Lebanese American University **Fall 2011**

Byblos

Discrete Structure I **Date:** 25/01/2012
Final Exam **Duration:** 2h 15

Name: **ID:**

Part I: Problems (70 pts)

1. **[10 pts]** Find a resolution proof and a formal proof for the following propositional logic formula:

$$\{C \longrightarrow (A \vee D)\,;\, \neg D \longrightarrow (B \vee \neg C)\,;\, C\,;\, \neg D\} \vdash \neg(A \longrightarrow \neg B)$$

(*Hint*: For the formal proof, assume $A \longrightarrow \neg B$ and prove a contradiction)

2. [**15 pts**] Prove, by induction, the following:

(a) $8^n - 3^n$ is divisible by 5 for all $n \in \mathbb{N}$.

(b) $\displaystyle\sum_{k=1}^{n} \frac{k}{(k+1)!} = 1 - \frac{1}{(n+1)!}$.

(c) $5^{2n+1} \cdot 2^{n+2} + 3^{n+2} \cdot 2^{2n+1}$ is divisible by 19 for all $n \in \mathbb{N}$.

3. [**10 pts**] Solve using pigeonhole principle the following two independent questions:

   (a) There are 25 students in a class. While doing a keyboarding test, each student made fewer than 12 mistakes. Show that at least 3 students made the same number of mistakes.

   (b) Five points are chosen from inside an equilateral triangle of side 2. Prove that two points must be within a distance of 1 of each other.

4. [**10 pts**] We consider the following program (input is n and output is r):

```
if n > 0 then
   begin r := 0;
         i := 1;
         while i <= n do
           begin r := r + 2*i;
                 i := i + 1;
           end;
   end;
```

(a) Find the output of the program for $n = 1$, $n = 2$, $n = 3$ and $n = 4$.

(b) Which function computes the program?

5. **[13 pts]** Verify, in the following cases, if the binary relation R defined over the set X is reflexive, symmetric and transitive. If R is an equivalence relation then find the set of classes π_R.

   (a) $X = \mathbb{Z}$ and $x\,R\,y$ iff $x + y$ is even.

   (b) $X = \mathbb{Z}$ and $x\,R\,y$ iff xy is even

6. **[12 pts]** Find for the following two ordered sets the maximal, minimal, greatest and least elements (if any):

   (a) $X = \{2, 6, 12, 18, 24, 36\}$ and the order is $x \, R \, y$ iff x divides y.

   (b) $X = \{[0, 1], [-1, 2], [-1, 3], [2, 3], [-1, 5]\}$ and the order is $x \, R \, y$ iff $x \subseteq y$.

   (c) $X = \{2, 3, 4, 8, 9, 16\}$ and the order is $x \, R \, y$ iff $\exists \, n \in \mathbb{N}^*$ such that $y = x^n$.