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Time : 65 minutes
Fall 2016-17

MATHEMATICS 218
QUIZ 2

NAME
ID#

Circle your section number :

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2 W	1 W	11 W	12 F	4 F	3 F	12 M	1 M	11M	4 M	3 M	11 Th	11 F	4 F	5 F

PROBLEM GRADE

PART I

1 16 / 16

2 16 / 16

3 4 / 6

4 6 / 8

PART II

5	6	7	8	9	10
a	a	a	a	a	a
b	b	b	b	b	b
c	c	c	c	c	c
d	d	d	d	d	d
e	e	e	e	e	e

5-10 30 / 30

PART III

11	12	13	14	15	16	17	18
T	T	T	T	T	T	T	T
F	F	F	F	F	F	F	F

11-18 24 / 24

TOTAL 96 / 100

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PART I. Answer each of the following problems in the space provided for each problem (Problem 1 to Problem 4).

1. For each one of the following vector spaces V and subsets W⊂V, prove or disprove whether W is a subspace of V.

(a) $V = M_{2x2}$, $W = \{A \in M_{2x2} \mid AA^t = A\}$;

[8 points]

• $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0 \in W \Rightarrow W \neq \phi$

~~• let $A \in W$ & $B \in W \Rightarrow AA^t = A$ and $BB^t = B$~~

~~$(A+cB)(A+cB)^t = (A+cB)(A^t + cB)^t$~~

~~$= (A+cB)(A^t + cB^t)$~~

~~$= AA^t + cAB^t + cBA^t + cBB^t$~~

~~$= A + cAB^t + cBA^t + cB$~~

~~$=$~~

But W is not a subspace !!

Counterexample:

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \in W$ & $B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \in W$

since $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ since $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

But $A+B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$: $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \Rightarrow (A+B)(A+B)^t \neq (A+B)$

$\Rightarrow$ Not closed under addition $\Rightarrow$ NOT subspace

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(b) $V = P_2$, $W = \{p(x) \in P_2 \mid xp'(x) - 2p(x) = 0\}$

[8 points]

• zero polynomial $0x^2 + 0x + 0 = 0 \in W \Rightarrow W \neq \phi$

• let $p(x) \in W$ & $q(x) \in W \Rightarrow \begin{cases} xp'(x) - 2p(x) = 0 \\ xq'(x) - 2q(x) = 0 \end{cases}$

$x(p+cq)'(x) - 2(p+cq)(x)$

$= x\left[p'(x) + (cq)'(x)\right] - 2\left[p(x) + (cq)(x)\right]$

$= x\left[p'(x) + cq'(x)\right] - 2\left[p(x) + cq(x)\right]$

$= xp'(x) + cxq'(x) - 2p(x) - 2cq(x)$

$= \left[xp'(x) - 2p(x)\right] + c\left[xq'(x) - 2q(x)\right]$

$= [0] + c[0]$

$= 0 \Rightarrow$ closed under addition & scalar multiplication.

So, W is subspace of P_2.

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2. Let A be the matrix

$$A=\begin{pmatrix}1&1&1\\2&3&1\\1&2&0\end{pmatrix}$$

(a) Find a basis for the null space N(A) of A. (no need to prove it is a basis)

[8 points]

Take $X=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$; N(A) is the solution of $AX=0 \Rightarrow$ augmented matrix:

$$\left[\begin{array}{ccc|c}1&1&1&0\\2&3&1&0\\1&2&0&0\end{array}\right] \xrightarrow[-R_1+R_3\to R_3]{-2R_1+R_2\to R_2} \left[\begin{array}{ccc|c}1&1&1&0\\0&1&-1&0\\0&1&-1&0\end{array}\right]$$

$$\xrightarrow{-R_2+R_3\to R_3} \left[\begin{array}{ccc|c}1&1&1&0\\0&1&-1&0\\0&0&0&0\end{array}\right] \Rightarrow$$

$x_3=t$ is a free variable & x_1 & x_2 are leading variables (pivots)

