LEBANESE AMERICAN UNIVERSITY
Division of Computer Science and Mathematics

Discrete Structures I

Exam II

Spring 2012 (April 23, 2012)

Name: Solutions. **ID:** ____________

Question Number	Grade
1. 9%	
2. 12%	
3. 12%	
4. 10%	
5. 8%	
6. 10%	
7. 12%	
8. 10%	
9. 7%	
10. 10%	
Total	

1. (9%) Let $A = \{a, \{b\}, \{a,b\}, \{\phi\}\}$. Answer the following by True or False:

   (a) $a \in A$ T

   (b) $b \subset A$ F

   (c) $\{a\} \in A$ F

   (d) $\{\{a,b\}\} \subset A$ T

   (e) $\{a,b\} \in A$ T

   (f) $\{a,b,\{\phi\}\} \subset A$ F

   (g) $\{\Phi\} \in A$ T

   (h) $\{\Phi\} \subset A$ F

   (i) $\{\{\Phi\}\} \subset A$ T

2. (12%) Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be defined by $f(m,n) = 3m + 2n$

   (a) Is f 1-1?

No, since

$f(2,3) = f(4,0) = 12$

(b) Is f onto? Yes since If $y \in \mathbb{Z} \Rightarrow$ Case ①: If y: even $\Rightarrow y = 2n \Rightarrow f(0, n) = 2n = m$. Case ② If y: odd. $\Rightarrow y = 2K+1$ or $2K+3 \Rightarrow f(1, K) = y$

$\therefore f$: onto.

3. (12%)Same question if $f : \mathbb{Z} \to \mathbb{Z}$ is given by $f(x) = 10 - x$. Show that f is 1-1 and onto and find its inverse.

1) Using Calculus, since $f \searrow \Rightarrow f$ is 1–1.

or: Assume $f(m) = f(n) = 10 - m = 10 - n \Rightarrow m = n \Rightarrow$ f: 1–1

2) f is onto since let $y \in \mathbb{Z}$ ∍.

$f(?) = y$

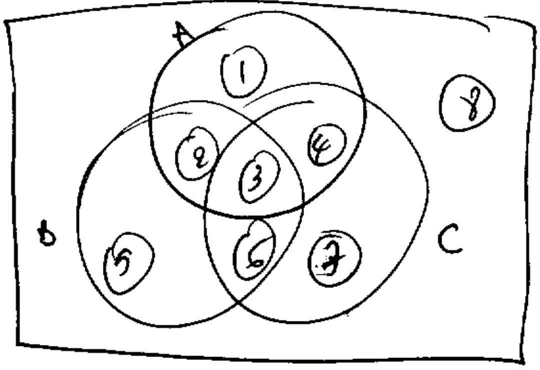
$f(10 - y) = 10 - [10 - y] = y \therefore$

the preimage of y is $10 - y$.

3) Therefore $f^{-1}(t) = 10 - t$ since

$f(f^{-1}(t)) = f(10 - t) = 10 - (10 - t) = t$.

4. (10%) Show that $A - (\overbrace{B \cap C}^{3,6}) = (A - B) \cup (A - C)$

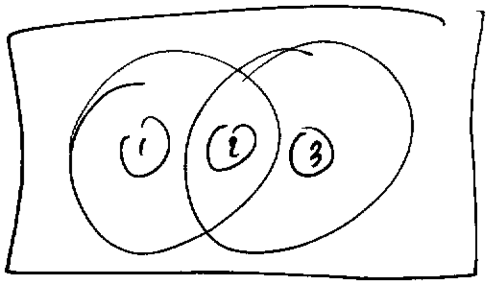


$A - \{3, 6\} = 1, 2, 4.$

$\left.\begin{array}{l} A - B = 1, 4 \\ A - C = 1, 2 \end{array}\right\} \; (A - B) \cup (A - C) = \{1, 2, 4\}$

$\therefore$ the 2 sets are EQUAL !!

5. (8%) Under what conditions is $A \oplus B = A \cup B$? when $A \cap B = \phi$.



since

$1, 2, 3 = 1, 3$ iff

$2 = \phi$

6. (10%) Let $A, B \subset U$, the universal set. Show that $|\overline{A} \cap \overline{B}| = |U| - |A| - |B| + |A \cap B|$

$\overline{A} \cap \overline{B} = \overline{A \cup B} \;\therefore$

$|\overline{A} \cap \overline{B}| = |\overline{A \cup B}| = |U| - |A \cup B|$

Since $|A \cup B| = |A| + |B| - |A \cap B|$

$\therefore \; |\overline{A} \cap \overline{B}| = |U| - |A \cup B| = |U| - \{|A| + |B| - |A \cap B|\}$

$= |U| - |A| - |B| + |A \cap B|$

7. (12%) If $f : A \to B$ be 1-1.Show that $f(S \cap T) = f(S) \cap f(T)$, where S and T are subsets of A.

Double inclusion (I) $f(S \cap T) \overset{?}{\subseteq} f(S) \cap f(T)$.

1) Let $y \in f(S \cap T) \Rightarrow y = f(x)$ for some $x \in S \cap T$

$\therefore x \in S$ and $x \in T \Rightarrow y \in f(S)$ and $y \in f(T)$

$\therefore y \in f(S \cap T)$

(II) $f(S) \cap f(T) \overset{?}{\subseteq} f(S \cap T)$

Let $y \in f(S) \cap f(T) \Rightarrow$ Since f is 1-1 $\Rightarrow$ there can't be 2 preimages for y $\therefore$ $\exists$ only one $x \in$ s.t. $f(x) = y$.

Therefore $x \in S \cap T$ $\therefore$ $y \in f(S \cap T)$.

8. (10%) If $f = O(h)$ and $g = O(h)$ show that $af + bg = O(h)$ also where a and b are positive real numbers

Since $f(x) < C_1 h(x)$ for x large $\Rightarrow$

$a f(x) < a C_1 h(x)$ " "

and $g(x) < C_2 h(x)$ for x large

$\therefore$ $b g(x) < b C_2 h(x)$ " " " $\therefore$

$a f(x) + b g(x) \leq a C_1 h(x) + b C_2 h(x)$ for x large

$\therefore$

$\leq C_3 h(x)$ $\therefore$

$\therefore$ $af + bg = O(h)$

9. (7%) Evaluate in an *efficient way* $\sum_{i=4}^{100}(3i+5)$ $= 3\sum_{i=4}^{100} i + \sum_{i=4}^{100} 5$

$$= 3\left(\sum_{i=1}^{100} i - \sum_{i=1}^{3} i\right) + 5(100-4+1)$$

$$= 3\left[\frac{(100)(101)}{2} - \frac{4(3)}{2}\right] + 5(97)$$

10. (10%) Show that $n = \lfloor n/2 \rfloor + \lceil n/2 \rceil$ for $n \in \mathbb{Z}$

1) Case 1 : If n : even $\Rightarrow$ $n = 2k$.

$\Rightarrow$ $n = \lfloor k \rfloor + \lceil k \rceil = 2k$

2) Case 2 : If n : odd $\Rightarrow$ $n = 2k+1$

$\Rightarrow \left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor k + \frac{1}{2} \right\rfloor = k$

$\left\lceil \frac{n}{2} \right\rceil = \left\lceil k + \frac{1}{2} \right\rceil = k+1$

$\therefore$ $\lfloor n/2 \rfloor + \lceil n/2 \rceil = 2k+1 = n$