## LEBANESE AMERICAN UNIVERSITY Division of Computer Science and Mathematics

## Discrete Structures I

## Exam I

Fall 2012 (October 31)

Name: Solutions II

Question Number	<u>Grade</u>
1. 10%	
2. 10%	
3.8%	
4. 7%	
5. 12%	
6. 12%	
7. 13%	
8. 13%	
9. 15%	
Total	

1. (10%) Given that the sets A, B and C are all countable, show that their union is also countable.

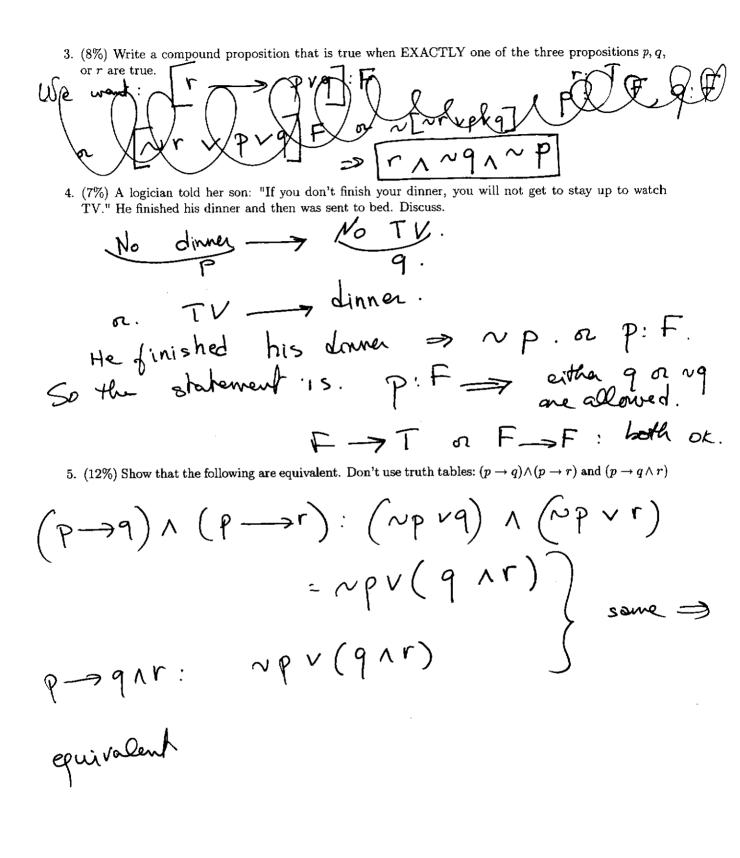
Let  $D = A \cup B \cup C$ .  $A = \{a_1, a_2, \dots\}$   $B = \{b_1, b_2, \dots\}$   $C = \{a_1, c_2, \dots\}$ Let  $D = \{d_1, d_2, \dots\}$ so that  $a_1 = d_1$   $a_2 = d_4$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_4$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_6$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_6$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_6$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_6$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_6$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_6$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_6$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_6$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_2$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_2$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_2$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_2$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_2$   $a_3 = d_7$   $a_1 = d_1$   $a_2 = d_2$   $a_3 = d_3$   $a_1 = d_1$   $a_2 = d_2$   $a_3 = d_3$   $a_1 = d_1$   $a_2 = d_2$   $a_3 = d_3$   $a_1 = d_1$   $a_2 = d_2$   $a_3 = d_3$   $a_4 = d_1$   $a_1 = d_1$   $a_2 = d_2$   $a_3 = d_3$   $a_4 = d_3$   $a_1 = d_2$   $a_1 = d_3$   $a_2 = d_3$   $a_1 = d_3$   $a_2 = d_4$   $a_1 = d_4$   $a_2 = d_4$   $a_1 = d_4$   $a_1 = d_4$   $a_2 = d_4$   $a_1 = d_4$   $a_1 = d_4$   $a_2 = d_4$   $a_1 = d_4$   $a_1 = d_4$   $a_2 = d_4$   $a_3 = d_4$   $a_4 = d_4$   $a_4 = d_4$   $a_4 = d_4$   $a_4 = d_4$   $a_5 = d_5$   $a_5 = d_5$   $a_5 = d_6$   $a_5 = d_6$   $a_5 = d_6$   $a_5 = d_6$ 

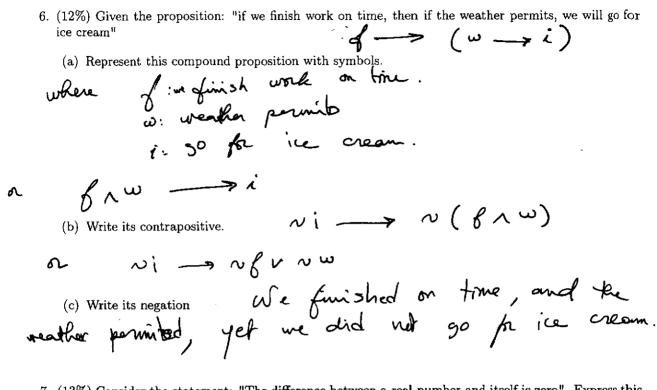
2. (10%) If you know that x is rational and y is irrational, show whether or not  $2x + \frac{3}{2}y$  is rational Specify the proof type you use.

Space not [pmy by contradiction]  $\Rightarrow$ .

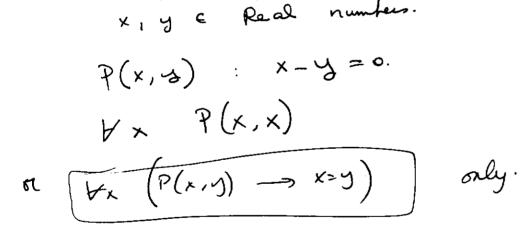
Space not [pmy by contradiction]  $\Rightarrow$ .

Space  $2=\frac{m}{n}$   $y = \frac{2}{3}[2-2x] = \frac{2}{3}[\frac{m}{m}-2x]$ but since x is rational  $\Rightarrow$   $x = \frac{a}{b}$ , for some  $a, b \in \mathbb{Z}$   $\Rightarrow$   $y = \frac{2}{3}[\frac{m}{n} - \frac{2a}{b}]$  also rational a contradiction, since y is irrational





7. (13%) Consider the statement: "The difference between a real number and itself is zero". Express this statement symbolically using predicates, quantifiers, logical connectives etc....



8. (13%) Given the proposition  $\forall x \exists y (F(x) \land P(x)) \rightarrow M(x,y)$ 

Make up your own English statement that agrees with the above. Make sure you specify the domains of x and y.

9. (15%) Show using induction that  $8|5^{n+1}+2*(3^n)+1$  for all n=1,2,3,...

$$5^{(k+1)} + 2 \cdot 3^{2} + 1 = 8 c$$
  $5^{(k+1)} = 8 c - 2 \cdot 3^{(k+1)} = 8 c - 2 \cdot 3^{(k+1$ 

$$= 40 c - 4 * 3 - 4$$

$$= 4 \left( 10 c - 3 - 1 \right) = 4 * even$$

$$= 8d$$

$$= 8d$$