So, $x_2-t=0 \Rightarrow x_2=t$

$x_1+t+t=0 \Rightarrow x_1=-2t$

So, $N(A)=\left\{\begin{pmatrix}-2t\\t\\t\end{pmatrix} \middle/ t\in\mathbb{R}\right\}=\left\{t\begin{pmatrix}-2\\1\\1\end{pmatrix} \middle/ t\in\mathbb{R}\right\} \Rightarrow$ basis of $N(A) = \left\{\begin{pmatrix}-2\\1\\1\end{pmatrix}\right\}$

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(b) Find a basis of the column space Col(A) of A. (no need to prove it is a basis)

[8 points]

we notice from the row echelon form

$$\left[\begin{array}{ccc|c}1&1&1&0\\0&1&-1&0\\0&0&0&0\end{array}\right]$$ ~~that ...~~ !

that the third column has no pivot among its entries ~~⇒ the~~ and pivots are only in the 1^{st} & 2^{nd} columns.

$\Rightarrow$ basis of $Col(A)=\left\{\begin{pmatrix}1\\2\\1\end{pmatrix},\begin{pmatrix}1\\3\\2\end{pmatrix}\right\}$ which are the two first columns of A (initially).

3. Show that Span$\{1, x, x^2\}$= Span$\{x^2-x, x+1, 5\}$ in P_2

[6 points]

- $\text{Span}\{1, x, x^2\} = c_1(1) + c_2(x) + c_3(x^2)$

$= c_1 + c_2 x + c_3 x^2$

$\Rightarrow \{1, x, x^2$ span P_2 since $c_1 + c_2 x + c_3 x^2 = a + bx + cx^2$??

~~for $c_1 = a$, $c_2 = b$, $c_3 = c$ $\Rightarrow$ any val~~

- $\text{Span}\{x^2-x, x+1, 5\} = c_1'(x^2-x) + c_2'(x+1) + c_3'(5)$

$= c_2' + 5c_3' + (-c_1' + c_2')x + c_1' x^2$

$\Rightarrow x^2-x, x+1, 5$ span P_2 since $c_2' + 5c_3' + (-c_1' + c_2')x + c_1' x^2$

So, $\text{Span}\{1, x, x^2\} = \text{Span}\{x^2-x, x+1, 5\} = P_2$ $\quad = a + bx + cx^2$ where $\begin{cases} a = c_2' + 5c_3' \\ b = -c_1' + c_2' \\ c = c_1' \end{cases}$

$c_1' =$
$c_2' =$
$c_3' =$

4. Let **V** be a vector space such that $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}\}$ is a basis of **V**. Let **U**=Span$\{\mathbf{u_1}, \mathbf{u_2}\}$and **W**= Span $\{\mathbf{u_3}, \mathbf{u_4}\}$. Show that U$\cap$W=$\{\mathbf{0}\}$.

[8 points]

- $\{u_1, u_2, u_3, u_4\}$ is basis of V

$\Rightarrow u_1, u_2, u_3, u_4$ are linearly independent.

$\Rightarrow c_1 u_1 + c_2 u_2 + c_3 u_3 + c_4 u_4 = 0$ only for $c_1 = c_2 = c_3 = c_4 = 0$

- $U = \text{Span}\{u_1, u_2\}$

$\Rightarrow k_1 u_1 + k_2 u_2 = u$ where $u \in U$ & $k_1, k_2 \in R$

~~but since $k_1 = k_2 = 0$ (since u_1, u_2 linearly independent) $\Rightarrow u = 0$~~

- $W = \text{Span}\{u_3, u_4\}$

$\Rightarrow l_1 u_3 + l_2 u_4 = u'$ where $u' \in W$ & $l_1, l_2 \in R$

~~but since $l_1 = l_2 = 0$ (since u_3, u_4 linearly independent) $\Rightarrow u' = 0$~~

~~So, $U = \text{Span}\{u_1, u_2\} = 0$ & $W = \text{Span}\{u_3, u_4\} = 0$~~

~~$\Rightarrow U \cap W = \{0\}$~~

$U \cap W$ corresponds to $u + u'$

$= k_1 u_1 + k_2 u_2 + l_1 u_3 + l_2 u_4$

$= 0$

So, $U \cap W = 0$

PART II. Circle the correct answer for each of the following problems (Problem 5 to Problem 10) IN THE TABLE OF THE FRONT PAGE . [5 points for each correct answer].

