Discrete Mathematical Structures I
Quiz 1
November 23, 2012

Name: Solutions

1. Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be given by: $f(m,n) = m^2 + n$

   (a) is f 1-1?

   $f(-1, 2) = f(1, 2) \Rightarrow f$ not 1-1

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   (b) is f onto? Yes. since $\forall a \in \mathbb{Z}$. $f(0, a) = a$.

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2. Same for $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be given by: $f(m,n) = 3^m 4^n$.

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1) f is 1-1

since $\forall$

$$f(m,n) = f(m',n')$$

$\Rightarrow$

$$3^m 4^n = 3^{m'} 4^{n'}$$

$$3^{m-m'} = 4^{n'-n} \Rightarrow m - m' = n' - n = 0$$

$\Rightarrow$ $m = m'$

$n = n'$ $\Rightarrow$

f : 1-1 ✓

2) f is not onto since $\nexists\, (m,n)$ s.t. $f(m,n) = 11$

3. Write your own example of a function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ that is 1-1 and onto. discuss

$f(m, n) = (n, m)$

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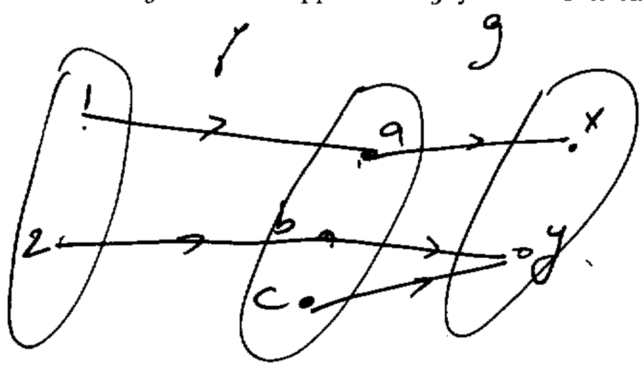
4. Consider the two functions $f : A \to B$ and $g : B \to C$.Suppose that $gof : A \to C$ is onto, then do both g and f need to be onto? Discuss.

No :



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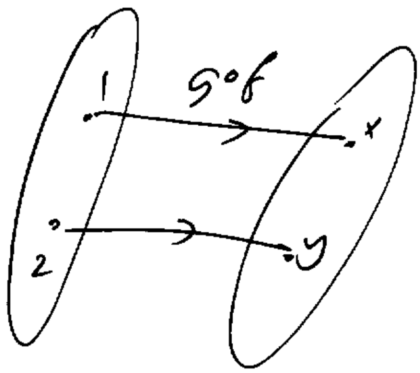
f not onto,
although gof onto



5. Determnine if $f(x) : \mathbb{R} \to \mathbb{Z} = \lceil x/2 \rceil$ is 1-1 or onto.

1) no 1-1.

$$f(1.2) = f(1.3)$$

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2) Yes onto $\forall$ $n \in \mathbb{N}$ $f(2n) = n$
$\Rightarrow$ onto.

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6. Show taht if f is 1-1 then $f(S \cap T) = f(S) \cap f(T)$.

1) $\overset{?}{\subseteq}$ Let $x \in S \cap T$ so that $f(x) \in f(S \cap T) \Rightarrow$

$\Rightarrow$ $f(x) \in f(S)$, since $x \in S$

and $f(x) \in f(T)$ since $x \in T \Rightarrow$

$f(x) \in f(S) \cap f(T)$

2) $\overset{?}{\supseteq}$ Spse $y \in f(S) \cap f(T) -$

$y \in f(S) \Rightarrow$ $y = f(x)$. for some $x \in S$.

and $y \in f(T) \Rightarrow y = f(x')$, $x' \in T$

$\Rightarrow f(x') = f(x) = y$
but since f is 1-1 $\Rightarrow$

$x = x' \in S \cap T \Rightarrow$ $y \in f(S \cap T)$.

7. Prove by **induction** that $n(n^2+5)$ is divisible by 6.

$6 \mid n(n^2+5)$

1) Basic Step: $6 \mid 1.(1+5)$ $n \geq 1$ ✓

2) Assume $6 \mid K(K^2+5)$, show $6 \mid (K+1)((K+1)^2+5)$

$K(K^2+5) = 6c$; $\Rightarrow K^3 + 5K = 6c$ or $\boxed{K^3 = 6c - 5K}$

show $(K+1)(K^2+2K+\underbrace{1+5}_{6}) \stackrel{?}{=} 6d$ for some $d \in \mathbb{Z}$

$K^3 + 2K^2 + 6K + K^2 + 2K + 6 = K^3 + 3K^2 + 8K + 6$

$= (6c - 5K) + 3K^2 + 8K + 6$

$= 6c + 3K^2 + 3K + 6.$

$= 6c + 3\underbrace{K(K+1)}_{\text{even}} + 6$ $= 6c + 3(2e) + 6$

$= 6d$ ✓

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