C\,;\, C \longrightarrow D\} \vdash \neg B \longrightarrow D$

(b) $\{A \longrightarrow B\,;\, B \vee C\,;\, A \longrightarrow C\,;\, B \longrightarrow A\,;\, C \longrightarrow A\} \vdash B \wedge C$

(c) $\vdash [(A \longrightarrow B) \vee (A \longrightarrow C)] \longrightarrow [A \longrightarrow (B \vee C)]$

Lebanese American University **Fall 2012**

Byblos

Discrete Structure I **Date:** 07/11/2012
Test #1 **Duration:** 1h 45

Name: **ID:**

---

1. **(20 pts)** Let φ be the following predicate logic formula:

$$[\forall x \underline{P}(x,y) \vee \exists t \underline{Q}(x,t)] \rightarrow [\exists y \underline{R}(y) \longrightarrow \underline{S}(y,a,t)]$$

   (a) List free and bound occurrences of variables in φ.
   (b) Give the parse tree of φ.
   (c) Find the prenex form of φ.

2. **(22 pts)** Assume you want to throw a party, respecting people's incompatibilities. You know that:

   (1) John comes if Mary or Ann comes.
   (2) Ann comes if Mary does not come.
   (3) If Ann comes, John does not.

   Consider the following propositional logic variables:

   - J: John comes.
   - A: Ann comes.
   - M: Mary comes.

   (a) Write propositions (1), (2) and (3) using J, A, M and logical connectives.
   (b) Now let Γ be the set of formulas of the preceding question. Prove, without using truth table method, that

$$\Gamma \vdash (J \wedge \neg A \wedge M)$$

   Conclusion?

3. **(16 pts)** Express the following in predicate logic:

   (a) If the square of an integer is even then that integer is also even.
   (b) Everybody has a mother.

4. **(36 pts)** Find a resolution and a formal proof for the following:

   (a) $\{p \longrightarrow r\,;\, q \longrightarrow r\,;\, \neg p \longrightarrow (q \vee r)\} \vdash r$
   (b) $\vdash (p \vee q) \longrightarrow [((p \longrightarrow r) \wedge (q \longrightarrow r)) \longrightarrow r]$
   (c) $\vdash \neg(p \wedge q) \longrightarrow (q \longrightarrow \neg p)$

5. **(16 pts)** Find using Karnaugh maps a DNF and a CNF for the formula φ which has the following truth table:

$\longrightarrow$

p	q	r	s	φ
T	T	T	T	F
T	T	T	F	F
T	T	F	T	F
T	F	T	T	T
F	T	T	T	T
T	T	F	F	F
T	F	T	F	T
T	F	F	T	T
F	T	T	F	T
F	T	F	T	F
F	F	T	T	T
T	F	F	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	T	T
F	F	F	F	T

Lebanese American University **Fall 2013**

Byblos

Discrete Structure I **Date:** 13/11/2013

Test #1 **Duration:** 2h

Name: **ID:**

---

1. **(20 pts)** Let φ be the following predicate logic formula:

$$[\forall x\, \underline{P}(x,z) \longrightarrow \exists t\, \underline{Q}(x,t)] \vee \neg[\underline{R}(y) \longrightarrow \exists y\, \underline{S}(y,t)]$$

   (a) List free and bound occurrences of variables in φ.
   (b) Give the parse tree of φ.
   (c) Find the prenex form of φ.

2. **(30 pts)** You are walking in a labyrinth and all of a sudden you find yourself in front of three possible roads: the road on your left is paved with gold, the one in front of you is paved with marble, while the one on your right is made of small stones. Each street is protected by a guardian. You talk to the guardians and this is what they tell you:

   - The guardian of the gold street: This road will bring you straight to the center. Moreover, if the stones take you to the center, then also the marble takes you to the center.
   - The guardian of the marble street: Neither the gold nor the stones will take you to the center.
   - The guardian of the stone street: Follow the gold and you will reach the center, follow the marble and you will be lost.