5. Let U be the subset of P_4 given by

$$U=\{p(x)=ax^4+bx^3+cx^2+dx+e \in P_4 \mid p(0)=0 \, and \, p'''(x)=0\}.$$

Then dim U=

a. 1
b. 2
c. 3
d. 4
e. none of the above.

[5 points]

6. Let S={**u**, **v**, **w**} be a linearly dependent subset of a vector space V. Then

(a) **w** is a linear combination of **u** and v
(b) {**u+w**, **u+v+w**, **v**}is linearly independent in V
(c) **u=0 or v=0 or w=0**
(d) {**2u**, **v**, **3w**} is linearly dependent in V.
(e) None of the above

[5 points]

7. Let $T:\mathbf{R}\to\mathbf{R}$ be a linear transformation such that $T(3)=5$, *then* $T(8)=$

(a) 8/3
(b) 5/3
(c) 40/3
(d) 3
(e) none of the above.

[5 points]

8. Let U and W be the subspaces of $\mathbb{R}^3$ defined by:

$$U=\{\begin{pmatrix} a \\ b \\ c \end{pmatrix} \in \mathbb{R}^3 \mid a=b=c\}, \quad W=\{\begin{pmatrix} 0 \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid y, z \in \mathbb{R}\}, \text{ then dim } U \cap W =$$

a. 1
b. 2
c. 3
d. 0
e. none of the above.

[5 points]

9. Let V be a vector space of dimension n. Which one of the following statements is FALSE:

a. Any set of $(n+1)$ vectors in V is linearly dependent.
b. Any linearly independent set of n vectors in V is a basis of V.
c. If U and W are subspaces of V such that dim U=dim W, then U=W
d. Any set of n vectors spanning V is a basis of V.
e. If U and W are subspaces of V, then $U \cap W$ is linearly dependent

[5 points]

10. Let $S=\{\mathbf{v_1},\mathbf{v_2},\mathbf{v_3}\} \subset R^4$ be a linearly independent set. Which one of the following statements is **FALSE**:

(a) The set $\{\mathbf{v_1},\mathbf{v_3}\}$ is linearly independent.
(b) The subspace $\text{span}(S) \subset R^4$ has dimension 3.
(c) The set $\{2\mathbf{v_1},\mathbf{v_2},\mathbf{v_3}\}$ is linearly independent.
(d) $\text{span}(S)=R^4$.
(e) For any vector $\mathbf{v} \notin \text{span}(S)$, the set $\{\mathbf{v_1},\mathbf{v_2},\mathbf{v_3},\mathbf{v}\}$ is a basis for R^4.

[5 points]

PART III. Answer TRUE or FALSE only IN THE TABLE OF THE FRONT PAGE [3 points for each correct answer, -1 point penalty for each wrong answer]

11. If $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is a linearly independent subset of $\mathbf{R}^3$, then $\{\mathbf{v_1}+\mathbf{v_2}, \mathbf{v_1}+\mathbf{v_2}+\mathbf{v_3}, \mathbf{v_3}\}$ is linearly independent. F

12. The set of all 2×2 non-invertible matrices is a subspace of $M_{2\times2}$. F

13. Let $V=\left\{\begin{pmatrix} x \\ y \\ |x| \end{pmatrix} \in R^3 \middle| \ x, y \in R\right\}$. Then V is a subspace of R^3. F

14. The set of polynomials $\{1+x, x^2+3x+3, x^2\}$ is a spanning set for P_2. F

15. If **V** is a vector space having only finitely many elements, Then $\mathbf{V}=\{\mathbf{0}\}$. T

16. Let $W=\left\{\begin{pmatrix} a & b \\ b & a+2b \end{pmatrix} \in M_{2\times2} \middle| \ a, b \in R\right\}$, then W is a subspace of $M_{2\times2}$ T

17. Let A be an m×n matrix. Then the set $W=\{x \in R^n | Ax = 0\}$ is a subspace of R^n. T

18. If $T:V \to W$ is a linear transformation and if $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is a linearly dependent subset of V, then $\{T(\mathbf{u_1}), T(\mathbf{u_2}), T(\mathbf{u_3})\}$ is linearly dependent in W. T

[24 points]

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