1. Let A, B and C be sets. Who that $(A \oplus B) \oplus C = A \oplus (B \oplus C)$, using any method. (Remember $A \oplus B = \{x \ s.t. \ x \in A$ or $x \in B$ but not in both$\}$

2. Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be defined by $f(m, n) = 2^m 3^n$.

   (a) Is f 1-1?

   (b) Is f onto?

3. Same question if $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$

4. Let $\sim$ be a relation on Z given by $m \sim n$ if and only if $m^3 = n^3$ Show that $\sim$ is an equivalence relation and find the equivalence classes

5. Fill in the blanks with the appropriate symbols ($\in$, $\notin$,or $\subset$, $\nsubseteq$) in the following cases:

   (a) $1.....\{1, 2, \{1\}, \{1, 2\}\}$

   (b) $\{2\}....\{1, 2, \{1\}, \{1, 2\}\}$

   (c) $\{\{1\}\}...\{1, 2, \{1\}, \{1, 2\}\}$

   (d) $\{1, \{2\}\}...\{1, 2, \{1\}, \{1, 2\}\}$

6. Write the negation of the following statements

   (a) All students are given a second chance

   (b) If we relax, we will perform

   (c) If the weather permits and if we are lucky then we will meet in the park

7. Sketch a graph whose adjacency matrix is

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 3 & 0 & 0 & 1 \end{pmatrix}$$

   You can label the vertices with any letter you choose, ex $v_1, v_2, ..$

8. Write your own relation on the set $\{1, 2, 3, 4, 5\}$ that is (Reflexive) and (Anti Symmetric) and (Transitive).Draw its digraph.

9. Write an explicit formula;a for $a_n = 3a_{n-1} - 2a_{n-2}$; $n \geq 3$, where $a_1 = 5$, and $a_2 = 3$.

10. If $b_n = 4 + b_{n-1}$, for all $n \geq 1$, if $b_0 = 5$,show that $b_n = 5 + 4n$

11. Consider the matrices: $A = \begin{bmatrix} 4 & 5 & -3 \\ 2 & 1 & 4 \\ 3 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 2 & 2 \\ 1 & 0 & 3 \end{bmatrix}$. Evaluate the following:

    (a) $A + B$

(b) $2A - 3B$

(c) $A.B$

(d) $B.A$

(e) A^t

12. Prove **by induction** that $n(n^2 + 5)$ is divisible by 6.

13. A license plate for a car consists of 3 letters followed by 4 digits.

    (a) How many possible licence plates are there?

    (b) How many are there with no repeated letters and non repeating digits?

14. Partial Order

    Define the relation R on the plane by

    $$(x, y)\, R\, (w, u) \text{ if } x^2 + y^2 < w^2 + u^2 \text{ or } (x, y) = (w, u)$$

    - Show that this relation is a partial order
    - Is it a total order?

15. Given any four integers, explain why two of them must be congruent mod 3.

16. Let A and B be two finite sets with $|A| < |B|$. True or false.

    - There is a function mapping A onto B
    - There is a function mapping B onto A.
    - There is a one to one function from a A to B
    - There is a one to one function from B to A

17. True or false

    - The set of positive rationals is countable
    - The set of all positive real numbers is countable
    - The intersection of two countably infinite sets is countably infinite

18. Show that

    $$\sum_{k=0}^{m} a_k n^k = a_m n^m + O(n^{m-1}), \text{ if } a_m \neq 0$$

19. Fact1: If R is a relation on a set S, then R is transitive if and only if $R^2 \subseteq R$.

    Fact 2: If M_R is the boolean matrix corresponding to R, then the matrix given by the boolean product $M_R \odot M_R$ corresponds to the composition of the relation with itself i.e. $R \circ R$.

For two boolean matrices of the same size $m \times n$, M and N, we say that $M \leq N$ if $m_{ij} \leq n_{ij}$ for all $i = 1, \cdots m$ and $j = 1, \cdots n$.

Fact 3: $R_1 \subseteq R_2$ if and only if $M_{R1} \leq M_{R2}$

Use the 3 facts above to show that the relation R on the set $S = \{1, 2, 3\}$ whose matrix is

$$M_R = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

is transitive (compute a suitable boolean product)