**LEBANESE AMERICAN UNIVERSITY**
**DEPARTMENT OF COMPUTER SCIENCE AND MATHEMATICS**

# EXAM 1 - MTH 207 DISCRETE STRUCTURES 1 – FALL 2010

**DURATION: 75 MIN**

**NAME:**

**ID:**

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INSTRUCTIONS: This exam consists of 7 pages and 9 problems. Check that none is missing. Answer the questions in the space provided for each problem; if more space is needed, you may use the back pages.

**GOOD LUCK!**

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| QUESTION | GRADE |
|---|---|
| 1. 20% | |
| 2. 8% | |
| 3. 10% | |
| 4. 6% | |
| 5. 8% | |
| 6. 16% | |
| 7. 10% | |
| 8. 6% | |
| 9. 16% | |
| TOTAL | |

**1.**

**a.** **(6%)** Evaluate the following sum $\sum_{j=0}^{8}\left(2\cdot(-3)^{j}-j^{2}\right)$

**b.** **(5%)** Find the general term $b_n$ of the sequence $\left\{1,-1,\frac{1}{2},-\frac{1}{6},\frac{1}{24},-\frac{1}{120},\ldots\right\}$

**c.** **(6%)** Show that the sequence $\{5n-2\}_{n=1}$ is an algebraic progression. Identify its initial term and its common difference.

**d.** **(3%)** Find the first six terms of the sequence $\{a_n\}_{n=1}$ where $a_n=\left\lfloor \sqrt{2n}+\frac{1}{2}\right\rfloor$

**2.** **(8%)** Recall that if $A$ and $B$ are two sets, then $|A \cup B| = |A| + |B| - |A \cap B|$. Use this formula to show that if $C$ is a third set, then $|A \cup B \cup C| = |A| + |B| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

**3. (10%)** Prove the following De Morgan's Law $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by showing that $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$ and $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$

**4. (6%)** Using the law in the previous exercise, prove the following generalized De Morgan's Law one: $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$ <u>(This is a one line proof)</u>

**5. (8%)** Use the membership table to show the same generalized De Morgan's Law as in the previous exercise ( $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$ ).

**6.** Let $f: S \to T$ denote a function.

**a.** **(6%)** Which ones of the following statements mean that the function is 1-1:

i. If $f(s_1) = f(s_2)$, then $s_1 = s_2$

ii. If $s_1 = s_2$ then $f(s_1) = f(s_2)$

iii. If $s_1 \neq s_2$ then $f(s_1) \neq f(s_2)$

**b.** **(10%)** Let $f: P \times P \to P$ be given by $f(m,n) = 3^m 7^n$. Show that this function is 1-1 but not onto ($P$ is the set of positive integers)

**7.** **(10%)** Let $f: S \to T$ and $g: T \to U$ denote two 1-1 functions. Show that $g \circ f: S \to U$ is 1-1.

**8.** **(6%)** Define the <u>cardinality</u> of a set $S$ in case the set is finite and in case it is infinite.

**9.**

**a.** **(10%)** Find a 1-1 correspondence between the set of positive integers $\mathbb{N}^* = \{1, 2, 3, 4...\}$ and the set T of all multiples of 3 $\left(T = \{3, 6, 9, 12, ...\}\right)$

**b.** **(2%)** Is the set $T$ countable or uncountable? Justify.

**c.** **(4%)** Can two infinite sets have the same cardinality even though they may seem to have different sizes? Justify