   Consider the following propositional logic variables:

   - g: The gold road brings to the center
   - m: The marble road brings to the center
   - s: The stone road brings to the center

   (a) Write three propositional formulas that represent what the guardians told you using g, m, s and logical connectives.
   (b) Now we assume that each one of the three guardians is a lier.
      i. Prove that we have: $g \longrightarrow (s \wedge \neg m)$, $g \vee s$ and $g \longrightarrow m$.
      ii. We denote by Γ the set of the three formulas of the preceding question. Prove using three different methods (by contradiction, formal proof and resolution proof) that

$$\Gamma \vdash (s \wedge \neg g)$$

      Conclusion?

$\longrightarrow$

3. **(16 pts)** Express the following in predicate logic:
   (a) There is a computer which is not used by any student.
   (b) If John loves someone, then he loves Mary.

4. **(30 pts)** Find a resolution and a formal proof for the following:
   (a) $\vdash p \vee (p \longrightarrow q)$
   (b) $\{p \longrightarrow (q \longrightarrow r)\,;\, p \vee r\,;\, \neg q \longrightarrow \neg p\} \vdash r$
   (c) $\vdash (p \longrightarrow (q \longrightarrow r)) \longrightarrow ((p \longrightarrow q) \longrightarrow (p \longrightarrow r))$

5. **(14 pts)** Find using Karnaugh maps a DNF and a CNF for the formula φ which has the following truth table:

p	q	r	φ
T	T	T	T
T	T	F	F
T	F	T	T
F	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T
F	F	F	T

Lebanese American University **Spring 2014**

Byblos

Discrete Structure I **Date:** 01/04/2014
Test #1 **Duration:** 1h 30

Name: **ID:**

---

1. **(20 pts)** Let φ be the following predicate logic formula:

$$[\forall x \, \underline{P}(x,y) \vee \exists t \, \underline{Q}(x,t)] \longrightarrow \neg[\exists y \, \underline{R}(y) \wedge \underline{S}(x,t)]$$

   (a) List free and bound occurrences of variables in φ.
   (b) Give the parse tree of φ.
   (c) Find the prenex form of φ.

2. **(30 pts)** You are lost on a track in the desert. You arrive at a junction. Each of the two tracks is kept by a sphinx that you can question. The tracks can either lead to an oasis, or to lose itself in the deep desert.

   - The sphinx of the right track answers you: At least one of the two tracks leads to an oasis.
   - The sphinx of the left track answers you: The right track leads you to the deep desert.

   Consider the following propositional logic variables:

   - l: The left track leads to an oasis.
   - r: The right track leads to an oasis.

   (a) Write two propositional formulas that represent what the sphinxes told you using l, r and logical connectives.
   (b) Now we assume that you know that the sphinxes both say the truth, or both lie.
      i. Explain why we can deduce that: $(l \vee r) \longleftrightarrow \neg r$
      ii. We denote by Γ the set of the two formulas of the preceding question, that is, $\Gamma = \{(l \vee r) \longrightarrow \neg r \,;\, \neg r \longrightarrow (l \vee r)\}$. Prove using three different methods (by contradiction, formal proof and resolution proof) that

$$\Gamma \vdash l \wedge \neg r$$

      Conclusion?

3. **(18 pts)** Express the following in predicate logic:

   (a) Tamira bought everything that John bought.
   (b) Every student loves some student.
   (c) Every student loves some other student. $\longrightarrow$

4. **(24 pts)** Find a resolution and a formal proof for the following:

   (a) $\{\neg g \longrightarrow \neg(p \longrightarrow a)\,;\ \neg p\} \vdash g$

   (b) $\{p \longrightarrow q\,;\ (\neg p \vee s) \longrightarrow r\,;\ \neg s \longrightarrow \neg r\,;\ \neg q\} \vdash s$

5. **(18 pts)** Find using Karnaugh maps a DNF and a CNF for the formula φ which has the following truth table:

p	q	r	s	φ
T	T	T	T	F
T	T	T	F	F
T	T	F	T	F
T	F	T	T	T
F	T	T	T	F
T	T	F	F	F
T	F	T	F	F
T	F	F	T	T
F	F	T	T	T
F	T	F	T	F
F	T	T	F	F
T	F	F	F	F
F	T	F	F	F
F	F	T	F	T
F	F	F	T	T
F	F	F	F	F