1. Find the gcd d of 20 and 75, then write d as a linear combination of 20 and 75.

2. Consider the equivalence relation on $Z \times Z$ given by $(m,n)R(p,q)$ if and only if $mq = np$.

   (a) Find the equivalence class represented by $(2,5)$.

   (b) Describe the set S of the equivalence classes determined by R.

3. Consider the matrix $M_R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

   (a) Write the relation on the set $\{a,b,c\}$ corresponding to M_R.

   (b) Draw the directed graph corresponding to M

   (c) Calculate M^2, M^3

   (d) Find a formula for M^n, and prove it by induction.

   (e) Using the previous part, or otherwise, find the number of paths of length n from a to c.

4. Define the relation R on $\mathbb{N} \times \mathbb{N}$ by: $(x,y)\ R\ (z,w)$ if and only if $x - z = w - y$. Check whether R is an equivalence relation. Explain your answer

5. Define the relation R on $\mathbb{N}$ by, mRn if $3 \mid m - n$

   (a) Is R an equivalence relation? If so, what are its equivalence classes?

6. Let $\sim$ be the equivalence relation on $\mathbb{Z}$ given by $m \sim n$ if and only if $m^3 = n^3$

   (a) Show that R is a reflexive, symmetric and transitive

7. Show that if a prime number $p \mid a^n$, then$p \mid a$.

8. Let $a = 272$ and $b = 176$. Find $d = gcd(a,b)$. then write d as a linear combination of a and b.

9. Find the gcd d of 20 and 75, then write d as a linear combination of 20 and 75.