

1. (4 %) Show that $(A \oplus B) \oplus C = A \oplus (B \oplus C)$, using any method

2. (6 %) Let $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $f(m, n) = 2^m 3^n$.

(a) Is f 1-1?

(b) Is f onto?

3. (5 %) Same question if $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R}$

4. (5 %) Let \sim be a relation on Z given by $m \sim n$ if and only if $m^3 = n^3$. Show that \sim is an equivalence relation and find the equivalence classes

5. (6 %) Fill in the blanks with the appropriate symbols (\in , \notin , or \subset , $\not\subset$) in the following cases:

(a) $1 \dots \{1, 2, \{1\}, \{1, 2\}\}$

(b) $\{2\} \dots \{1, 2, \{1\}, \{1, 2\}\}$

(c) $\{\{1\}\} \dots \{1, 2, \{1\}, \{1, 2\}\}$

(d) $\{1, \{2\}\} \dots \{1, 2, \{1\}, \{1, 2\}\}$

6. (9%) If you know that the proposition $(p \wedge \sim (q \wedge \sim s)) \rightarrow (p \wedge r)$ is false, what can you say about the truth value of the propositions

(a) $q \vee r$

(b) $q \rightarrow s$

(c) $r \vee (q \vee s)$

7. (3 %) Write the negation of the following statements

(a) All students are given a second chance

(b) If we relax, we will perform

(c) If the weather permits and if we are lucky then we will meet in the park

8. (10%) Consider the following digraph:

(a) Write the matrix that represents the graph

(b) Find a path of length 3 that goes from y to w .

(c) Find a path of length 3 that goes from v to w .

(d) Find a path of length 4 that goes from v to y .

(e) Find a path of length 3 that goes from z to itself.

9. (6%) Write your own relation on the set $\{1, 2, 3, 4, 5\}$ that is (Reflexive) and (Anti Symmetric) and (Transitive). Draw its digraph.

10. (6%) Write an explicit formula; a for $a_n = 3a_{n-1} - 2a_{n-2}$; $n \geq 3$, where $a_1 = 5$, and $a_2 = 3$.

11. (7%) If $b_n = 4 + b_{n-1}$, for all $n \geq 1$, if $b_0 = 5$, show that $b_n = 5 + 4n$

12. (7%) Consider the matrices: $A = \begin{bmatrix} 4 & 5 & -3 \\ 2 & 1 & 4 \\ 3 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 2 & 2 \\ 1 & 0 & 3 \end{bmatrix}$. Evaluate

the following:

(a) $A + B$

(b) $2A - 3B$

(c) $A \cdot B$

(d) $B \cdot A$

(e) A^t

13. (7%) Prove **by induction** that $n(n^2 + 5)$ is divisible by 6.

14. (6%) A license plate for a car consists of 3 letters followed by 4 digits.

(a) How many possible licence plates are there?

(b) How many are there with no repeated letters and non repeating digits?

(c) What is the probability that the license plate has 3 repeated letters?

15. (6%) A die is rolled twice

(a) What is the probability of getting a least one even number on one of the dice?

(b) What is the probability of getting a double?

- (c) What is the probability of getting a sum of 9.
16. (7%) A study was done on a certain drugs, A, B, and C. It was shown that out of 40 people, 23 reported relief from drug A, 18 reported relief from drug B, 31 reported relief from drug C, 11 reported relief from drug A and B, 19 reported relief from drug A and C, 14 reported relief from drug B and C, and 37 reported relief from at least one of the drugs.
- (a) How many reported relief from none of the drugs?
- (b) How many reported relief from all three drugs?