Math 218 Quiz 1 Fall 2009

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Instructions:

1. You must show correct work to receive credit. Correct answers with inconsistent work will not be given credit.
2. Books, notes and calculators are not allowed.
3. Turn off and put away all cell phones.

Question	Points	Points Possible
1	1	5
2	1	5
3	1	10
4	1	10
5	1	5
6	1	5
7	1	5
8	1	5
Total	8	50

Excellent Work!!!

Math 218 **Quiz 1** **Fall 2009**

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1. (5 pts) Determine whether $\mathbf{v}$ is a linear combination of $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ where

$$\mathbf{v} = \begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}, \quad \mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v_3} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}.$$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = v$

$c_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 2 & 1 & 4 & 8 \\ 1 & 1 & 3 & 4 \end{array}\right] \xrightarrow[R_3 - R_1 \to R_3]{R_2 - 2R_1 \to R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 6 & 4 \\ 0 & 1 & 4 & 2 \end{array}\right] \xrightarrow{R_3 - R_2 \to R_3}$

$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 6 & 4 \\ 0 & 0 & -2 & -2 \end{array}\right]$ $R_3: -2c_3 = -2$, $c_3 = 1$ $R_2: c_2 + 6c_3 = 4$, $c_2 = 4 - 6 = -2$

$c_1 - c_3 = 2$, $c_1 = 2 + 1 = 3$

2. (5 pts) Determine the values of a, b, and c for which the linear system is consistent.

$$\begin{cases} x + 2y - 3z &= a \\ 2x + 3y + 3z &= b \\ 5x + 9y - 6z &= c \end{cases}$$

$\left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 2 & 3 & 3 & b \\ 5 & 9 & -6 & c \end{array}\right] \xrightarrow[R_3 - 5R_1 \to R_3]{R_2 - 2R_1 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & -1 & 9 & c-5a \end{array}\right] \xrightarrow{R_3}$

$\left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & 0 & 0 & c-b-3a \end{array}\right]$

$c - b - 3a = 0.$

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4. (5 pts each) If $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ and $\det(A) = 5$, find

(a) $\det(2A^2A^tA^{-1})$

$$|2A^2A^tA^{-1}| = |2A^2|\,|A^t|\,|A^{-1}| = \frac{2^3|A|^2|A|}{|A|} = 2^3|A|^2$$
$$= 2^3 5^2$$
$$= 8\times 25$$
$$= 200$$

(b) $\det\left(\begin{bmatrix} a_1 & a_1+2b_1 & 3a_1+c_1 \\ a_2 & a_2+2b_2 & 3a_2+c_2 \\ a_3 & a_3+2b_3 & 3a_3+c_3 \end{bmatrix}\right)$

$$\det(A) = \det\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \xrightarrow{2R_2 \to R_2}$$

$$= \frac{1}{2}\det\begin{bmatrix} a_1 & a_2 & a_3 \\ 2b_1 & 2b_2 & 2b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \xrightarrow[R_3+3R_1 \to R_3]{R_2+R_1 \to R_1}$$

$$= \frac{1}{2}\det\begin{bmatrix} a_1 & a_2 & a_3 \\ a_1+2b_1 & a_2+2b_2 & a_3+2b_3 \\ 3a_1+c_1 & 3a_2+c_2 & 3a_3+c_3 \end{bmatrix}$$

$$= \frac{1}{2}\det\begin{bmatrix} a_1 & a_1+2b_1 & 3a_1+c_1 \\ a_2 & a_2+2b_2 & 3a_2+c_2 \\ a_3 & a_3+3b_3 & 3a_3+c_3 \end{bmatrix}$$

$\det(\ \ )$
$= 10.$

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5. (5 pts) Find all values of a for which the inverse of $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & a & 0 \\ 1 & 2 & a \end{bmatrix}$ exists.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & a & 0 \\ 1 & 2 & a \end{bmatrix} \xrightarrow[R_3 - R_1 \to R_3]{R_2 - R_1 \to R_2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & a-1 & 0 \\ 0 & 1 & a \end{bmatrix} \xrightarrow[(a-1)R_1 - R_2 \to R_1]{(a-1)R_3 - R_2 \to R_3}$$

$$\begin{bmatrix} a-1 & 0 & 0 \\ 0 & a-1 & 0 \\ 0 & 0 & a(a-1) \end{bmatrix}$$

$a(a-1) \neq 0$

$a \neq 0 \quad a \neq 1$

6. (5 pts) Let A and B be two $n \times n$ matrices. If AB is invertible, must both A and B be invertible? Justify your answer.

If AB in invertible then AB is row equi to the identity matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & a \\ 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$$

If one of the 2 matrics is not invertible then it contain a row of zeros. So not possible.

AB inv

$\Rightarrow |AB| \neq 0$

$\Rightarrow |A|\,|B| \neq 0$

$\Rightarrow |A|$ and $|B|$ both not zero

$\Rightarrow A$ & B both inv.

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7. (5 pts) Let A and B be symmetric matrices. Show that AB is symmetric **if and only if** $AB = BA$.

If $AB = BA$ then
$A = A^t$ $\quad B = B^t$

$(AB)^t = B^t A^t = BA = AB.$

If AB is sym
$(AB)^t = B^t A^t = BA$
But $(AB)^t = AB$
the $AB = BA$

8. (5 pts) Let A be a 4×4 invertible matrix. Find $\det(A)$ if $A^t = 2A^{-1}$.

$|A^t| = |2A^{-1}|$

$|A| = \dfrac{2^4}{|A|}$

$|A|^2 = 2^4$

$|A| = 2^2 = 4.$