LEBANESE AMERICAN UNIVERSITY
Division of Computer Science and Mathematics

Discrete Structures I

Exam I

Spring 2012 (March 19, 2012)

Name: Solutions **ID:**

Question Number	**Grade**
1. 10%	
2. 9%	
3. 12%	
4. 6%	
5. 6%	
6. 5%	
7. 20%	
8. 11%	
9. 10%	
10. 11%	
Total	

1. Define what is a countable set then prove that the set $\mathbb{Z} \times \mathbb{Z}$ is countable.

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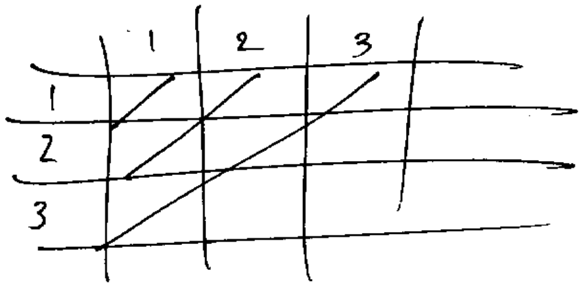
A is said to be countable if there is a 1-1 correspondence between A and $\mathbb{Z}$. (That is, if we are able to "enumerate" A by listing its elements:



We can "count" or enumerate the elements of $\mathbb{Z} \times \mathbb{Z}$ along "DIAGONALS" alternating + and - layers...

2. If you know that $(\sim p \wedge q) \rightarrow r$ is false, what can you say about the truth values of:

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$\sim p \wedge q$: T , ~~r: F~~

$\boxed{p: F;\ q: T;\ r: F}$

(a) $p \rightarrow q$

F → T : T

(b) $(p \wedge \sim q) \rightarrow (p \wedge r)$

F → F : T

(c) $p \wedge r \rightarrow r \vee q$

F → T   T

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3. Show *in two ways* that $(p \rightarrow r) \vee (q \rightarrow r)$ is equivalent to $(p \wedge q) \rightarrow r$

I $p \wedge q \rightarrow r$ is F where p, q : T r : F

$(p \rightarrow r) \vee (q \rightarrow r)$: F when $p \rightarrow r$: F, $q \rightarrow r$: F → p, q : T : r, F

∴ the 2 statements are F in the same situation

∴ They are equivalent.

II Truth Tables: Note the column 6 and 8 are the same ✓

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \vee (q \rightarrow r)$	$p \wedge q$	$p \wedge q \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	F	T
T	F	F	F	T	T	F	T
F	T	T	T	T	T	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

4. Give an example (in **English**) of a tautology and an absurdity.

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Tautology: I am either present or not present (absent)

Absurdity: I am present and not present (absent)

5. Write the negation of the statement: "If all goes well, and if she asks for my help, I will help her."

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All went well and she asked for my help, yet I did not help her

6. Mention all cases where the following statement is false: $(\sim p \vee q) \rightarrow r$

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$\sim p \vee q$: T r : F.

$\sim p \vee q$: T → $\sim p$:T p:F q:T
$\sim p$:T p:F q:F
$\sim p$:F p:T q:T

$(p,q,r) = (F,T,F)$ or $= (F,F,F)$ or $= (T,T,F)$

7. Let $l(a)$ be the statement "a has an internet connection", and $C(a,b)$ the statement "a and b chatted over the internet" where $a, b \in$ students in your class. Use quantifiers to express the following:

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(a) Not everyone in class has internet access.

$\exists a : \sim l(a)$

(b) No one in class has chatted with May

$$\sim \left[\exists a \left[C(a, May) \vee C(May, a)\right]\right]$$

(c) Some one in your class has internet connection but has not chatted with anyone.

$$\exists x \forall y : \left(\ell(x) \wedge \sim C(x,y) \wedge \sim C(y,x) \right)$$

(d) There is a student who has chatted with everyone in class on the internet.

$$\exists x \forall y : \left[C(x,y) \vee C(y,x) \right]$$

8. Prove that the sum of a rational number and an irrational number is irrational. *Specify the type of proof you are using.*

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Let $x \in \mathbb{Q}$, $y \in \mathbb{R} - \mathbb{Q}$.

then (by Contradiction), assume $x + y \in \mathbb{Q}$.

(reach a contradiction) $\Rightarrow$

$$x + y = \frac{m}{n}, \quad m, n \in \mathbb{Z}, \quad n \neq 0$$

$\Rightarrow y = \frac{m}{n} - x$ But since $x \in \mathbb{Q} \Rightarrow x = \frac{a}{b}$

$a, b \in \mathbb{Z}$

$b \neq 0$

$$y = \frac{m}{n} - \frac{a}{b} = \frac{mb - an}{bn} = \frac{c}{d} \in \mathbb{Q}.$$

$\therefore$ y is also rational, a contradiction $\Rightarrow$

our result is proven by contradiction !!

9. Use induction to prove that $n^2 + n$ is even for all $n = 1, 2, 3, \ldots$

$P(n)$: n^2+n : even.

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1) Basic Step $n=1$

$1^2+1=2$: even

2) Ind. Step

Assume $P(K)$ true. Show $P(K+1)$

$P(K)$: K^2+K : even $K^2+K=2c$.

Show $P(K+1)$: $(K+1)^2+(K+1) \stackrel{?}{=} 2d$

$K^2+2K+1+K+1 = K^2+3K+2 = K^2+K+2K+2$

$= 2c+2K+2 = 2d$ ✓

$\therefore$ $P(K+1)$: T

10. Use mathematical induction to show that $1+2+3+..n < \frac{(n+1)^2}{2}$ $n = 1, 2, 3, \ldots$

I) Method 1

1) Basic Step:

$P(1)$: $1 \overset{?}{<} \frac{2^2}{2} = 2$ ✓

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2) Ind. Step:

Assume $P(K)$, show $P(K+1)$

Assume $1+2+\cdots+K < \frac{(K+1)^2}{2}$, show $1+2+\cdots K+(K+1) \overset{?}{<} \frac{(K+2)^2}{2}$.

$$1+2+\cdots+K+(K+1) < \frac{(K+1)^2}{2} + (K+1)$$

Remains to show:

$$\frac{(K+1)^2}{2} + (K+1) \overset{?}{<} \frac{(K+2)^2}{2}$$

$$\overset{\alpha}{=} \quad (K+1)^2 + 2K+2 \overset{?}{<} (K+2)^2,$$

$$K^2+2K+1+2K+2 = K^2+4K+3 \overset{?}{<} K^2+4K+4$$

Yes since $3<4$ ✓

II) Method 2 (not ind) Use: $1+2+\cdots+n = \frac{n(n+1)}{2}$.

Then show $\frac{n(n+1)}{2} < \frac{(n+1)^2}{2}$, yes since $n < n+1$