1. Let $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \times \mathbb{N}$ be defined by $f(m, n) = (2^m, 3^n)$.

   (a) Show that f is 1-1.

   (b) Show that f is not onto.

2. Consider the function: $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = \lfloor x \rfloor$. Check whether or not f is $1-1$.Explain.

3. Let $f : \mathbb{R} - \{1\} \to \mathbb{R} - \{1\}$ where $f(x) = \dfrac{x}{1+x}$

   (a) Justify why $-1 \notin$domain of f.

   (b) Show that f is 1-1 and onto

   (c) Find its inverse

4. Consider the function f_E, where E =the set of even integers, that is, $E = \{2n : n \in \mathbb{Z}\}$.Consider the function $g(x) = 1 - f_E$.

   (a) Find $g(n)$.

   (b) show that $g(n)$ is the characteristic function of a certain set F. identify F.

5. how that if $a\,|b$ or $a\,|c$ then $a\,|bc$. Show that the converse is not true.

6. Let $m = 165, n = 275$.

   (a) Compute $d = \gcd(m, n)$ using two different ways.

   (b) Find $\operatorname{lcm}(m, n)$

   (c) DEDUCE that $\gcd(165, 275) = \gcd(165, 110)$

7. Suppose that $m \equiv n (\bmod 4)$ and $m \equiv n (\bmod 6)$, show $m \equiv n (\bmod 12)$

8. Consider $\sum\limits_{k=1}^{n} k$

   (a) Write the formula for $\sum\limits_{k=1}^{n} k$

   (b) Evaluate $\sum\limits_{k=1}^{20} k$

   (c) Show that $\sum\limits_{k=1}^{n} k = O(n^2)$