Discrete Structures I
Practice exercices for
Exam 2

Fall 09

1. Find 2 matrices A and B,of size $3x3$, such that $AB = 0$, yet $A \neq 0$,and $B \neq 0$.

2. Find A^3, where $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

3. Show that if A, B and AB are symmetric, then $AB = BA$.

4. Let $A = \begin{bmatrix} -2 & 4 & 7 \\ 6 & 1 & 5 \\ 2 & 3 & -4 \end{bmatrix}$

   Calculate:

   (a) A^t

   (b) AA^t

   (c) A^tA

5. Show that $1^2 + 3^2 + 5^2 ... + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$ for all n.

6. Let $P(n)$ be the inequality $n^2 < 2^n$.

   (a) Write $P(5), P(k), P(k+1)$

   (b) Show that $P(n)$ holds for all $n \geq 5$. Show all the details

7. Consider the digraph G below on the set $A = \{a, b, c, d, e\}$. Let R be the relation corresponding to G.

   (a) list paths of length 3 starting at a

   (b) Find paths of length 5.

   (c) Write the Matrix representation M_R of R.

8. Let R be the relation on N given by mRn if and only if $m < 3n$.Find a path of length 5 starting at 20.

9. Consider the relation R on A=$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, aRb if and only if $a+b \leq 10$. State all properties of R.

10. Define M_{R^∞},then answer the question: If R is a relation on the set $A = \{1, 2, 3, 4, 5\}$. How do you interpret the fact that on M_{R^∞}, we have $a_{2,4} = 1$?

11. Find, if possible, using a digraph, a relation on the set $A = \{a, b, c, d, e\}$ that is (AR),(S) and (T). Comment on your construction.

12. Let $A = \mathbb{Z}$, and R be the relation on A given by aRb if and only if $a^2 \equiv b^2 (\bmod 5)$.

    (a) Show (in a very efficient way) that the relation R is an equivalence relation.

    (b) Find all equivalence classes determined by R.

13. Answer the following. In case the answer is yes, prove your statement, and in case it is no, find a real counterexample.

    (a) If R and S are two transitive relations, is $R \cup S$ also transitive?

    (b) If R and S are two symmetric relations, is $R \cup S$ also symmetric?

(c) If R and S are two transitive relations, is $R \cap S$ also transitive?

14. Write your own equivalence relation on the set Z.

15. Find an equivalence relation and a function that determine the following partition on the set $A = \{1,2,3,4,5,6,7\}$: $\{1,3\}, \{2,4,5,\}, \{7\}, \{6\}$.

16. We consider the equivalence relation R on the set N of positive integers defined by nRm if and only if $|a| = |b|$. Find the equivalence classes determined by R.

17. Show that $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$.

18. Let $A = \{a, b, c, d\}$ and consider the matrix $M = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

   (a) Find all the paths of length 2 from vertex a to each of the other vertices.

   (b) Find $R(a)$ and $R(\{b, c, d\})$

   (c) Find, if possible, a relation on the set $\{a, b, c, d\}$ that is (R), (S) but not (AS) and not (T)

19. Consider the relation R on Z given by aRb if and only if $\frac{a}{b}$ is a multiple of 3. Determine wether R is reflexive, symmetric or transitive. Justify.

20. Let $M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ and $M_S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$.

   (a) Find $M_{R \cup S}$, $M_{R \cap S}$, $M_{R^{-1}}$, and $M_{\overline{S}}$.